PHY 113 C General Physics I
11 AM -12:15 PM TR Olin 101

Plan for Lecture 18:
Chapter 17 – Sound Waves, Doppler Effect
Chapter 18 – Superposition, Standing Waves, Interference

Reminder!

The linear wave equation
Solutions are of the form:

$y(x, t) = y_0 \sin \left( \frac{2\pi x}{\lambda} \right) \sin \left( \frac{2\pi t}{T} \right)$

Periodic traveling waves are solutions of the Wave Equation

$y(x, t) = y_0 \sin \left( \frac{2\pi x}{\lambda} \right) \sin \left( \frac{2\pi t}{T} \right)$

Amplitude

Phase factor (radians)

Period $T = \frac{1}{f}$ (s)

Wave length ($m$)
SOUND WAVES obey the linear wave equation:

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

We’re neglecting attenuation. Attenuation is very important in ultrasonic imaging.

Periodic sound waves, general form

$$y(x, t) = A \sin \left( \frac{2\pi}{\lambda} x - 2\pi ft + \phi \right)$$

- Amplitude (determines intensity or loudness)
- Phase factor (radians)
- Frequency $$f$$ (s^-1) determines pitch
- Wave length $$\lambda$$ (m)

SOUND: longitudinal mechanical vibrations transmitted through an elastic solid or a liquid or gas, with frequencies in the approximate range of 20 to 20,000 hertz. How are sound waves generated? What is the velocity of sound waves in air?

A simple model with the essential physics (for 113)

**STEP ONE**

Before the piston moves, the gas is undisturbed.

**STEP TWO**

The gas is compressed by the motion of the piston.

Questions:
what has moved in the gas? 
in what direction have gas molecules moved? 
why isn’t the gas uniformly compressed?
Notice that the velocity of the compressed zone is not the same as the velocity of the piston.

What happens if the piston oscillates at frequency f? Use animation...

A sound wave may be considered as:
- a displacement wave:
  \[ s = s_{max} \cos(kt - \omega t) \]
  
  Or nodes at \( \pi / 2, \ 3\pi / 2, \ldots \)

- a pressure wave:
  \[ \Delta P = \Delta P_{max} \sin(kt - \omega t) \]
  
  Antinodes at 0, \( \pi, \ 2\pi, \ldots \)

The velocity of sound is easy to measure. \( v = 343 \text{ m/s} \) at 20ºC in air. How to relate the velocity of sound to the known properties of gases?

Newton (1643-1727) worked out an analytic model, assuming isothermal compressions:

\[ v = 298 \text{ m/s} \]

Laplace (1749-1827) pointed out that on the timescale of sound frequencies, compressions are not isothermal, they’re adiabatic (isentropic)

If we use the linearized fluid equations** in the ideal gas approximation, we find that the density fluctuations \( \delta\rho \) obey:

\[ \frac{\partial^2 \delta\rho}{\partial x^2} + \frac{\partial^2 \delta\rho}{\partial t^2} = 0 \quad ; \quad \frac{\delta P}{\rho_0} = 343 \text{ m/s} \quad \text{here } \gamma \text{ is the ratio } \frac{C_P}{C_V} \]

** These follow from conservation of mass, momentum, and energy.
Speed of Sound in a Gas

Consider an element of the gas between the piston and the dashed line. Initially, this element is in equilibrium under the influence of forces of equal magnitude.

- There is a force from the piston on the left.
- There is another force from the rest of the gas.
- These forces have equal magnitudes of PA.
  - P is the pressure of the gas.
  - A is the cross-sectional area of the tube.

After a time period, \( \Delta t \), the piston has moved to the right at a constant speed \( \dot{v}_s \).

The force has increased from \( PA \) to \( (P + \Delta P)A \).

The gas to the right of the element is undisturbed since the sound wave has not reached it yet.

Impulse and Momentum, cont.

- The change in momentum of the element of gas of mass \( m \) is
  \[
  \Delta p = m \dot{v} - (p \dot{v} A \Delta x)
  \]

- Setting the impulse side of the equation equal to the momentum side and simplifying, the speed of sound in a gas becomes.
  \[
  \dot{v} = \frac{B}{\rho}
  \]
  - The bulk modulus of the material is \( B \).
  - The density of the material is \( \rho \).
Velocity of sound:

Recall:

For a string, \( v = \sqrt{\frac{T}{\mu}} \), mass per unit length = inertial property

So, for sound, \( v = \sqrt{\frac{M}{\rho}} \), bulk modulus = elastic property

Definition of \( B \): stress = modulus x strain

\( \sigma = B \varepsilon \), definition of bulk modulus

Intensity of sound waves;
Power decibels (dB)

SUPERPOSITION of WAVES (Chap. 18)

So far, we’ve considered only one travelling wave. However, a sum of two or more travelling waves will also satisfy the wave equation because the wave equation is linear

**The Superposition Principle:**
If waves \( y_1(x,t) \) and \( y_2(x,t) \) are traveling through the same medium at the same time, the resultant wave function \( y_3(x,t) = y_1(x,t) + y_2(x,t) \).

(This is not always true! If \( y_1 \) or \( y_2 \) become large enough, response of medium becomes nonlinear and the superposition principle is violated). Examples: music amplifiers; lasers.)

CASE 1. Superposition of 2 sinusoids, \( f_1 = f_2, A_1 = A_2 \), traveling in the same direction, but different phase

\( y_1 = y_0 \sin(kx - \omega t) \); \( y_2 = y_0 \sin(kx - \omega t + \phi) \)

use trig identity: \( \sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right) \)

\( y_1 + y_2 = 2\sin(kx - \omega t + \frac{\phi}{2})\cos\left(\frac{\phi}{2}\right) \)

Constructive interference: \( \phi = 0, \pi, ... \)
Destructive interference: \( \phi = \pi/2, 3\pi/2, ... \)
**Case 2: 2 pulses traveling in opposite directions:**

When the pulses collide, they cancel out each other.

When the pulses cancel, the amplitude is reduced.

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When the pulses cancel, the amplitude is reduced.

**Case 3: Standing waves.** Two sinusoidal waves, same amplitude, same $f$, but opposite directions

$$y_{\text{right}}(x,t) = y_0 \sin \left[ 2\pi \left( \frac{x}{\lambda} - ft \right) \right]$$

$$y_{\text{left}}(x,t) = y_0 \sin \left[ 2\pi \left( \frac{x}{\lambda} + ft \right) \right]$$

Use trig identity again:

$$\sin A + \sin B = 2 \sin \left[ \frac{1}{2} (A + B) \right] \cos \left[ \frac{1}{2} (A - B) \right]$$

get

$$y_{\text{right}}(x,t) + y_{\text{left}}(x,t) = 2y_0 \sin \left( \frac{2\pi x}{\lambda} \right) \cos (2\pi ft)$$

Standing wave:

**Check that the standing wave satisfies the Wave Equation.**

Wave equation:

$$\frac{\partial^2 y}{\partial t^2} - v^2 \frac{\partial^2 y}{\partial x^2} = 0$$

$$y_{\text{standing}}(x,t) = A \sin \left( \frac{2\pi x}{\lambda} \right) \cos (2\pi ft)$$

Check:

$$\frac{\partial^2 y_{\text{standing}}}{\partial t^2} = -\left( 2\pi \right)^2 A \sin \left( \frac{2\pi x}{\lambda} \right) \cos (2\pi ft)$$

Check:

$$\frac{\partial^2 y_{\text{standing}}}{\partial x^2} = -\frac{2\pi^2 A}{\lambda^2} \sin \left( \frac{2\pi x}{\lambda} \right) \cos (2\pi ft)$$

$$\frac{\partial^2 y}{\partial t^2} - v^2 \frac{\partial^2 y}{\partial x^2} = 0$$

Equality satisfied if $f = v$. Okay? Yes.
How to generate standing waves

1. Aim two loud speakers at one another.
2. Have two boundaries causing one wave to be reflected back on itself. Each boundary is a node. The reflected wave travels in the opposite direction.

Standing waves between reflecting walls

Possible spatial shapes: \( A \sin \left( \frac{2\pi n x}{L} \right) \) \( n = 1, 2, 3, 4, \ldots \)

Standing wave form: \( A \sin \left( \frac{2\pi n x}{L} \right) \cos \left( \frac{2\pi n t}{L} \right) \)

\[ \lambda_n = \frac{2L}{n} \]

Demo:
standing transverse waves in a string

**iclicker question after string demo:**
Which of the following statements are true about the string motions described above?
A. The human ear can directly hear the string vibrations.
B. The human ear could only hear the string vibration if it occurs in vacuum.
C. The human ear can only hear the string vibration if it produces a sound wave in air.
Standing waves in air:

- Fundamental
- Second harmonic
- Third harmonic
- Fourth harmonic
- Fifth harmonic

Standing longitudinal waves in a gas-filled pipe:

Do Eric's supersized fireplace log demo.

Sound board
Sound hole
Frets
6 Strings (primary elastic elements)
Bridge (couples string vibration to sound board)
Coupling standing wave resonances in materials with sound:

**iclicker question:**
Suppose an “A” is played on a guitar string. The standing wave on the string has a frequency (f_g), wavelength (\( \lambda_g \)) and speed (c_g). Which properties of the resultant sound wave are the same as wave on the guitar string?
A. Frequency f_s
B. Wavelength \( \lambda_s \)
C. Speed c_s
D. A-C
E. None of these

Comments about waves in solids and fluids:
- Solids can support both transverse and longitudinal wave motion. Earthquake signals
- Fluids, especially gases, can support only longitudinal wave motion

1820 harmonics
The Doppler effect. Funny things happen if wave source and wave detector are moving with respect to one another.

Case 1. Source stationary, observer moving toward source.

Let \( v' \) = speed of wave relative to observer

 Observer, \( v_S \), \( f' \) \( v' = v - v_S \) but wavelength \( \lambda' \) is unchanged

so for source, \( v = \lambda f \), but for observer, \( v = \lambda' f' \)

\[ f' = \frac{v + v_S}{v} \lambda \]

Case 2. It’s easy to see that if observer moves away from source

\[ f' = \frac{v - v_S}{v} \lambda \]

What happens as observer goes past the source?

Case 3. The Doppler effect for stationary observer, source moving toward observer A. Let \( v = \) velocity of sound, \( f = \) frequency of source

During each vibration of source, source moves distance \( v_S T = v_S / f \).

Therefore observed wavelength \( \lambda' = \lambda - \frac{v_S}{f} \)

But \( f' = \frac{\lambda}{\lambda'} \) for Observer A

so \( f' = \frac{v}{\lambda - \frac{v_S}{f}} = \frac{v}{\frac{v_S}{f}} = \frac{v v_S}{f} \)

\[ f' = \frac{v}{v + v_S} \lambda \]

Case 4. Source moving away from observer B

\[ f' = \frac{v - v_S}{v} \lambda \]

Summary of sound Doppler effect:

\[ f_o = f_s \frac{v \pm v_S}{v \mp v_S} \]

Doppler effect for electromagnetic waves:

\[ f_o = f_s \frac{v + v_R}{v - v_R} \]

Relative velocity of source toward observer
Case 4. Source and observer both moving.

The figure on the left shows a car traveling at a velocity $v_{o}$ being followed by a police car traveling at a velocity $v_{p} = 30 \text{ m/s}$. The police car has a siren at frequency $f_{s} = 500 \text{ Hz}$. The observer in the front car hears the siren at a frequency of $f_{o} = 520 \text{ Hz}$. 

(a) Is the front car moving faster or slower than the police car? 
(b) What is the velocity of the front car $v_{o}$?

\[ f_{o} = f_{s} \frac{v - v_{o}}{v + v_{p}} \]

Velocity of sound: 
\[ v = 343 \text{ m/s} \]

In this case:
\[ f_{o} = f_{s} \frac{v - v_{o}}{v - v_{p}} \]
\[ f_{o} < f_{s} \Rightarrow v_{o} > v_{p} \]
\[ v_{o} \approx v - (v - v_{s}) \frac{f_{o}}{f_{s}} = 40 \text{ m/s} \]

Doppler effect for electromagnetic waves

For sound waves, model and equations used velocity of source or observer with respect to the medium. For electromagnetic (EM) waves (light, radio waves), there is no medium and the velocity of light is $c$. 

\[ f' = f \frac{c + v_{\text{relative}}}{c - v_{\text{relative}}} \]

Applications: wavelengths of spectral lines from stars, e.g. H atom, are slightly shifted.

Doppler radar: make sketch on board

iclicker question

Is Doppler radar described by the equations given above for sound Doppler?

(A) yes \hspace{1cm} (B) no

Is “ultra sound” subject to the sound form of the Doppler effect?

(A) yes \hspace{1cm} (B) no
\[
\sin A \pm \sin B = 2 \sin \left( \frac{A \pm B}{2} \right) \cos \left( \frac{A \mp B}{2} \right)
\]

"Standing" wave:
\[
y_{\text{right}}(x, t) + y_{\text{left}}(x, t) = 2y_0 \sin \left( \frac{2\pi x}{\lambda} \right) \cos(2\pi ft)
\]