PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 7:

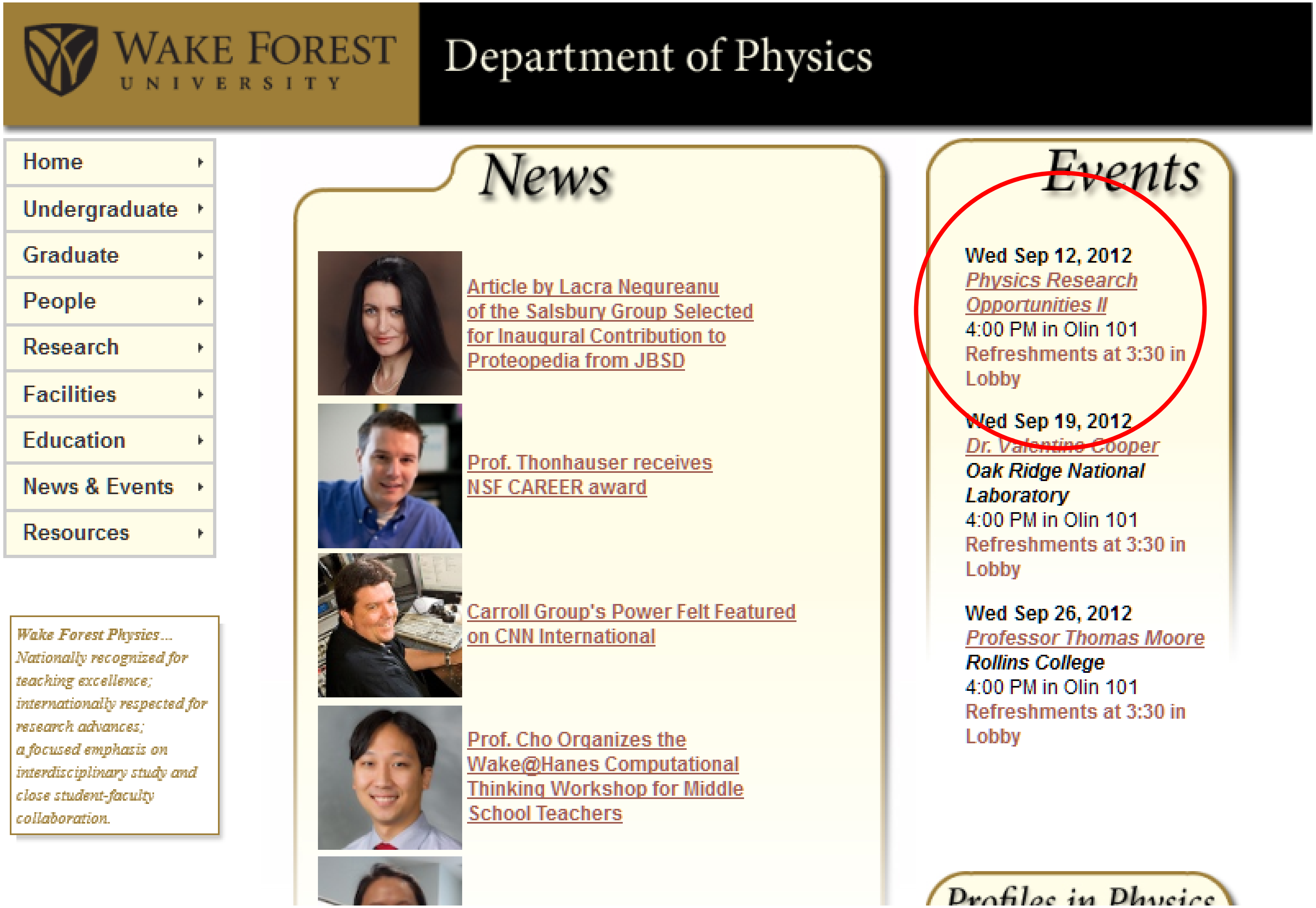
Continue reading Chapter 3

Further development of the "calculus of variation"

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment
1	Wed, 8/29/2012	Chap. 1	Review of basic principles;Scattering theory	#1
2	Fri, 8/31/2012	Chap. 1	Scattering theory continued	#2
3	Mon, 9/03/2012	Chap. 1	Scattering theory continued	#3
4	Wed, 9/05/2012	Chap. 1 & 2	Scattering theory/Accelerated coordinate frame	#4
5	Fri, 9/07/2012	Chap. 2	Accelerated coordinate frame	#5
6	Mon, 9/10/2012	Chap. 3	Calculus of Variation	#6
7	Wed, 9/12/2012	Chap. 3	Calculus of Variation continued	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News
Article by Lacra Negureanu of the Salsbury Group Selected for Inaugural Contribution to Proteopedia from JBSD
Prof. Thonhauser receives NSF CAREER award
Carroll Group's Power Felt Featured on CNN International
Prof. Cho Organizes the Wake@Hanes Computational Thinking Workshop for Middle School Teachers
Events
Wed Sep 12, 2012
Physics Research Opportunities II
4:00 PM in Olin 101
Refreshments at 3:30 in Lobby
Wed Sep 19, 2012
Dr. Valentino Cooper
Oak Ridge National Laboratory
4:00 PM in Olin 101
Refreshments at 3:30 in Lobby
Wed Sep 26, 2012
Professor Thomas Moore
Rollins College
4:00 PM in Olin 101
Refreshments at 3:30 in Lobby
Profiles in Physics

FOREST
SITY

Department of Physics

WFU Physics Colloquium

TITLE: "WFU Physics Research -- Part II"

TIME: Wednesday Sept. 12, 2012 at 4:00 PM

PLACE: George P. Williams, Jr. Lecture Hall, (Olin 101)

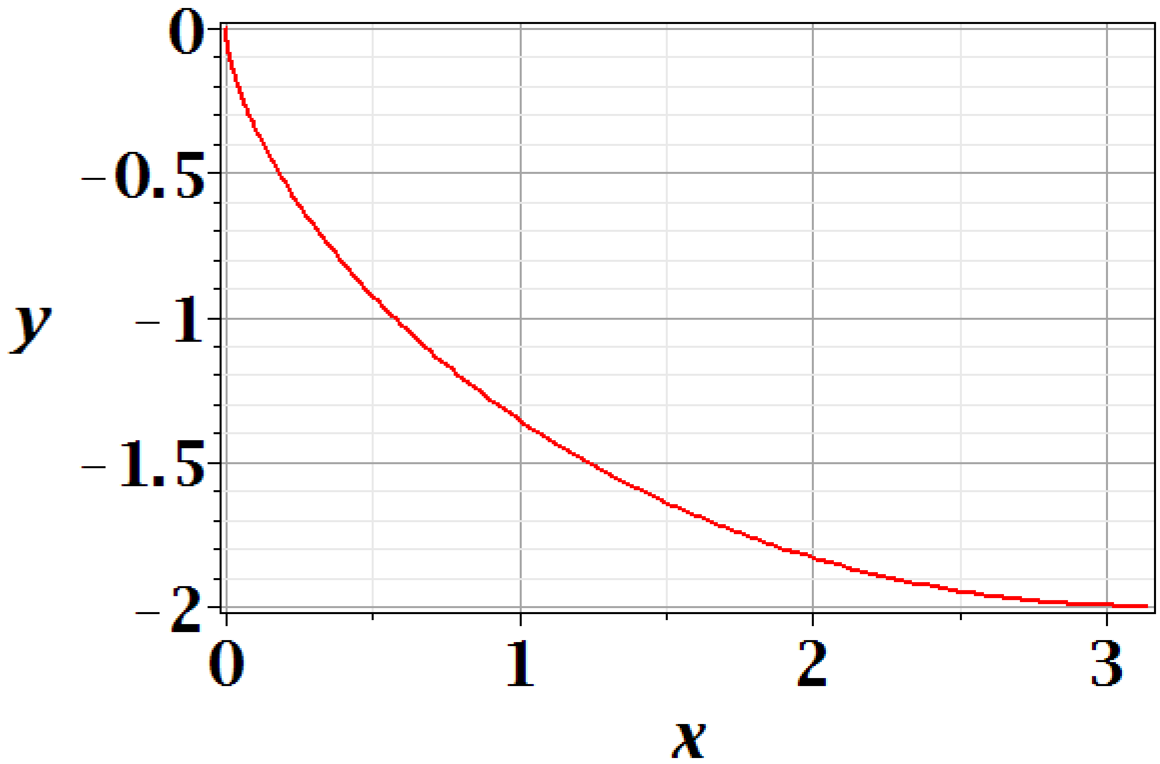
Refreshments will be served at 3:30 PM in the lounge. All interested persons are cordially invited to attend.

PROGRAM

This colloquium is the second of two which will highlight physics research at Wake Forest University. During the colloquium, Physics Department faculty members will present short overviews of their research programs in the Physics Department. This forum for sharing ideas will hopefully inspire collaborations between students and faculty and between research groups.

Brachistochrone problem: (solved by Newton in 1696)

http://mathworld.wolfram.com/BrachistochroneProblem.html



A particle of weight *mg* travels frictionlessly down a path of shape *y(x).* What is the shape of the path *y(x)* that minimizes the travel time from $y(0)=0$ to $y(\pi)=-2$?

$$T=\int_{x_i y_i}^{x_f y_f}\frac{ds}{v}=\int_{x_i}^{x_f}\frac{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\sqrt{-2gy}}dx \quad \text{because} \quad \tfrac{1}{2}mv^2=-mgy$$

$$f\left(\left\{y(x),\frac{dy}{dx}\right\},x\right)=\sqrt{\frac{1+\left(\frac{dy}{dx}\right)^2}{-y}}$$

Alternative relationships for extremization :

$$\left(\frac{\partial f}{\partial y}\right)-\frac{d}{dx}\left[\left(\frac{\partial f}{\partial(dy/dx)}\right)\right]=0$$

$$\frac{d}{dx}\left(f-\frac{\partial f}{\partial(dy/dx)}\frac{dy}{dx}\right)=\left(\frac{\partial f}{\partial x}\right)$$

$$f\left(\left\{y(x),\frac{dy}{dx}\right\},x\right)=\sqrt{\frac{1+\left(\frac{dy}{dx}\right)^2}{-y}}$$

$$\frac{d}{dx}\left(f-\frac{\partial f}{\partial(dy/dx)}\frac{dy}{dx}\right)=\left(\frac{\partial f}{\partial x}\right)$$

$$\Rightarrow\frac{d}{dx}\left(\frac{1}{\sqrt{-y\left(1+\left(\frac{dy}{dx}\right)^2\right)}}\right)=0 \qquad -y\left(1+\left(\frac{dy}{dx}\right)^2\right)=K\equiv 2a$$

$$-y\left(1+\left(\frac{dy}{dx}\right)^2\right)=K\equiv 2a$$

Let $y=-2a\sin^2\frac{\theta}{2}=a(\cos\theta-1)$

$$\frac{dy}{dx}=-\sqrt{\frac{2a}{-y}-1}$$

$$-\frac{dy}{\sqrt{\frac{2a}{-y}-1}}=\frac{2a\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,d\theta}{\sqrt{\frac{2a}{2a\sin^2\frac{\theta}{2}}-1}}=dx$$

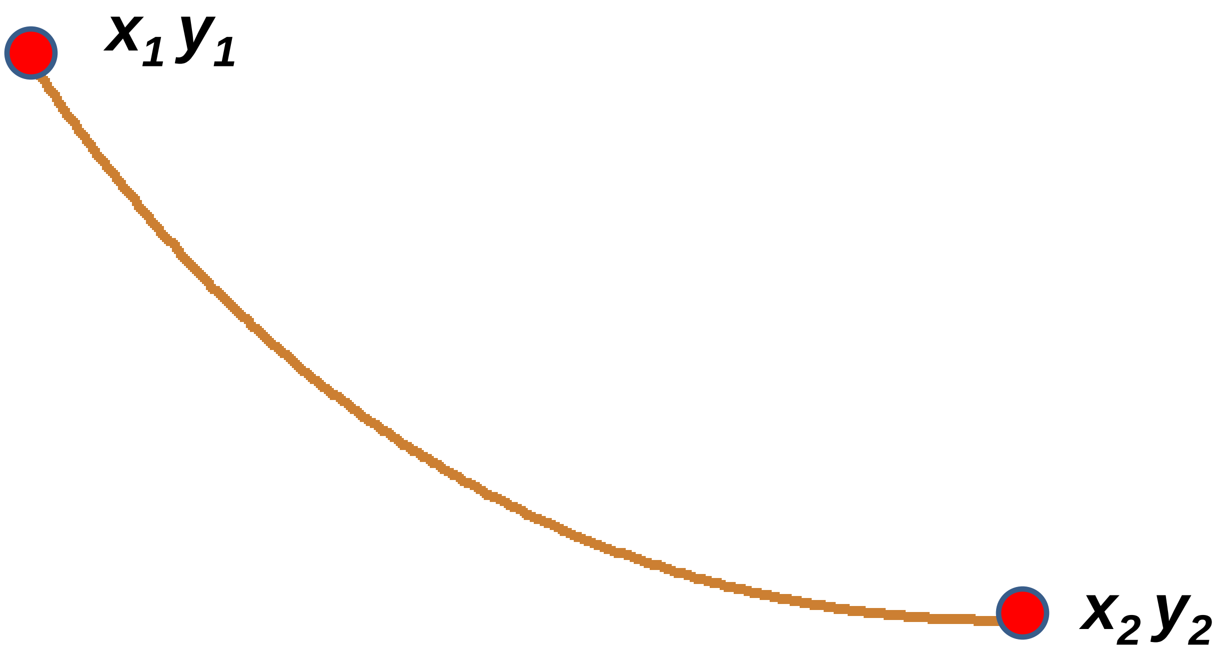
$$-\frac{dy}{\sqrt{\frac{2a}{-y}-1}}=dx$$

$$x=\int_0^\theta a(1-\cos\theta')d\theta'=a(\theta-\sin\theta)$$

Parametric equations for Brachistochrone:

$$x=a(\theta-\sin\theta)$$
$$y=a(\cos\theta-1)$$

Shape of a rope of length L and mass density ρ hanging between two points



Potential energy of hanging rope:

$$E = \rho g \int_{x_1}^{x_2} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

Length of rope:

$$L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

Define a composite function to minimize:

$$W \equiv E + \lambda L$$

Lagrange multiplier

$$W = \int_{x_1}^{x_2} (\rho g y + \lambda)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$f\left(\left\{y, \frac{dy}{dx}\right\}\right) = (\rho g y + \lambda)\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\frac{d}{dx}\left(f - \frac{\partial f}{\partial(dy/dx)}\frac{dy}{dx}\right) = \left(\frac{\partial f}{\partial x}\right)$$

$$\Rightarrow (\rho g y + \lambda)\left(\sqrt{1+\left(\frac{dy}{dx}\right)^2} - \frac{\left(\frac{dy}{dx}\right)^2}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}\right) = K$$

$$\left(\rho g y+\lambda\right)\left(\sqrt{1+\left(\frac{dy}{dx}\right)^2}-\frac{\left(\frac{dy}{dx}\right)^2}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}\right)=K$$

$$\left(\rho g y+\lambda\right)\left(\frac{1}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}\right)=K$$

$$y(x)=-\frac{1}{\rho g}\left(\lambda+K\cosh\left(\frac{x-a}{K/\rho g}\right)\right)$$

$$y(x) = -\frac{1}{\rho g}\left(\lambda + K\cosh\left(\frac{x-a}{K/\rho g}\right)\right)$$

Integration constants $: K, a, \lambda$

Constraints : $\quad y(x_1) = y_1$

$$y(x_2) = y_2$$

$$\int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = L$$

Summary of results

For the class of problems where we need to perform an extremization on an integral form:

$$I = \int_{x_i}^{x_f} f\left(\left\{y(x), \frac{dy}{dx}\right\}, x\right) dx$$

A necessary condition is the Euler-Lagrange equations:

$$\left(\frac{\partial f}{\partial y}\right) - \frac{d}{dx}\left[\left(\frac{\partial f}{\partial (dy/dx)}\right)\right] = 0$$

$$\frac{d}{dx}\left(f - \frac{\partial f}{\partial (dy/dx)}\frac{dy}{dx}\right) = \left(\frac{\partial f}{\partial x}\right)$$

Application to particle dynamics

$$x \to t \quad \text{(time)}$$

$$y \to q \quad \text{(generalized coordinate)}$$

$$f \to L \quad \text{(Lagrangian)}$$

$$I \to A \quad \text{(action)}$$