PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 2:

1. Comments on Maple software
2. Chapter 1 – scattering theory
   a) Rutherford scattering
   b) Scattering for arbitrary potential

PHY 711 Classical Mechanics and Mathematical Methods

MWF 10 AM-10:50 PM	OPL 103	http://www.wfu.edu/~natalie/f12phy711/

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Course schedule

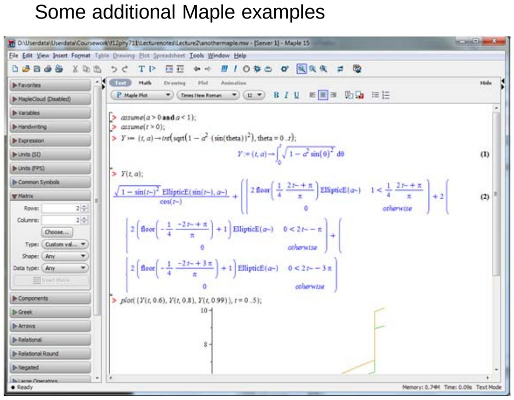
(Preliminary schedule -- subject to frequent adjustment.)

Date	F&W Reading	Topic	Assi gnment
Wed, 8/29/2012	Chap. 1	Review of basic principles;Scattering theory	#1
Fri, 8/31/2012	Chap. 1	Scattering theory continued	#2
Fri, 9/03/2012	Chap. 1	Scattering theory continued	#3

PHY 711 - Assignment #1

e differential cross section for Rutherford scattering, it is necessary to evaluate the following rela

Some additional Maple examples



Scattering theory:

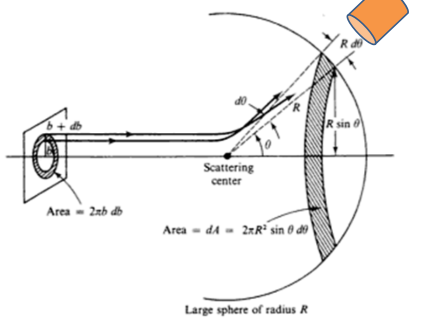


Figure 5.5 The scattering problem and relation of cross section to impact parameter.

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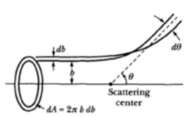
Differential cross section

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\text{Number of detected particles at } \theta \text{ per target particle}}{\text{Number of incident particles per unit area}}$$

$= $ Area of incident beam that is scattered into detector at angle θ



$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{2\pi\, b\, db}{2\pi \sin\theta\, d\theta} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Figure from Marion & Thorton, Classical Dynamics

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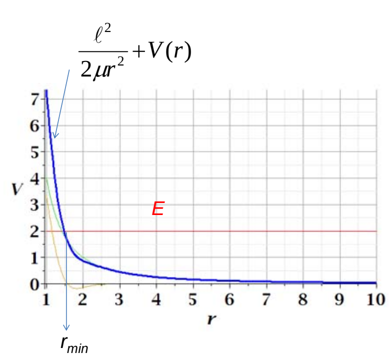

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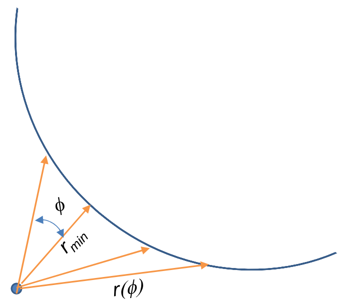

Conservation of angular momentum:

$$\ell = \mu r^2 \left(\frac{d\phi}{dt}\right)$$

Transformation of trajectory variables:

$$r(t) \Leftrightarrow r(\phi)$$

$$\frac{dr}{dt} = \frac{dr}{d\phi}\frac{d\phi}{dt} = \frac{dr}{d\phi}\frac{\ell}{\mu r^2}$$

Conservation of energy in the center of mass frame:

$$E = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

$$= \frac{1}{2}\mu\left(\frac{dr}{d\phi}\frac{\ell}{\mu r^2}\right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

$$\Rightarrow E = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

$$= \frac{1}{2}\mu\left(\frac{dr}{d\phi}\frac{\ell}{\mu r^2}\right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

Solving for $r(\phi) \Leftrightarrow \phi(r)$

$$\left(\frac{dr}{d\phi}\right)^2 = \left(\frac{2\mu r^4}{\ell^2}\right)\left(E - \frac{\ell^2}{2\mu r^2} - V(r)\right)$$

$$d\phi = dr\left(\frac{\ell / r^2}{\sqrt{2\mu\left(E - \frac{\ell^2}{2\mu r^2} - V(r)\right)}}\right)$$

$$d\phi = dr\left(\frac{\ell / r^2}{\sqrt{2\mu\left(E - \frac{\ell^2}{2\mu r^2} - V(r)\right)}}\right)$$

Further simplification at large separation:

$$\ell = \mu v_\infty b$$

$$E = \frac{1}{2}\mu v_\infty^2$$

$$\Rightarrow \ell = \sqrt{2\mu E}\, b$$

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When the dust clears:

$$d\phi = dr\left(\frac{\ell / r^2}{\sqrt{2\mu\left(E - \frac{\ell^2}{2\mu r^2} - V(r)\right)}}\right)$$

$$d\phi = dr\left(\frac{b / r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}}\right)$$

$$\Rightarrow \phi(b, E)$$

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$$\int_0^{\phi_{\max}} d\phi = \int_{r_{\min}}^{\infty} dr\left(\frac{b / r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}}\right)$$

where:

$$1 - \frac{b^2}{r_{\min}^2} - \frac{V(r_{\min})}{E} = 0$$

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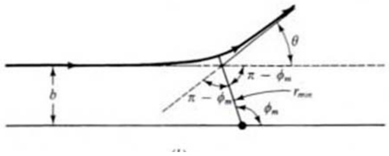
Relationship between ϕ_{max} and θ:



$$2(\pi-\phi_{\max})+\theta=\pi$$

$$\Rightarrow \phi_{\max}=\frac{\pi}{2}-\frac{\theta}{2}$$

$$\phi_{\max}=\frac{\pi}{2}-\frac{\theta}{2}=\int_{r_{\min}}^{\infty}dr\left(\frac{b/r^2}{\sqrt{1-\frac{b^2}{r^2}-\frac{V(r)}{E}}}\right)$$

$$\theta=\pi-2b\int_{r_{\min}}^{\infty}dr\left(\frac{1/r^2}{\sqrt{1-\frac{b^2}{r^2}-\frac{V(r)}{E}}}\right)$$

$$\theta=\pi-2b\int_{0}^{1/r_{\min}}du\left(\frac{1}{\sqrt{1-b^2u^2-\frac{V(1/u)}{E}}}\right)$$

Scattering angle equation :

$$\theta=\pi-2b\int_{0}^{1/r_{\min}}du\left(\frac{1}{\sqrt{1-b^2u^2-\frac{V(1/u)}{E}}}\right)$$

Rutherford scattering example :

$$\frac{V(r)}{E}\equiv\frac{\kappa}{r}\qquad \frac{1}{r_{\min}}=\frac{1}{b}\left(-\frac{\kappa}{2b}+\sqrt{\left(\frac{\kappa}{2b}\right)^2+1}\right)$$

$$\theta=\pi-2b\int_{0}^{1/r_{\min}}du\left(\frac{1}{\sqrt{1-b^2u^2-\kappa u}}\right)=2\sin^{-1}\left(\frac{1}{\sqrt{(2b/\kappa)^2+1}}\right)$$

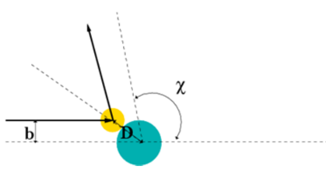
Rutherford scattering continued :

$$\theta = 2\sin^{-1}\left(\frac{1}{\sqrt{(2b/\kappa)^2+1}}\right)$$

$$\frac{2b}{\kappa} = \left|\frac{\cos(\theta/2)}{\sin(\theta/2)}\right|$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{\kappa^2}{16}\frac{1}{\sin^4(\theta/2)}$$

Hard sphere scattering



For your homework you will show that

$$b = D\cos\left(\frac{\chi}{2}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\chi}\left|\frac{db}{d\chi}\right| = \frac{D^2}{4}$$