

**PHY 711 Classical Mechanics and
Mathematical Methods
10-10:50 AM MWF Olin 103**

| Course schedule | | | | |
|--|-------------|--|-------------------|----------------|
| (Preliminary schedule – subject to frequent adjustment.) | | | | |
| Date | F&W | Reading | Topic | Assignment |
| 1 Wed, 8/29/2012 | Chap. 1 | Review of basic principles | Scattering theory | #1 |
| 2 Fri, 8/31/2012 | Chap. 1 | Scattering theory continued | | #2 |
| 3 Mon, 9/03/2012 | Chap. 1 | Scattering theory continued | | #3 |
| 4 Wed, 9/05/2012 | Chap. 1 & 2 | Scattering theory/Accelerated coordinate frame | | #4 |
| 5 Fri, 9/07/2012 | Chap. 2 | Accelerated coordinate frame | | #5 |
| 6 Mon, 9/10/2012 | Chap. 3 | Calculus of Variation | | #6 |
| 7 Wed, 9/12/2012 | Chap. 3 | Calculus of Variation continued | | #7 |
| 8 Fri, 9/14/2012 | Chap. 3 | Lagrangian | | #8 |
| 9 Mon, 9/17/2012 | Chap. 3 & 6 | Lagrangian | | #9 |
| 10 Wed, 9/19/2012 | Chap. 3 & 6 | Lagrangian | | #10 |
| 11 Fri, 9/21/2012 | Chap. 3 & 6 | Lagrangian | | #11 |
| 12 Mon, 9/24/2012 | Chap. 3 & 6 | Lagrangian and Hamiltonian | | #11 |
| 13 Wed, 9/26/2012 | Chap. 6 | Lagrangian and Hamiltonian | | #12 |
| 14 Fri, 9/28/2012 | Chap. 6 | Lagrangian and Hamiltonian | | #13 |
| 15 Mon, 10/01/2012 | Chap. 4 | Small oscillations | | #14 |
| 16 Wed, 10/03/2012 | Chap. 4 | Small oscillations | | #15 |
| 17 Fri, 10/05/2012 | Chap. 4 | Small oscillations | | |
| 18 Mon, 10/08/2012 | Chap. 7 | Wave equation | | Take Home Exam |
| 19 Wed, 10/10/2012 | Chap. 7 | Wave equation | | Take Home Exam |
| 20 Fri, 10/12/2012 | Chap. 7 | Wave equation | | Take Home Exam |
| 21 Mon, 10/15/2012 | Chap. 7 | Wave equation | | Exam due |
| 22 Wed, 10/17/2012 | Chap. 7, 5 | Moment of inertia | | |
| 23 Fri, 10/19/2012 | | Fall break | | |
| 23 Mon, 10/22/2012 | Chap. 5 | Rigid body rotation | | #16 |
| 24 Wed, 10/24/2012 | Chap. 5 | Rigid body rotation | | #17 |

WAKE FOREST
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News

Workshop for Middle School Teachers Organized by Prof. Che is Featured in Mashable, Huffington Post, and Fox 8 News

Article in WS Journal on Tech Expo Features Beet-Root Juice

Article by Lacra Neureanu of the Salisbury Group Selected for Inaugural Contribution to Proteopedia from JRSO

Events

Wed Oct 24, 2012
Prof. Pasha Logunov
Georgia Tech
Black Holes
4:00 PM in Clin 101
Refreshments at 3:30 in Lobby

Wed Oct 31, 2012
Dr. Paul Klem
Orion
Lithium Ion Batteries
4:00 PM in Clin 101
Refreshments at 3:30 in Lobby

Oct 28-30, 2012
Sumanth Ramadas
Nanocarbon Technology Conference

Wake Forest Physics
Nationally recognized for teaching excellence;
internationally recognized for research.

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FOREST Department of Physics

WFU Physics Colloquium

TITLE: Black holes and gravitational waves: The quest to show if Einstein was right

SPEAKER: Professor Pablo Laguna,

Center for Relativistic Astrophysics
School of Physics
Georgia Institute of Technology

TIME: Wednesday October 24, 2012

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

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Comment about Exam problem #1:

Consider a Rutherford scattering experiment analyzed in the center of mass frame of reference in which the ratio of the (repulsive) potential to the center-of-mass energy is described by a modified interaction of the form:

$$\frac{V(r)}{E} = \begin{cases} \frac{\kappa}{r} & \text{for } r < r_{\max} \\ 0 & \text{for } r > r_{\max} \end{cases}$$

(a) Using the approach described in Problem 1.15 of your textbook, find an expression for $\theta(b)$, the scattering angle θ as a function of the impact parameter b . (Simplify the expression, but you need not explicitly solve for b .)

(b) Show that in the limit $r_{\max} \rightarrow \infty$, your result is consistent with the Rutherford scattering result.

(c) For the modified interaction, assume that

$$\frac{\kappa}{r_{\max}} \gg \frac{b}{r_{\max}},$$

so that terms including (b/r_{\max}) may be neglected. Find the form of the scattering cross section for this case and compare it with the pure Rutherford scattering cross section.

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Motivation for model:

Au target particles consist of Au nuclei which are screened by electrons.

http://tap.iop.org/atoms/rutherford/img_full_47190.gif

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Calculation of the scattering angle as a function of impact parameter b :

$$\begin{aligned}\theta(b) &= \pi - 2b \int_{r_{\min}}^{r_{\max}} dr \frac{1}{r\sqrt{r^2 - \kappa r - b^2}} - 2b \int_{r_{\max}}^{\infty} dr \frac{1}{r\sqrt{r^2 - b^2}} \\ &= \pi - 2 \tan^{-1} \left(\frac{-2b^2 - \kappa r}{2b\sqrt{r^2 - \kappa r - b^2}} \right) \Big|_{r_{\min}}^{r_{\max}} - 2 \tan^{-1} \left(\frac{-2b^2}{2b\sqrt{r^2 - b^2}} \right) \Big|_{r_{\max}}^{\infty} \\ &= 2 \tan^{-1} \left(\frac{2b^2 + \kappa r_{\max}}{2b\sqrt{r_{\max}^2 - \kappa r_{\max} - b^2}} \right) + 2 \tan^{-1} \left(\frac{2b^2}{2b\sqrt{r_{\max}^2 - b^2}} \right) \\ &= 2 \tan^{-1} \left(\frac{(b/r_{\max}) + (\kappa/2b)}{\sqrt{1 - (\kappa/r_{\max}) - (b/r_{\max})^2}} \right) + 2 \tan^{-1} \left(\frac{(b/r_{\max})}{\sqrt{1 - (b/r_{\max})^2}} \right)\end{aligned}$$

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$$\begin{aligned}\tan \left(\frac{\theta}{2} - \alpha \right) &= \frac{(b/r_{\max}) + (\kappa/2b)}{\sqrt{1 - (\kappa/r_{\max}) - (b/r_{\max})^2}} \\ \tan^2 \left(\frac{\theta}{2} - \alpha \right) &= \frac{(b/r_{\max})^2 + (\kappa/r_{\max}) + (\kappa/2b)^2}{1 - (\kappa/r_{\max}) - (b/r_{\max})^2} \\ \text{where } \tan \alpha &= \frac{(b/r_{\max})}{\sqrt{1 - (b/r_{\max})^2}}\end{aligned}$$

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Back to rotational motion

Time rate of change of angular momentum

$$\frac{d\mathbf{L}}{dt} = \left(\frac{d\mathbf{L}}{dt} \right)_{\text{body}} + \boldsymbol{\omega} \times \mathbf{L}$$

For (body fixed) coordinate system that diagonalizes moment of inertia tensor:

$$\begin{aligned}\tilde{\mathbf{I}} \cdot \hat{\mathbf{e}}_i &= I_i \hat{\mathbf{e}}_i \quad \boldsymbol{\omega} = \tilde{\omega}_1 \hat{\mathbf{e}}_1 + \tilde{\omega}_2 \hat{\mathbf{e}}_2 + \tilde{\omega}_3 \hat{\mathbf{e}}_3 \\ \mathbf{L} &= I_1 \tilde{\omega}_1 \hat{\mathbf{e}}_1 + I_2 \tilde{\omega}_2 \hat{\mathbf{e}}_2 + I_3 \tilde{\omega}_3 \hat{\mathbf{e}}_3 \\ \frac{d\mathbf{L}}{dt} &= I_1 \dot{\tilde{\omega}}_1 \hat{\mathbf{e}}_1 + I_2 \dot{\tilde{\omega}}_2 \hat{\mathbf{e}}_2 + I_3 \dot{\tilde{\omega}}_3 \hat{\mathbf{e}}_3 + \tilde{\omega}_2 \tilde{\omega}_3 (I_3 - I_2) \hat{\mathbf{e}}_1 \\ &\quad + \tilde{\omega}_3 \tilde{\omega}_1 (I_1 - I_3) \hat{\mathbf{e}}_2 + \tilde{\omega}_1 \tilde{\omega}_2 (I_2 - I_1) \hat{\mathbf{e}}_3\end{aligned}$$

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Descriptions of rotation about a given origin -- continued

Note that the torque equation

$$\frac{d\mathbf{L}}{dt} = \left(\frac{d\mathbf{L}}{dt} \right)_{body} + \boldsymbol{\omega} \times \mathbf{L} = \boldsymbol{\tau}$$

is very difficult to solve directly in the body fixed frame.

For $\boldsymbol{\tau} = 0$ we can solve the Euler equations :

$$\begin{aligned} \frac{d\mathbf{L}}{dt} = I_1 \dot{\tilde{\omega}}_1 \hat{\mathbf{e}}_1 + I_2 \dot{\tilde{\omega}}_2 \hat{\mathbf{e}}_2 + I_3 \dot{\tilde{\omega}}_3 \hat{\mathbf{e}}_3 + \tilde{\omega}_2 \tilde{\omega}_3 (I_3 - I_2) \hat{\mathbf{e}}_1 \\ + \tilde{\omega}_3 \tilde{\omega}_1 (I_1 - I_3) \hat{\mathbf{e}}_2 + \tilde{\omega}_1 \tilde{\omega}_2 (I_2 - I_1) \hat{\mathbf{e}}_3 = 0 \end{aligned}$$

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Euler equations for rotation in body fixed frame :

$$I_1 \dot{\tilde{\omega}}_1 + \tilde{\omega}_2 \tilde{\omega}_3 (I_3 - I_2) = 0$$

$$I_2 \dot{\tilde{\omega}}_2 + \tilde{\omega}_3 \tilde{\omega}_1 (I_1 - I_3) = 0$$

$$I_3 \dot{\tilde{\omega}}_3 + \tilde{\omega}_1 \tilde{\omega}_2 (I_2 - I_1) = 0$$

Solution for asymmetric top -- $I_3 \neq I_2 \neq I_1$:

$$\text{Suppose : } \dot{\tilde{\omega}}_3 \approx 0 \quad \text{Define : } \Omega_1 \equiv \tilde{\omega}_3 \frac{I_3 - I_2}{I_1}$$

$$\text{Define : } \Omega_2 \equiv \tilde{\omega}_3 \frac{I_3 - I_1}{I_2}$$

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$$I_1 \dot{\tilde{\omega}}_1 + \tilde{\omega}_2 \tilde{\omega}_3 (I_3 - I_2) = 0$$

$$I_2 \dot{\tilde{\omega}}_2 + \tilde{\omega}_3 \tilde{\omega}_1 (I_1 - I_3) = 0$$

$$I_3 \dot{\tilde{\omega}}_3 + \tilde{\omega}_1 \tilde{\omega}_2 (I_2 - I_1) = 0$$

$$\dot{\tilde{\omega}}_3 \approx 0 \quad \text{Define : } \Omega_1 \equiv \tilde{\omega}_3 \frac{I_3 - I_2}{I_1} \quad \Omega_2 \equiv \tilde{\omega}_3 \frac{I_3 - I_1}{I_2}$$

$$\dot{\tilde{\omega}}_1 = -\Omega_1 \tilde{\omega}_2 \quad \dot{\tilde{\omega}}_2 = \Omega_2 \tilde{\omega}_1$$

If Ω_1 and Ω_2 are both positive or both negative :

$$\tilde{\omega}_1(t) \approx A \cos(\sqrt{\Omega_1 \Omega_2} t + \varphi)$$

$$\tilde{\omega}_2(t) \approx A \sqrt{\frac{\Omega_2}{\Omega_1}} \sin(\sqrt{\Omega_1 \Omega_2} t + \varphi)$$

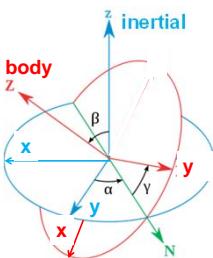
\Rightarrow If Ω_1 and Ω_2 have opposite signs, solution is unstable.

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Transformation between body-fixed and inertial coordinate systems – Euler angles



http://en.wikipedia.org/wiki/Euler_angles

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The diagram illustrates a rigid body in three-dimensional space, defined by a fixed coordinate system $\hat{\mathbf{e}}_3^0$ (blue) and a body-fixed coordinate system $\hat{\mathbf{e}}_3'$ (red). The body rotates through three successive rotations about the $\hat{\mathbf{e}}_3^0$ axis (vertical), the $\hat{\mathbf{e}}_3'$ axis (middle), and the $\hat{\mathbf{e}}_2'$ axis (bottom). The Euler angles are labeled as α , β , and γ . A green vector \mathbf{N} is shown at the origin.

$\tilde{\boldsymbol{\omega}} = \dot{\alpha} \hat{\mathbf{e}}_3^0 + \dot{\beta} \hat{\mathbf{e}}_3' + \dot{\gamma} \hat{\mathbf{e}}_2'$

Need to express all components in body-fixed frame:

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$\tilde{\omega} = \dot{\alpha} \hat{e}_3^0 + \dot{\beta} \hat{e}_2 + \dot{\gamma} \hat{e}_3$

$\hat{e}_2 = \sin \gamma \hat{e}_1 + \cos \gamma \hat{e}_2$

Matrix representation :

$$\hat{e}_2 = \begin{pmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \sin \gamma \\ \cos \gamma \\ 0 \end{pmatrix}$$

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$$\tilde{\boldsymbol{\omega}} = \dot{\alpha} \hat{\mathbf{e}}_3^0 + \dot{\beta} \hat{\mathbf{e}}_2^+ + \dot{\gamma} \hat{\mathbf{e}}_3^-$$

$$\hat{\mathbf{e}}_3^0 = -\sin \beta \hat{\mathbf{e}}'_1 + \cos \beta \hat{\mathbf{e}}'_3$$

Matrix representation :

$$\begin{aligned}\hat{\mathbf{e}}_3^0 &= \begin{pmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} -\sin \beta \cos \gamma \\ \sin \beta \sin \gamma \\ \cos \beta \end{pmatrix}\end{aligned}$$

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$$\tilde{\boldsymbol{\omega}} = \dot{\alpha} \hat{\mathbf{e}}_3^0 + \dot{\beta} \hat{\mathbf{e}}_2' + \dot{\gamma} \hat{\mathbf{e}}_3$$

$$\tilde{\mathbf{o}} = \dot{\alpha} \begin{pmatrix} -\sin \beta \cos \gamma \\ \sin \beta \sin \gamma \\ \cos \beta \end{pmatrix} + \dot{\beta} \begin{pmatrix} \sin \gamma \\ \cos \gamma \\ 0 \end{pmatrix} + \dot{\gamma} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\tilde{\boldsymbol{\omega}} = \tilde{\omega}_1 \hat{\mathbf{e}}_1 + \tilde{\omega}_2 \hat{\mathbf{e}}_2 + \tilde{\omega}_3 \hat{\mathbf{e}}_3$$

$$\tilde{\mathbf{v}} = \dot{\alpha} \begin{pmatrix} -\sin \beta \cos \gamma \\ \sin \beta \sin \gamma \\ \cos \beta \end{pmatrix} + \dot{\beta} \begin{pmatrix} \sin \gamma \\ \cos \gamma \\ 0 \end{pmatrix} + \dot{\gamma} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\tilde{\omega}_1 = \dot{\alpha}(-\sin \beta \cos \gamma) + \dot{\beta} \sin \gamma$$

$$\tilde{\omega}_2 = \dot{\alpha}(\sin \beta \sin \gamma) + \dot{\beta} \cos \gamma$$

$$\tilde{\omega}_3 = \dot{\alpha} \cos \beta + \dot{\gamma}$$

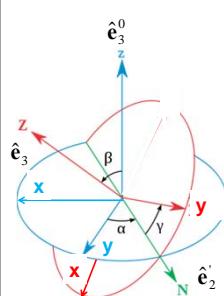
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$$\tilde{\boldsymbol{\omega}} = \dot{\alpha} \hat{\mathbf{e}}_3^0 + \dot{\beta} \hat{\mathbf{e}}_2^+ + \dot{\gamma} \hat{\mathbf{e}}_3^-$$

$$\tilde{\omega} = \left[\dot{\alpha}(-\sin \beta \cos \gamma) + \dot{\beta} \sin \gamma \right] \hat{e}_1 \\ + \left[\dot{\alpha}(\sin \beta \sin \gamma) + \dot{\beta} \cos \gamma \right] \hat{e}_2 \\ + \left[\dot{\alpha} \cos \beta + \dot{\gamma} \right] \hat{e}_3$$



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Rotational kinetic energy

$$T(\alpha, \beta, \gamma, \dot{\alpha}, \dot{\beta}, \dot{\gamma}) = \frac{1}{2} I_1 \tilde{\omega}_1^2 + \frac{1}{2} I_2 \tilde{\omega}_2^2 + \frac{1}{2} I_3 \tilde{\omega}_3^2$$

$$= \frac{1}{2} I_1 [\dot{\alpha}(-\sin \beta \cos \gamma) + \dot{\beta} \sin \gamma]^2$$

$$+ \frac{1}{2} I_2 [\dot{\alpha}(\sin \beta \sin \gamma) + \dot{\beta} \cos \gamma]^2$$

$$+ \frac{1}{2} I_3 [\dot{\alpha} \cos \beta + \dot{\gamma}]^2$$

If $I_1 = I_2$:

$$T(\alpha, \beta, \gamma, \dot{\alpha}, \dot{\beta}, \dot{\gamma}) = \frac{1}{2} I_1 (\dot{\alpha}^2 \sin^2 \beta + \dot{\beta}^2) + \frac{1}{2} I_3 [\dot{\alpha} \cos \beta + \dot{\gamma}]^2$$

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