PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 5:

1. Chapter 2 – Physics described in an accelerated coordinate frame

PHY 711 Classical Mechanics and Mathematical Methods

MWF 10 AM-10:50 PM	OPL 103	http://www.wfu.edu/~natalie/f12phy711/

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Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment
1	Wed, 8/29/2012	Chap. 1	Review of basic principles;Scattering theory	#1
2	Fri, 8/31/2012	Chap. 1	Scattering theory continued	#2
3	Mon, 9/03/2012	Chap. 1	Scattering theory continued	#3
4	Wed, 9/05/2012	Chap. 1 & 2	Scattering theory/Accelerated coordinate frame	#4
5	Fri, 9/07/2012	Chap. 2	Accelerated coordinate frame	#5

Special seminar today: 11-11:50 AM Olin 103

GROWTH OF SILICENE

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Application of Newton's laws in a coordinate system which has an angular velocity $\boldsymbol{\omega}$ and linear acceleration $\left(\frac{d^2\mathbf{a}}{dt^2}\right)_{inertial}$

Newton's laws; Let **r** denote the position of particle of mass *m*:

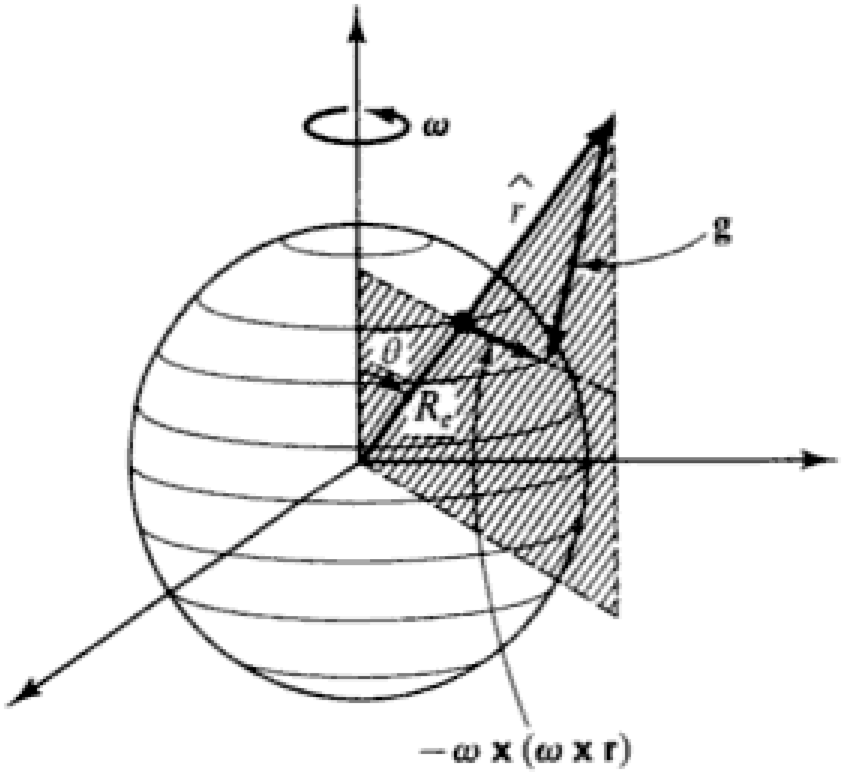
$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{inertial} = \mathbf{F}_{ext}$$

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{body} = \mathbf{F}_{ext} - m\left(\frac{d^2\mathbf{a}}{dt^2}\right)_{inertial} - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{body} - m\frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$

Coriolis force

Centrifugal force

Motion on the surface of the Earth:



$$\omega = \frac{2\pi}{\tau} \approx 7.3\times 10^{-5}\,\text{rad/s}$$

$$\mathbf{F}_{ext} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}'$$

Main contributions:

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}' - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth} - m\frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$

Non-inertial effects on effective gravitational “constant”

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}' - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth} - m\frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$

$$\text{For } \left(\frac{d\mathbf{r}}{dt}\right)_{earth} = 0 \quad \text{and} \quad \left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} = 0,$$

$$0 = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}' - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$

$$\mathbf{F}' = -m\mathbf{g}$$

$$\Rightarrow \mathbf{g} = -\frac{GM_e}{r^2}\hat{\mathbf{r}} - \boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}\Big|_{r\approx R_e}$$

$$= \left(-\frac{GM_e}{R_e^{\,2}} + \omega^2 R_e \sin^2\theta\right)\hat{\mathbf{r}} + \sin\theta\cos\theta\,\omega^2 R_e\hat{\boldsymbol{\theta}}$$

0.03 m/s²

9.80 m/s²

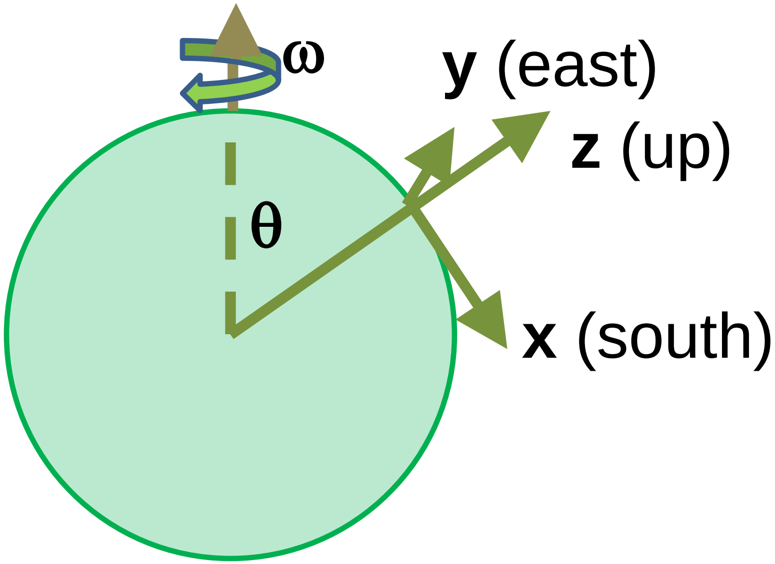
Foucault pendulum

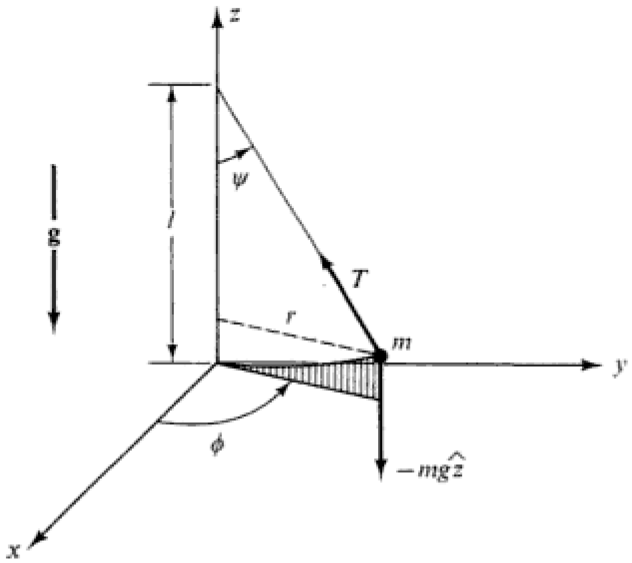
http://www.si.edu/Encyclopedia_SI/nmah/pendulum.htm

The Foucault pendulum was displayed for many years in the Smithsonian's National Museum of American History. It is named for the French physicist Jean Foucault who first used it in 1851 to demonstrate the rotation of the earth.

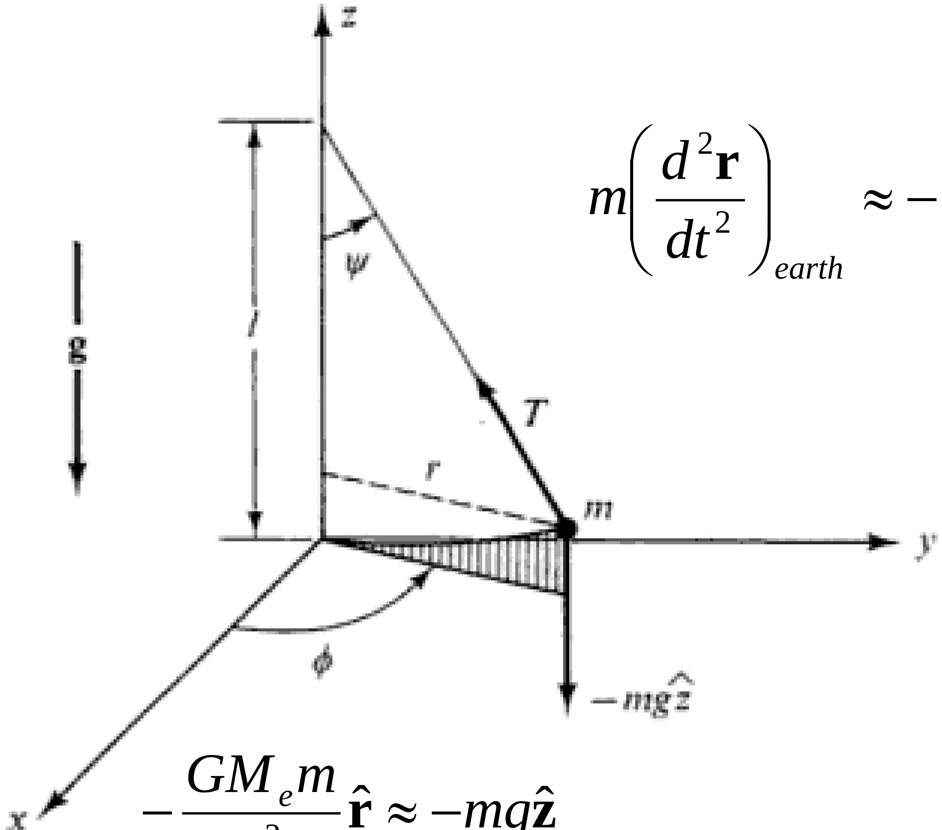
Equation of motion on Earth's surface

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}' - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth} - m\frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$





Foucault pendulum continued – keeping leading terms:



$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} \approx -\frac{GM_e m}{R_e^2}\hat{\mathbf{r}} + \mathbf{F}' - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth}$$

$$-\frac{GM_e m}{r^2}\hat{\mathbf{r}} \approx -mg\hat{\mathbf{z}}$$

$$\mathbf{F}' \approx -T\sin\psi\cos\phi\hat{\mathbf{x}} - T\sin\psi\sin\phi\hat{\mathbf{y}} + T\cos\psi\hat{\mathbf{z}}$$

$$\boldsymbol{\omega} \approx -\omega\sin\theta\hat{\mathbf{x}} + \omega\cos\theta\hat{\mathbf{z}}$$

$$\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth} \approx \omega\left(-\dot{y}\cos\theta\hat{\mathbf{x}} + \left(\dot{x}\cos\theta + \dot{z}\sin\theta\right)\hat{\mathbf{y}} - \dot{y}\cos\theta\hat{\mathbf{z}}\right)$$

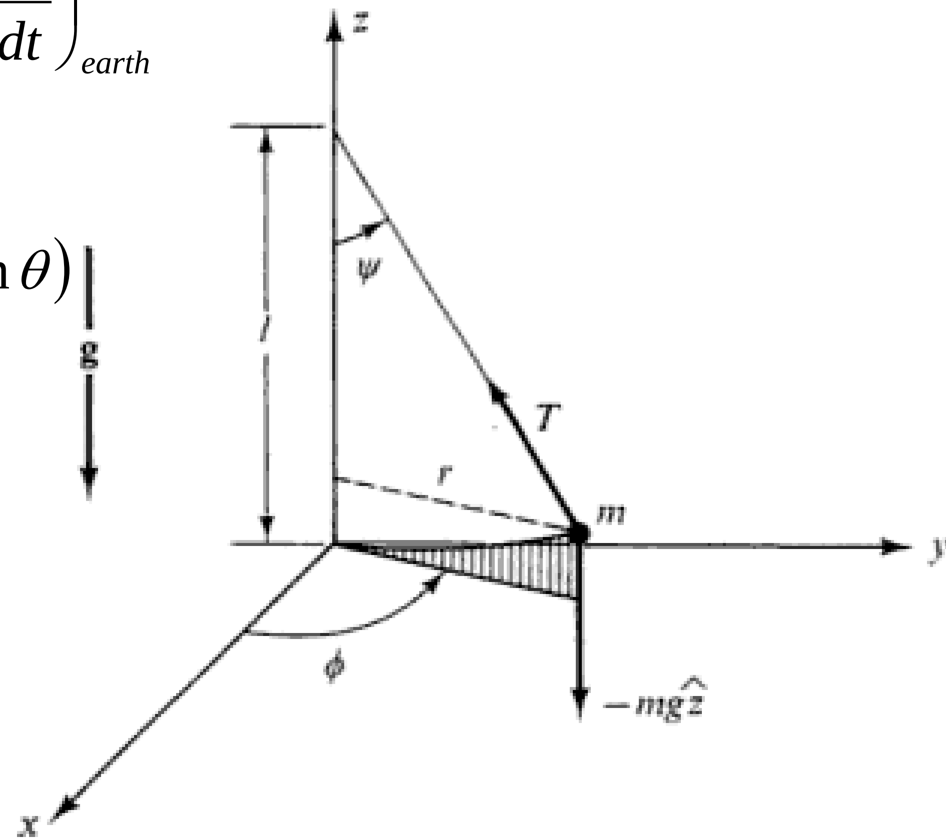
Foucault pendulum continued – keeping leading terms:

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} \approx -\frac{GM_e m}{R_e^{\ 2}}\hat{\mathbf{r}} + \mathbf{F}' - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth}$$

$$m\ddot{x} \approx -T\sin\psi\cos\phi + 2m\omega\dot{y}\cos\theta$$

$$m\ddot{y} \approx -T\sin\psi\sin\phi - 2m\omega\left(\dot{x}\cos\theta + \dot{z}\sin\theta\right)$$

$$m\ddot{z} \approx T\cos\psi - mg + 2m\omega\dot{y}\cos\theta$$



Further approximation :

$$\psi << 1; \quad \ddot{z} \approx 0; \quad T \approx mg$$

$$m\ddot{x} \approx -mg\sin\psi\cos\phi + 2m\omega\dot{y}\cos\theta$$

$$m\ddot{y} \approx -mg\sin\psi\sin\phi - 2m\omega\dot{x}\cos\theta$$

Also note that :

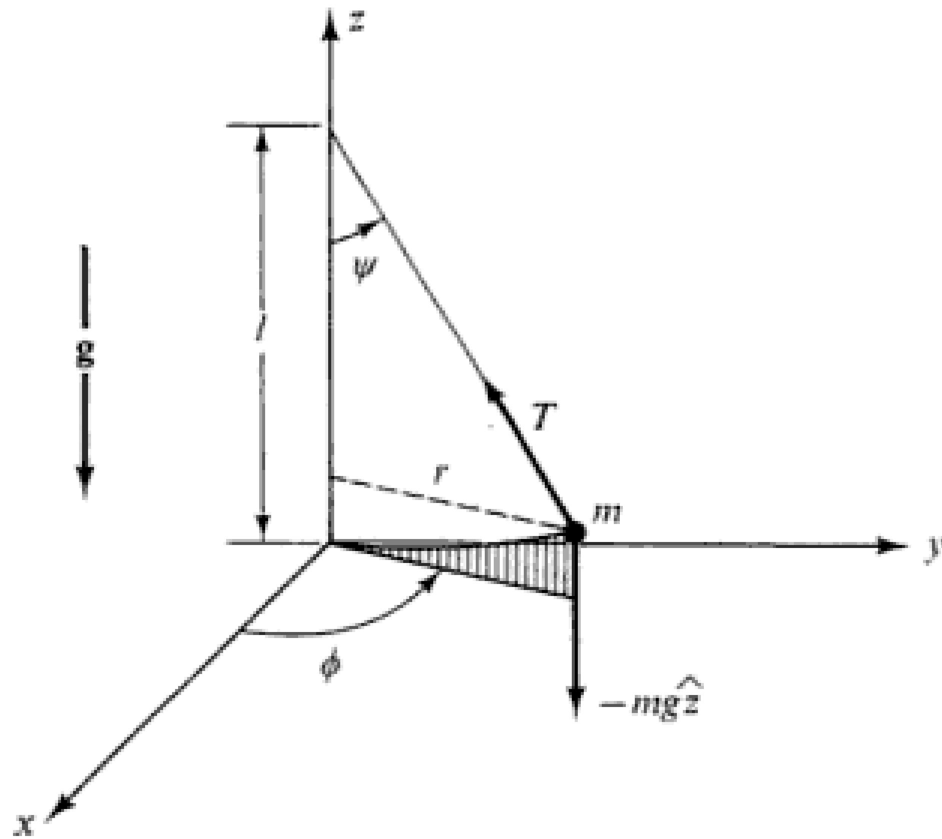
$$x \approx \ell\sin\psi\cos\phi$$

$$y \approx \ell\sin\psi\sin\phi$$

Foucault pendulum continued – coupled equations:

$$\ddot{x} \approx -\frac{g}{\ell} x + 2\omega \cos\theta \dot{y}$$

$$\ddot{y} \approx -\frac{g}{\ell} y - 2\omega \cos\theta \dot{x}$$



Try to find a solution of the form:

$$x(t) = Xe^{-iqt} \quad y(t) = Ye^{-iqt}$$

Denote $\omega_\perp \equiv \omega\cos\theta$

$$\begin{pmatrix} -q^2 + \frac{g}{\ell} & i2\omega_\perp q \\ -i2\omega_\perp q & -q^2 + \frac{g}{\ell} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = 0$$

Non-trivial solutions:

$$q_\pm = \alpha \pm \beta \equiv \omega_\perp \pm \sqrt{\omega_\perp{}^2 + \frac{g}{\ell}}$$

Foucault pendulum continued – coupled equations:

Solution continued:

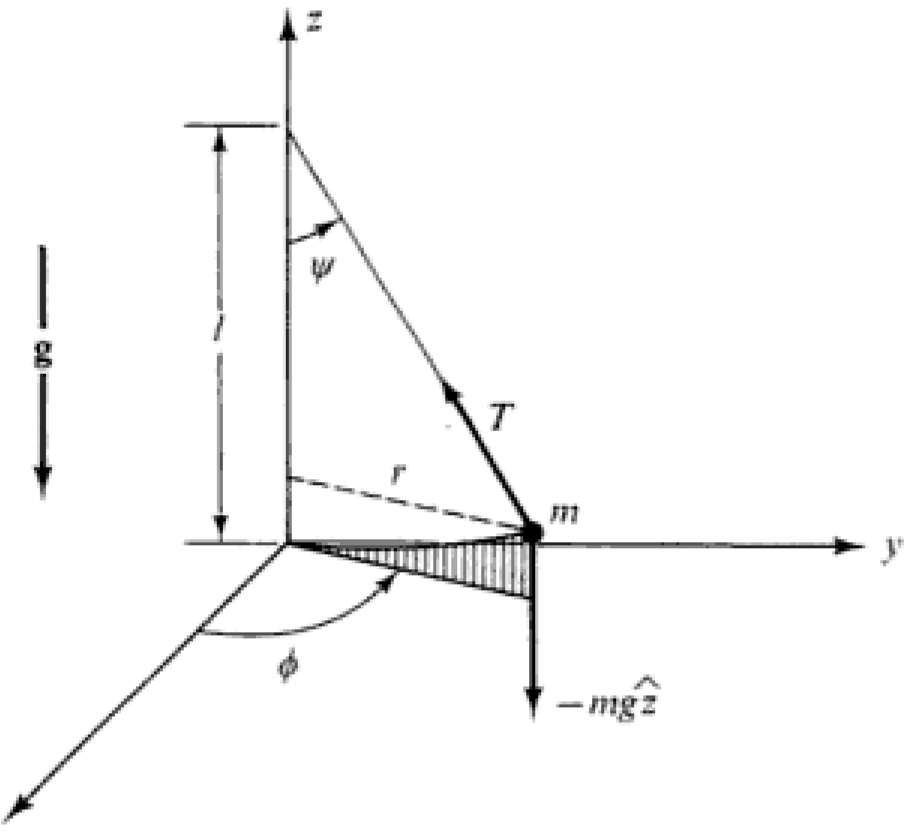
$$x(t) = Xe^{-iqt} \qquad y(t) = Ye^{-iqt}$$

$$\begin{pmatrix} -q^2 + \frac{g}{\ell} & i2\omega_\perp q \\ -i2\omega_\perp q & -q^2 + \frac{g}{\ell} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = 0$$

Non-trivial solutions:

$$q_\pm = \alpha \pm \beta \equiv \omega_\perp \pm \sqrt{\omega_\perp{}^2 + \frac{g}{\ell}}$$

Amplitude relationship: $X = iY$



General solution with complex amplitudes C and D:

$$x(t) = \text{Re}\left\{iCe^{-i(\alpha+\beta)t} + iDe^{-i(\alpha-\beta)t}\right\}$$

$$y(t) = \text{Re}\left\{Ce^{-i(\alpha+\beta)t} + De^{-i(\alpha-\beta)t}\right\}$$

General solution with complex amplitudes C and D:

$$x(t) = \mathrm{Re}\left\{iCe^{-i(\alpha+\beta)t} + iDe^{-i(\alpha-\beta)t}\right\}$$

$$y(t) = \mathrm{Re}\left\{Ce^{-i(\alpha+\beta)t} + De^{-i(\alpha-\beta)t}\right\}$$

$$q_{\pm} = \alpha \pm \beta \equiv \omega_{\perp} \pm \sqrt{\omega_{\perp}{}^{2} + \frac{g}{\ell}}$$

$$\omega_{\perp} \approx 7\times 10^{-5} \cos\theta \;\; rad/s << \sqrt{\frac{g}{\ell}}$$