PHY 711 Classical Mechanics and Mathematical Methods 10-10:50 AM MWF Olin 103

Plan for Lecture 7:

Continue reading Chapter 3

Further development of the "calculus of variation"

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Course schedule (Preliminary schedule -- subject to frequent adjustment.) Date F&W Reading Topic Assignment 1 | Wed, 8/29/2012 | Chap. 1 | Review of basic principles; Scattering theory | #1 | 2 | Fri, 8/31/2012 | Chap. 1 | Scattering theory continued | #2 | 3 | Mon. 9/03/2012 | Chap. 1 | Scattering theory continued | #3 | 4 | Wed, 9/05/2012 | Chap. 1 & 2 | Scattering theory (continued | #3 | 5 | Fri, 9/07/2012 | Chap. 2 | Accelerated coordinate frame | #4 | 6 | Mon. 9/10/2012 | Chap. 3 | Calculus of Variation | #6 | 7 | Wed, 9/12/2012 | Chap. 3 | Calculus of Variation | #6 | 7 | Wed, 9/12/2012 | Chap. 3 | Calculus of Variation | #6 | 9/12/2012 | PHY711 | Fall 2012 - Lecture 7 | 2

Home	News	Events
Undergraduate		
Graduate	Article by Lacra Negureanu	Wed Sep 12, 2012 Physics Research
People	of the Salsbury Group Selected for Inaugural Contribution to	Opportunities // 4:00 PM in Olin 101
Research	Proteopedia from JBSD	Refreshments at 3:30 in Lobby
Facilities		Wed Sep 19, 2012
Education	Prof. Thonhauser receives	Dr. Valending Cooper Oak Ridge National
News & Events •	NSF CAREER award	Laboratory
Resources		4:00 PM in Olin 101 Refreshments at 3:30 in
Wake Forest Physics	Carroll Group's Power Felt Featured on CNN International	Lobby Wed Sep 26, 2012
Nationally recognized for	on CNN International	Professor Thomas Moore Rollins College
teaching excellence; internationally respected for		4:00 PM in Olin 101 Refreshments at 3:30 in
research advances; a focused emphasis on	Prof. Cho Organizes the Wake@Hanes Computational	Lobby
interdisciplinary study and close student-faculty	Thinking Workshop for Middle	
collaboration.	School Teachers	

Department of Physics

WFU Physics Colloquium

TITLE: "WFU Physics Research -- Part II" TIME: Wednesday Sept. 12, 2012 at 4:00 PM PLACE: George P. Williams, Jr. Lecture Hall, (Olin 101)

Refreshments will be served at 3:30 PM in the lounge. All interested persons are cordially invited to attend.

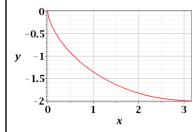
This colloquium is the second of two which will highlight physics research at Wake Forest University. During the colloquium, Physics Department faculty members will present short overviews of their research programs in the Physics Department. This forum for sharing ideas will hopefully inspire collaborations between students and faculty and between research groups.

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Brachistochrone problem: (solved by Newton in 1696)

http://mathworld.wolfram.com/BrachistochroneProblem.html



A particle of weight mg travels frictionlessly down a path of shape y(x). What is the shape of the path y(x) that minimizes the travel time from y(0)=0 to $y(\pi)=-2$?

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$$T = \int_{x_i y_i}^{x_j y_i} \frac{ds}{v} = \int_{x_i}^{x_i} \frac{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}{\sqrt{-2gy}} dx \quad \text{because } \frac{1}{2}mv^2 = -mgy$$

$$f\left(\left\{y(x), \frac{dy}{dx}\right\}, x\right) = \sqrt{\frac{1 + \left(\frac{dy}{dx}\right)^2}{-y}}$$

 $Alternative\ relationships\ for\ extremization:$

$$\left(\frac{\partial f}{\partial y}\right) - \frac{d}{dx} \left[\left(\frac{\partial f}{\partial \left(dy/dx\right)}\right) \right] = 0$$

$$\frac{d}{dx}\left(f - \frac{\partial f}{\partial (dy/dx)}\frac{dy}{dx}\right) = \left(\frac{\partial f}{\partial x}\right)$$

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$$f\left(\left\{y(x), \frac{dy}{dx}\right\}, x\right) = \sqrt{\frac{1 + \left(\frac{dy}{dx}\right)^2}{-y}}$$

$$\frac{d}{dx} \left(f - \frac{\partial f}{\partial (dy/dx)} \frac{dy}{dx}\right) = \left(\frac{\partial f}{\partial x}\right)$$

$$\Rightarrow \frac{d}{dx} \left(\frac{1}{\sqrt{-y\left(1 + \left(\frac{dy}{dx}\right)^2\right)}}\right) = 0 \quad -y\left(1 + \left(\frac{dy}{dx}\right)^2\right) = K \equiv 2a$$

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$$-y\left(1+\left(\frac{dy}{dx}\right)^{2}\right) = K = 2a \qquad \text{Let} \qquad y = -2a\sin^{2}\frac{\theta}{2} = a(\cos\theta - 1)$$

$$-\frac{dy}{dx} = -\sqrt{\frac{2a}{-y}} - 1 \qquad -\frac{\frac{dy}{\sqrt{\frac{2a}{-y}}} - \frac{2a\sin\frac{\theta}{2}\cos\frac{\theta}{2}d\theta}{\sqrt{\frac{2a}{2a\sin^{2}\frac{\theta}{2}}} - 1} = dx$$

$$-\frac{dy}{\sqrt{\frac{2a}{-y}} - 1} = dx \qquad x = \int_{0}^{\theta} a(1-\cos\theta')d\theta' = a(\theta-\sin\theta)$$

Parametric equations for Brachistochrone:

$$x = a(\theta - \sin \theta)$$
$$y = a(\cos \theta - 1)$$

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Shape of a rope of length L and mass density ρ hanging between two points

• x₁ y₁

• x₂ y₂

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Potential energy of hanging rope:

$$E = \rho g \int_{x}^{x_2} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Length of rope:

$$L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

Define a composite function to minimize:

$$W \equiv E + \lambda L$$
 Lagrange multiplier

$$W = \int_{x_1}^{x_2} (\rho g y + \lambda) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$f\left(\left\{y, \frac{dy}{dx}\right\}\right) = (\rho g y + \lambda) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\frac{d}{dx} \left(f - \frac{\partial f}{\partial (dy/dx)} \frac{dy}{dx}\right) = \left(\frac{\partial f}{\partial x}\right)$$

$$\Rightarrow (\rho g y + \lambda) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} - \frac{\left(\frac{dy}{dx}\right)^2}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} = K$$

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$$(\rho g y + \lambda) \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2} - \frac{\left(\frac{dy}{dx}\right)^2}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} \right) = K$$

$$(\rho g y + \lambda) \left(\frac{1}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} \right) = K$$

$$y(x) = -\frac{1}{\rho g} \left(\lambda + K \cosh\left(\frac{x - a}{K / \rho g}\right) \right)$$
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$$y(x) = -\frac{1}{\rho g} \left(\lambda + K \cosh \left(\frac{x - a}{K / \rho g} \right) \right)$$

Integration constants: K, a, λ

Constraints: $y(x_1) = y_1$

$$y(x_2) = y_2$$

$$\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = L$$

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Summary of results

For the class of problems where we need to perform an extremization on an integral form:

$$I = \int_{x_1}^{x_2} f\left(\left\{y(x), \frac{dy}{dx}\right\}, x\right) dx$$
A necessary condition is the Euler - Lagrange equations:

$$\left(\frac{\partial f}{\partial y}\right) - \frac{d}{dx} \left[\left(\frac{\partial f}{\partial (dy/dx)}\right) \right] = 0$$

$$\frac{d}{dx}\left(f - \frac{\partial f}{\partial (dy/dx)}\frac{dy}{dx}\right) = \left(\frac{\partial f}{\partial x}\right)$$

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Application to particle dynamics

 $x \rightarrow t$ (time)

 $y \rightarrow q$ (generalized coordinate)

 $f \rightarrow L$ (Lagrangian)

 $I \rightarrow A$ (action)

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