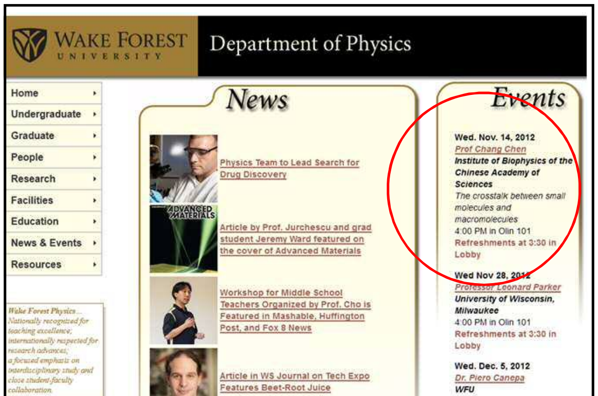
PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

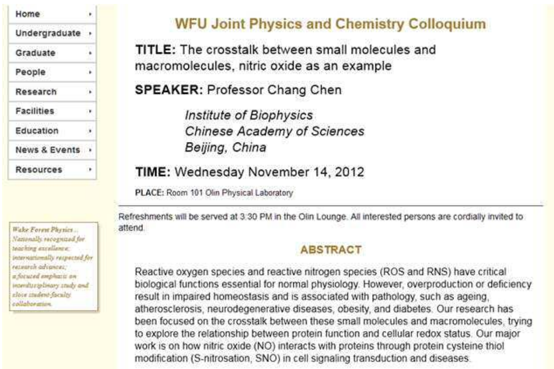
Plan for Lecture 33:

Wave equation for sound

1. **Example of linear sound**
2. **Non-linear effects in sound**

22	Wed, 10/17/2012	Chap. 7, 5	Moment of inertia	
	Fri, 10/19/2012		Fall break	
23	Mon, 10/22/2012	Chap. 5	Rigid body rotation	#16
24	Wed, 10/24/2012	Chap. 5	Rigid body rotation	#17
25	Fri, 10/26/2012	Chap. 5	Rigid body rotation	#18
26	Mon, 10/29/2012	Chap. 8	Waves in elastic membranes	#19
27	Wed, 10/31/2012	Chap. 9	Introduction to hydrodynamics	
28	Fri, 11/01/2012	Chap. 9	Introduction to hydrodynamics	
29	Mon, 11/05/2012	Chap. 9	Introduction to hydrodynamics	#20
30	Wed, 11/07/2012	Chap. 9	Sound waves	
31	Fri, 11/09/2012	Chap. 9	Linear sound waves	#21
32	Mon, 11/12/2012	Chap. 9	Green's function for linear sound	
33	Wed, 11/14/2012	Chap. 9	Non-linear sound	
34	Fri, 11/16/2012	Chap. 9	Non-linear sound	Take Home Exam
35	Mon, 11/19/2012	Chap. 10	Surface waves	Take Home Exam
	Wed, 11/21/2012		*Thanksgiving Holiday*	
	Fri, 11/23/2012		*Thanksgiving Holiday*	
36	Mon, 11/26/2012	Chap. 10	Surface waves	Exam due
37	Wed, 11/28/2012	Chap. 10	Surface waves	
38	Fri, 11/30/2012	Chap. 10	Surface waves	
39	Mon, 12/03/2012		Student presentations I	
40	Fri, 12/05/2012		Student presentations II	





WFU Joint Physics and Chemistry Colloquium

TITLE: The crosstalk between small molecules and macromolecules, nitric oxide as an example

SPEAKER: Professor Chang Chen

Institute of Biophysics
Chinese Academy of Sciences
Beijing, China

TIME: Wednesday November 14, 2012

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

Reactive oxygen species and reactive nitrogen species (ROS and RNS) have critical biological functions essential for normal physiology. However, overproduction or deficiency result in impaired homeostasis and is associated with pathology, such as ageing, atherosclerosis, neurodegenerative diseases, obesity, and diabetes. Our research has been focused on the crosstalk between these small molecules and macromolecules, trying to explore the relationship between protein function and cellular redox status. Our major work is on how nitric oxide (NO) interacts with proteins through protein cysteine thiol modification (S-nitrosation, SNO) in cell signaling transduction and diseases.

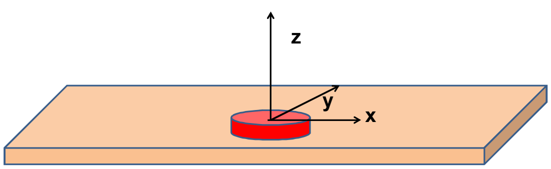
11/14/2012 PHY 711 Fall 2012 -- Lecture 33 4

Wave equation with source:

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -f(\mathbf{r},t)$$

Example :

$f(\mathbf{r},t) \Rightarrow$ time harmonic piston of radius a, amplitude $\varepsilon\hat{\mathbf{z}}$
can be represented as boundary value of $\Phi(\mathbf{r},t)$



11/14/2012 PHY 711 Fall 2012 -- Lecture 33 5

Treatment of boundary values for time-harmonic force:

$$\tilde{\Phi}(\mathbf{r},\omega) = \int_V \tilde{G}(|\mathbf{r}-\mathbf{r}'|,\omega)\tilde{f}(\mathbf{r}',\omega)d^3r' +$$
$$\oint_S \left(\tilde{\Phi}(\mathbf{r}',\omega)\nabla'\tilde{G}(|\mathbf{r}-\mathbf{r}'|,\omega) - \tilde{G}(|\mathbf{r}-\mathbf{r}'|,\omega)\nabla'\tilde{\Phi}(\mathbf{r}',\omega)\right)\cdot\hat{\mathbf{n}}d^2r'$$

Boundary values for our example :

$$\left(\frac{\partial\tilde{\Phi}}{\partial z}\right)_{z=0} = \begin{cases} 0 & \text{for } x^2+y^2 > a^2 \\ i\omega\varepsilon a & \text{for } x^2+y^2 < a^2 \end{cases}$$

Note : Need Green's function with vanishing gradient at $z=0$:

$$\tilde{G}(|\mathbf{r}-\mathbf{r}'|,\omega) = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} + \frac{e^{ik|\mathbf{r}-\bar{\mathbf{r}}'|}}{4\pi|\mathbf{r}-\bar{\mathbf{r}}'|} \quad \text{where } \bar{z}' = -z'; \quad z>0$$

11/14/2012 PHY 711 Fall 2012 -- Lecture 33 6

$$\tilde{\Phi}(\mathbf{r},\omega) = -\oint_{S:\,z'=0} \tilde{G}(|\mathbf{r}-\mathbf{r}'|,\omega)\frac{\partial\tilde{\Phi}(\mathbf{r}',\omega)}{\partial z}dx'\,dy'$$

$$\tilde{G}(|\mathbf{r}-\mathbf{r}'|,\omega) = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} + \frac{e^{ik|\mathbf{r}-\bar{\mathbf{r}}'|}}{4\pi|\mathbf{r}-\bar{\mathbf{r}}'|} \qquad \text{where } \bar{z}' = -z'; \quad z > 0$$

$$\tilde{G}(|\mathbf{r}-\mathbf{r}'|,\omega)_{z'=0} = \left.\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{2\pi|\mathbf{r}-\mathbf{r}'|}\right|_{z'=0}; \quad z > 0$$

$$\tilde{\Phi}(\mathbf{r},\omega) = -\oint_{S:\,z'=0} \tilde{G}(|\mathbf{r}-\mathbf{r}'|,\omega)\frac{\partial\tilde{\Phi}(\mathbf{r}',\omega)}{\partial z}dx'\,dy'$$

$$= -i\omega\varepsilon a\int_0^a r'\,dr'\int_0^{2\pi} d\phi' \left.\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{2\pi|\mathbf{r}-\mathbf{r}'|}\right|_{z'=0}$$

Integration domain: $x' = r'\cos\phi'$

$y' = r'\sin\phi'$

For $r >> a$; $\quad |\mathbf{r}-\mathbf{r}'| \approx r - \hat{\mathbf{r}}\cdot\mathbf{r}'$

Assume $\hat{\mathbf{r}}$ is in the yz plane; $\phi = \frac{\pi}{2}$

$$\hat{\mathbf{r}} = \sin\theta\hat{\mathbf{y}} + \cos\theta\hat{\mathbf{z}}$$

$$|\mathbf{r}-\mathbf{r}'| \approx r - \hat{\mathbf{r}}\cdot\mathbf{r}' = r - r'\sin\theta\sin\phi'$$

$$\tilde{\Phi}(\mathbf{r},\omega) = -\frac{i\omega\varepsilon a}{2\pi}\frac{e^{ikr}}{r}\int_0^a r'\,dr'\int_0^{2\pi} d\phi'\, e^{-ikr'\sin\theta\sin\phi'}$$

Note that: $\quad \frac{1}{2\pi}\int_0^{2\pi} d\phi'\, e^{-iu\sin\phi'} = J_0(u)$

$$\Rightarrow \tilde{\Phi}(\mathbf{r},\omega) = -i\omega\varepsilon a\frac{e^{ikr}}{r}\int_0^a r'\,dr'\,J_0(kr'\sin\theta)$$

$$\int_0^w u\,du\,J_0(u) = wJ_1(w)$$

$$\Rightarrow \tilde{\Phi}(\mathbf{r},\omega) = -i\omega\varepsilon a^3\frac{e^{ikr}}{r}\frac{J_1(ka\sin\theta)}{ka\sin\theta}$$

Energy flux : $\mathbf{j}_e = \delta\mathbf{v}p$

Taking time average : $\langle \mathbf{j}_e \rangle = \frac{1}{2}\Re\left(\delta\mathbf{v}p^*\right)$

$$= \tfrac{1}{2}\rho_0 \Re\left(\left(-\nabla\Phi\right)\left(-i\omega\Phi\right)^*\right)$$

Time averaged power per solid angle :

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \langle \mathbf{j}_e \rangle \cdot \hat{\mathbf{r}} r^2 = \frac{1}{2}\rho_0 \varepsilon^2 c^3 k^4 a^6 \left| \frac{J_1(ka\sin\theta)}{ka\sin\theta} \right|^2$$

Effects of nonlinearities in fluid equations
-- one dimensional case

Newton - Euler equation of motion :

$$\frac{\partial \mathbf{v}}{\partial t} + \left(\mathbf{v}\cdot\nabla\right)\mathbf{v} = \mathbf{f}_{applied} - \frac{\nabla p}{\rho}$$

Continuity equation : $\dfrac{\partial \rho}{\partial t} + \nabla\cdot\left(\rho\mathbf{v}\right) = 0$

Assume that spatial variation confined to x direction ; assume that $\mathbf{v} = v\hat{\mathbf{x}}$ and $\mathbf{f}_{applied} = 0$.

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial x} = 0$$

Expressing p in terms of ρ : $p = p(\rho)$

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial \rho}\frac{\partial \rho}{\partial x} \equiv c^2(\rho)\frac{\partial \rho}{\partial x} \qquad \text{where} \quad \frac{\partial p}{\partial \rho} \equiv c^2(\rho)$$

For adiabatic ideal gas :

$$c^2(\rho) = \frac{\gamma p}{\rho} = c_0^2\left(\frac{\rho}{\rho_0}\right)^{\gamma-1} \qquad \text{where} \quad c_0^2 \equiv \frac{\gamma p_0}{\rho_0}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{c^2(\rho)}{\rho}\frac{\partial \rho}{\partial x} = 0$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial x} = 0$$

Expressing variation v in terms of variation of ρ :

$$\frac{\partial v}{\partial \rho}\frac{\partial \rho}{\partial t} + v\frac{\partial v}{\partial \rho}\frac{\partial \rho}{\partial x} + \frac{c^2(\rho)}{\rho}\frac{\partial \rho}{\partial x} = 0$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial \rho}\frac{\partial \rho}{\partial x} = 0$$

Traveling wave solution:

Assume : $\rho = \rho_0 + f(x - u(\rho)t)$

Need to find self-consistent equations for propagation velocity $u(\rho)$ using equations

$$\frac{\partial v}{\partial \rho}\frac{\partial \rho}{\partial t} + v\frac{\partial v}{\partial \rho}\frac{\partial \rho}{\partial x} + \frac{c^2(\rho)}{\rho}\frac{\partial \rho}{\partial x} = 0$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial \rho}\frac{\partial \rho}{\partial x} = 0$$