PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 38:

1. Comment on Exam questions
2. Remaining concepts in classical mechanics
   a. Stokes equation
3. Class evaluation

22	Wed, 10/17/2012	Chap. 7, 5	Moment of inertia	
	Fri, 10/19/2012		Fall break	
23	Mon, 10/22/2012	Chap. 5	Rigid body rotation	#16
24	Wed, 10/24/2012	Chap. 5	Rigid body rotation	#17
25	Fri, 10/26/2012	Chap. 5	Rigid body rotation	#18
26	Mon, 10/29/2012	Chap. 8	Waves in elastic membranes	#19
27	Wed, 10/31/2012	Chap. 9	Introduction to hydrodynamics	
28	Fri, 11/01/2012	Chap. 9	Introduction to hydrodynamics	
29	Mon, 11/05/2012	Chap. 9	Introduction to hydrodynamics	#20
30	Wed, 11/07/2012	Chap. 9	Sound waves	
31	Fri, 11/09/2012	Chap. 9	Linear sound waves	#21
32	Mon, 11/12/2012	Chap. 9	Green's function for linear sound	
33	Wed, 11/14/2012	Chap. 9	Non-linear sound	
34	Fri, 11/16/2012	Chap. 9	Non-linear sound	Take Home Exam
35	Mon, 11/19/2012	Chap. 10	Surface waves	Take Home Exam
	Wed, 11/21/2012		*Thanksgiving Holiday*	
	Fri, 11/23/2012		*Thanksgiving Holiday*	
36	Mon, 11/26/2012	Chap. 10	Surface waves	Exam due
37	Wed, 11/28/2012	Chap. 10	Surface waves	
38	Fri, 11/30/2012	Chap. 10	Surface waves	
39	Mon, 12/03/2012		Student presentations I	
40	Wed, 12/05/2012		Student presentations II	

Schedule for PHY 711 presentations

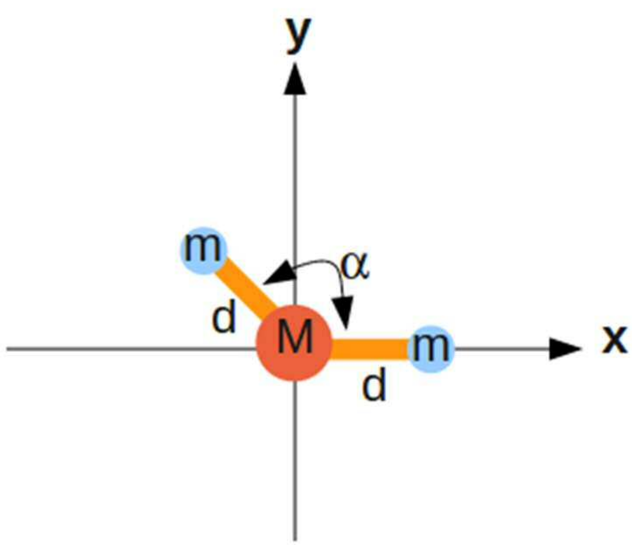
Monday, Dec. 3, 2012

10:00	Pete Diemer -- examples of hydrodynamic phenomena
10:15	Katelyn Goetz - Physics and Golf
10:30	
10:45	
11:00	Xiaohua Liu---

Wednesday, Dec. 5, 2012

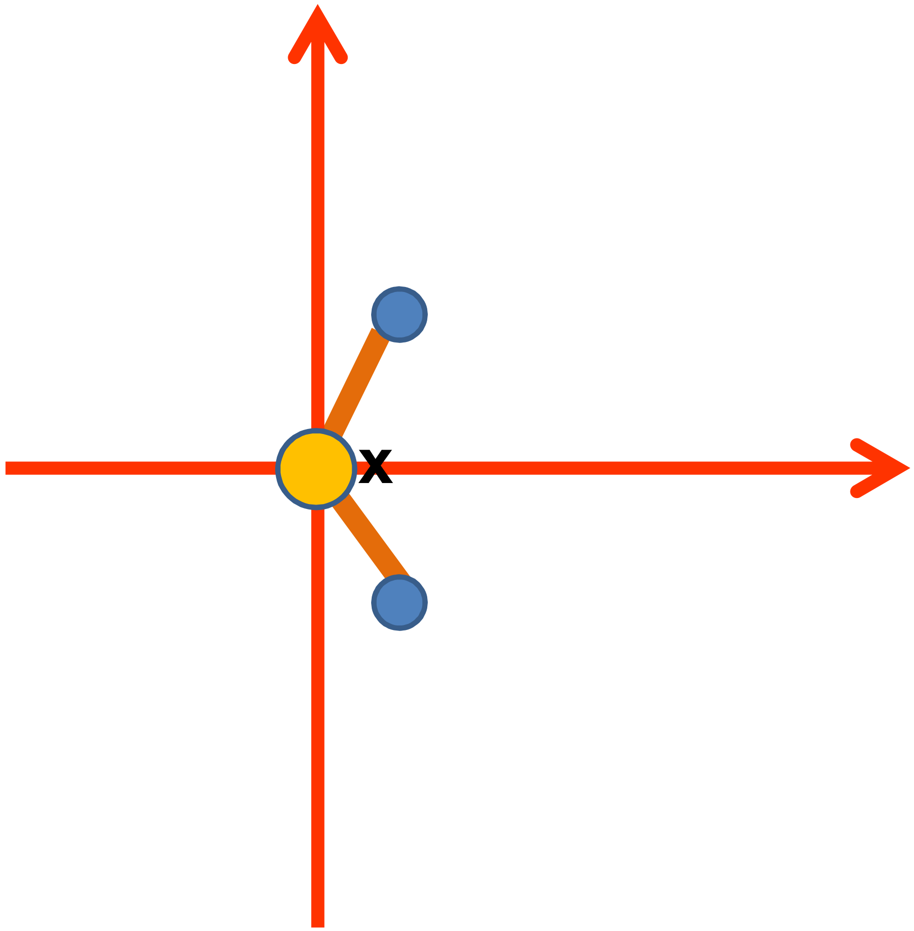
10:00	Zach Lamport -- The Physics of Hockey
10:15	Jiajie Xiao -- Physics of Chinese Flute
10:30	David Montgomery--
10:45	Chaochao Dun-Membranes
11:00	

Comment on exam problem 2

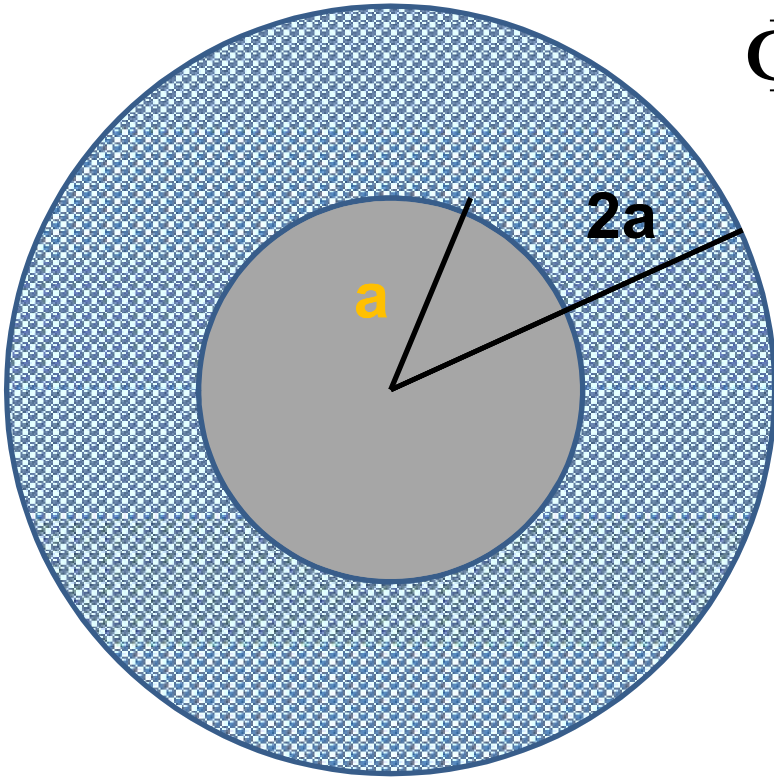


$$I = md^2 \begin{pmatrix} \sin^2\alpha & -\sin\alpha\cos\alpha & 0 \\ -\sin\alpha\cos\alpha & 1+\cos^2\alpha & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Principal moments of inertia

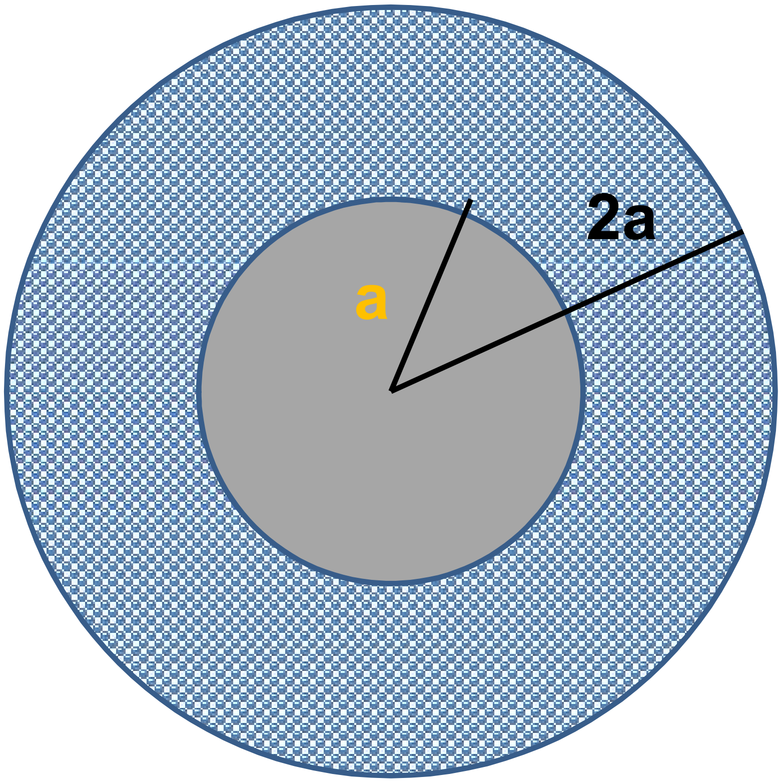


Comment on exam problem 4



$$\Phi(\mathbf{r},t) = \phi_l(r) Y_{lm}(\hat{\mathbf{r}}) e^{-i\omega t}$$

$$\phi_l(r) = C_l \left(j_l(kr) + x_l y_l(kr) \right)$$



$$\phi_l(r) = C_l\left(j_l(kr) + x_l y_l(kr)\right)$$

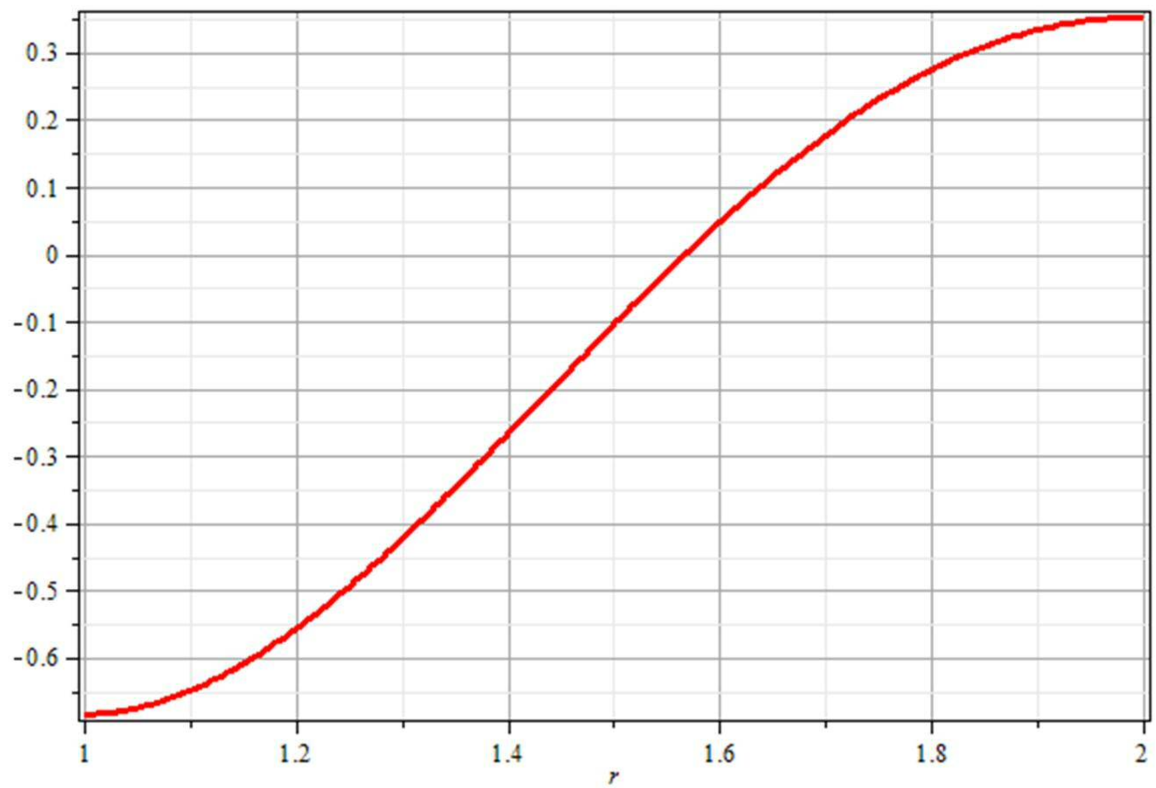
Boundary values:

$$\frac{d\phi_l(a)}{dr} = 0 = \frac{d\phi_l(2a)}{dr}$$

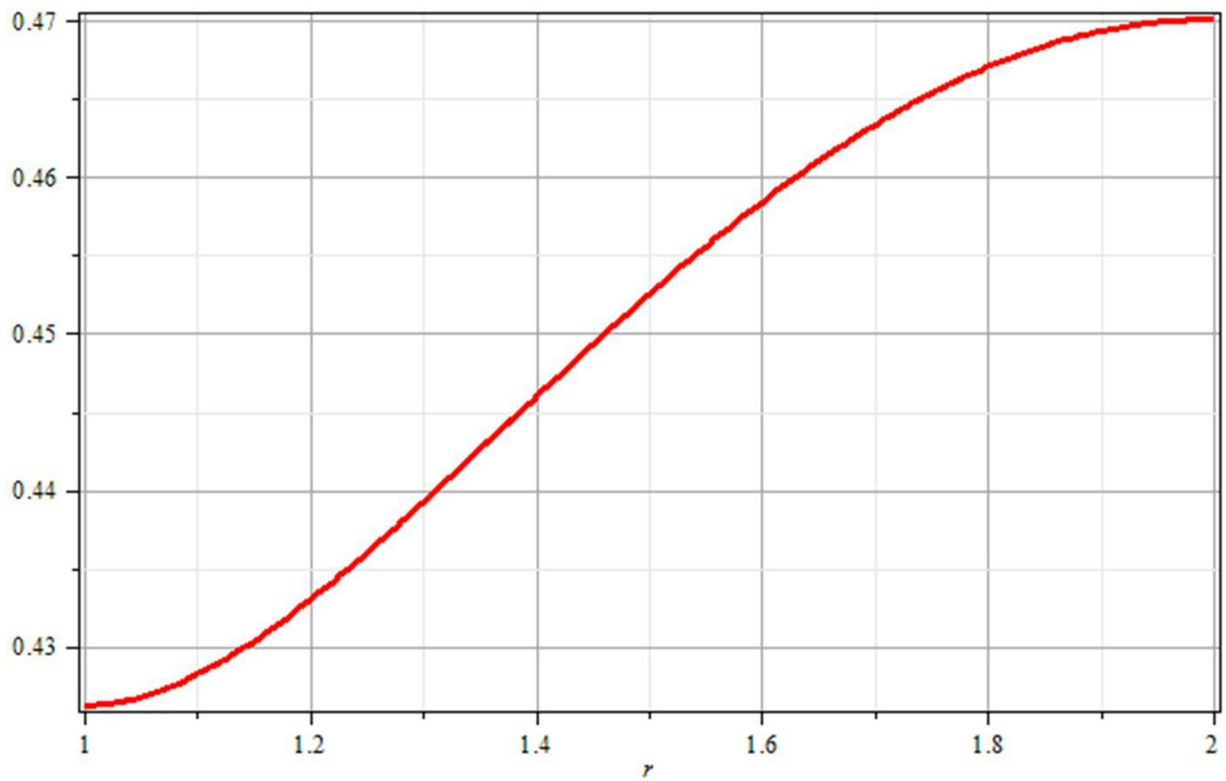
$$j'_l(ka) + x_l y'_l(ka) = 0 = j'_l(2ka) + x_l y'_l(2ka)$$

$$x_l = -\frac{j'_l(ka)}{y'_l(ka)} = -\frac{j'_l(2ka)}{y'_l(2ka)}$$

For l=0: k=3.286/a x=-2.125



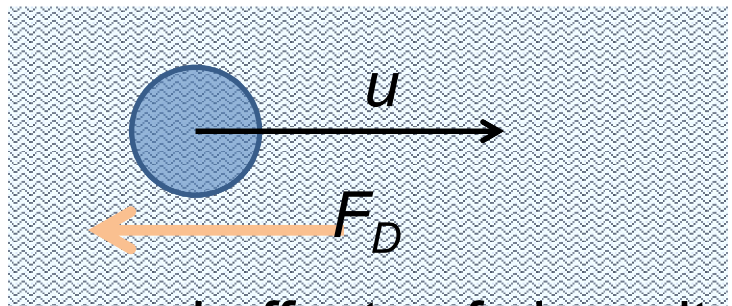
For l=1: k=0.920/a x=-0.092/a



Brief introduction to viscous effects in incompressible fluids

Stokes' analysis of viscous drag on a sphere of radius R moving at speed u in medium with viscosity η:

$$F_D = -\eta(6\pi R u)$$



Plan:

1. Consider the general effects of viscosity on fluid equations
2. Consider the solution to the linearized equations for the case of steady-state flow of a sphere of radius R
3. Infer the drag force needed to maintain the steady-state flow

Newton - Euler equation for incompressible fluid, modified by viscous contribution (Navier - Stokes equation):

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \mathbf{f}_{applied} - \frac{\nabla p}{\rho} + \frac{\eta}{\rho}\nabla^2\mathbf{v}$$

Continuity equation: $\nabla\cdot\mathbf{v} = 0$ Irrotational flow: $\nabla\times\mathbf{v} = 0$

Assume steady state: $\Rightarrow \frac{\partial \mathbf{v}}{\partial t} = 0$

Assume non - linear effects small

Initially set $\mathbf{f}_{applied} = 0$; $\quad \nabla p = \eta\nabla^2\mathbf{v}$

Assume $\mathbf{v} = \nabla\times(\nabla\times f(r)\mathbf{u}) + \mathbf{u}$

where $f(r)\xrightarrow[r\to\infty]{} 0$

$$\mathbf{v} = \nabla \times \left(\nabla \times f(r)\mathbf{u}\right) + \mathbf{u}$$

$$\mathbf{u} = u\hat{\mathbf{z}}$$

$$\nabla \times \left(\nabla \times f(r)\hat{\mathbf{z}}\right) = \nabla\left(\nabla \cdot f(r)\hat{\mathbf{z}}\right) - \nabla^2 f(r)\hat{\mathbf{z}}$$

$$\nabla \times \mathbf{v} = 0 \qquad \Rightarrow \nabla^2\left(\nabla \times \mathbf{v}\right) = 0$$

$$\nabla^4\left(\nabla \times f(r)\hat{\mathbf{z}}\right) = 0 \quad \Rightarrow \nabla^4\left(\nabla f(r) \times \hat{\mathbf{z}}\right) = 0 \quad \Rightarrow \nabla^4 f(r) = 0$$

$$f(r) = C_1 r^2 + C_2 r + C_3 + \frac{C_4}{r}$$

$$v_r = u\cos\theta\left(1 - \frac{2}{r}\frac{df}{dr}\right) = u\cos\theta\left(1 - 4C_1 - \frac{2C_2}{r} - \frac{2C_4}{r^3}\right)$$

$$v_\theta = -u\sin\theta\left(1 - \frac{d^2 f}{dr^2} - \frac{1}{r}\frac{df}{dr}\right) = -u\sin\theta\left(1 - 4C_1 - \frac{C_2}{r} + \frac{C_4}{r^3}\right)$$

$$v_r = u\cos\theta\left(1 - \frac{2}{r}\frac{df}{dr}\right) = u\cos\theta\left(1 - 4C_1 - \frac{2C_2}{r} - \frac{2C_4}{r^3}\right)$$

$$v_r = u\cos\theta\left(1 - \frac{2}{r}\frac{df}{dr}\right) = -u\sin\theta\left(1 - 4C_1 - \frac{C_2}{r} + \frac{C_4}{r^3}\right)$$

To satisfy $\mathbf{v}(r \to \infty) = \mathbf{u}: \qquad \Rightarrow C_1 = 0$

To satisfy $\mathbf{v}(R) = 0$ solve for C_2, C_4

$$v_r = u\cos\theta\left(1 - \frac{3R}{2r} + \frac{R^3}{2r^3}\right)$$

$$v_\theta = -u\sin\theta\left(1 - \frac{3R}{4r} - \frac{R^3}{4r^3}\right)$$

$$v_r = u\cos\theta\left(1 - \frac{3R}{2r} + \frac{R^3}{2r^3}\right)$$

$$v_\theta = -u\sin\theta\left(1 - \frac{3R}{4r} - \frac{R^3}{4r^3}\right)$$

Determining pressure:

$$\nabla p = \eta\nabla^2\mathbf{v} = -\eta\nabla\left(u\cos\theta\left(\frac{3R}{2r^2}\right)\right)$$

$$\Rightarrow p = p_0 - \eta u\cos\theta\left(\frac{3R}{2r^2}\right)$$

$$p = p_0 - \eta u \cos\theta \left(\frac{3R}{2r^2} \right)$$

Corresponds to :

$$F_D = \left(p(R) - p_0 \right) \frac{4\pi R^2}{\cos\theta} = -\eta u \left(6\pi R \right)$$