PHY 113 C General Physics I
11 AM – 12:15 PM TR Olin 101

Plan for Lecture 2:
Chapter 2 – Motion in one dimension
1. Time and Position
2. Time and Velocity
3. Time and Acceleration
4. Special relationships for constant acceleration

Some updates/announcements

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Tutorials starting September 2, 2013
- Mondays 5:00-7:00 PM — John Byrum
- Tuesdays 6:00-7:00 PM — Billy Nicholson; 7:00-9:00 PM — John Byrum
- Wednesdays 5:30-7:30 PM — Junwei Xu
- Thursdays 5:00-7:00 PM — Travis Jones; 7:00-9:00 PM — Junwei Xu.

iclicker exercises:
Webassign Experiences so far
A. Have not tried it
B. Cannot login
C. Can login
D. Have logged in and have completed one or more example problems.

Textbook Experiences
A. Have no textbook (yet)
B. Have complete physical textbook
C. Have electronic version of textbook
D. Textbook is on order
E. Other
Comment on Webassign #1 problem:

7. A pet lamb grows rapidly, with its mass proportional to the cube of its length. When the lamb's length changes by 16%, its mass increases by 16.8 kg. Find the lamb's mass at the end of this process.

Problem solving steps:
1. Visualize problem - labeling variables
2. Determine which basic physical principle(s) apply
3. Write down the appropriate equations using the variables defined in step 1.
4. Check whether you have the correct amount of information to solve the problem (same number of knowns and unknowns).
5. Solve the equations.
6. Check whether your answer makes sense (units, order of magnitude, etc.).

\[ M_i = kL_i^3, \quad M_f = kL_f^3 \]
\[ M_f - M_i = \Delta M = 16.8 \text{ kg} \]
\[ L_f - L_i = p = 0.16 \]

When the dust clears:
\[ \frac{M_f}{\Delta M} = \frac{kL_f^3}{kL_i^3} = \left( \frac{L_f}{L_i} \right)^3 \]
\[ M_f = \Delta M \left( \frac{L_f}{L_i} \right)^3 = 16.8 \text{ kg} \left( \frac{1.16}{1} \right)^3 = 46.8 \text{ kg} \]

**clicker exercise:**
What did you learn from this problem?

A. It is a bad idea to have a pet lamb
B. This was a very hard question
C. I hope this problem will not be on an exam
D. My physics instructor is VERY mean
E. All of the above.
Motion in one-dimension

<table>
<thead>
<tr>
<th>Table 2.1</th>
<th>Position of the Car at Various Times</th>
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<tbody>
<tr>
<td>Time (s)</td>
<td>Position (m)</td>
</tr>
<tr>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>0.10</td>
<td>10</td>
</tr>
<tr>
<td>0.20</td>
<td>20</td>
</tr>
<tr>
<td>0.30</td>
<td>30</td>
</tr>
<tr>
<td>0.40</td>
<td>40</td>
</tr>
<tr>
<td>0.50</td>
<td>50</td>
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</tbody>
</table>

Graphical representation of position (displacement) \( x(t) \)

Comment:
Your text mentions the notion of a "scalar quantity" in contrast to a "vector quantity" which will be introduced in Chapter 3. In most contexts, a scalar quantity – like one-dimensional distance or displacement can be positive or negative.

Another comment:
Your text describes the time rate of change of displacement as "velocity" which, in one-dimension is a signed scalar quantity. In general "speed" is the magnitude of velocity – a positive scalar quantity.
Graphical representation of position (displacement): \( x(t) \)  
\[ \text{time rate of change of displacement} = \text{velocity: } v(t) \]

\[ v(t) = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} \]

\[ v(t) = \frac{x(40) - x(20)}{40s - 20s} = \frac{40m - 40m}{20s} = -4 \text{ m/s} \]

Instantaneous velocity

![Graph showing instantaneous velocity with different points labeled A through F]

Demonstration of tangent line limit

Instantaneous velocity:

\[ v(t) = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} \]

\[ = \frac{dx}{dt} \]
Average velocity versus instantaneous velocity

Instantaneous velocity:
\[ v(t) = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} \]

Average velocity:
\[ \langle v \rangle_A^B = \frac{\int_{t_A}^{t_B} v(t) \, dt}{t_B - t_A} \]

\[ = \frac{x(t_B) - x(t_A)}{t_B - t_A} \]

Average velocity

Previously stated result:
\[ \langle v \rangle_A^B = \frac{x(t_B) - x(t_A)}{t_B - t_A} \]

This result is:
A. Exact
B. Approximate

Why:
\[ \langle v \rangle_A^B = \frac{\sum_i^n s(t_i)}{N} = \frac{\int_s(t) \, dt}{(t_B - t_A)} = \frac{\int_s(t) \, dt}{(t_B - t_A)} = \frac{x(t_B) - x(t_A)}{t_B - t_A} \]

Webassign Example
### Instantaneous velocity using calculus

Suppose:

\[ x(t) = 5 + 6t + 7t^2 - 2t^4 \]

\[ v(t) = \frac{dx}{dt} = 6 + 14t - 8t^3 \]

### Anti-derivative relationship

Constant velocity motion

Suppose: \( \frac{dx}{dt} = v_0 \)

Then: \( x(t) = x_0 + v_0 t \)

Example -- suppose \( x_0 = 0 \) and \( v_0 = 0.3 \text{ m/s} \):

### Velocity

Instantaneous velocity:

\[ v(t) = \frac{dx}{dt} \]

Average velocity:

\[ \langle v \rangle_A^B = \frac{x(t_B) - x(t_A)}{t_B - t_A} \]
Acceleration

Instantaneous acceleration:
\[ a(t) = \frac{dv}{dt} = \frac{d}{dt} \frac{dx}{dt} = \frac{d^2x}{dt^2} \]

Average acceleration:
\[ \langle a \rangle = \frac{v(t_f) - v(t_i)}{t_f - t_i} \]

Rate of acceleration

Instantaneous rate of acceleration:
\[ \frac{da}{dt} = \frac{d}{dt} \frac{dv}{dt} = \frac{d^2v}{dt^2} = \frac{d}{dt} \frac{d}{dt} \frac{dx}{dt} = \frac{d^3x}{dt^3} \]

Instantaneous rate of rate of acceleration:
\[ \frac{d}{dt} \frac{da}{dt} = \frac{d}{dt} \frac{d^2v}{dt^2} = \frac{d^3v}{dt^3} = \frac{d}{dt} \frac{d}{dt} \frac{d}{dt} \frac{dx}{dt} = \frac{d^4x}{dt^4} \]

iclicker exercise

How many derivatives of position are useful for describing motion:
A. 1 \( (dx/dt) \)
B. 2 \( (d^2x/dt^2) \)
C. 3 \( (dx^3/dt^3) \)
D. 4 \( (dx^4/dt^4) \)
E. \( \infty \)

Anti-derivative relationship

Constant acceleration motion

Suppose:
\[ \frac{dv}{dt} = a_0 \quad \text{and} \quad v(0) = v_0, \quad x(0) = x_0 \]

Then:
\[ v(t) = v_0 + a_0 t \]
\[ x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2 \]
Examples

\[ v(t) = v_0 + a_0 t \]
\[ x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2 \]

\[ x_0 = 0 \quad v_0 = 5 \text{ m/s} \quad a_0 = -9.8 \text{ m/s}^2 \]

Summary

\[ v(t) = \frac{dx}{dt} \quad \Leftrightarrow \quad x(t) = \int_{t_0}^{t} v(t') dt' \]
\[ a(t) = \frac{dv}{dt} \quad \Leftrightarrow \quad v(t) = \int_{t_0}^{t} a(t') dt' \]
Another example

\[ x(t) \]

\[ v(t) \]

From webassign:

velocity at \( t = 5 \text{s} \):
\[ v(5) = 2 \text{m/s} \cdot 5 \text{s} = 10 \text{m/s} \]

position at \( t = 5 \text{s} \):
\[ x(5) = \frac{1}{2} 2 \text{m/s} \cdot (5 \text{s})^2 = 25 \text{m} \]

Another example

\[ x(t) = \int_{t_2}^{t} v(t') \, dt' \]
Another example -- continued

\[ a(t) = \frac{dv(t)}{dt} \]

Special relationships between \( t, x, v, a \) for constant \( a \):

**General relationship:**

\[ v(t) = \frac{dx}{dt} \iff x(t) = \int v(t')dt' \]

\[ a(t) = \frac{dv}{dt} \iff v(t) = \int a(t')dt' \]

**Special relationship:**

\[ v(t) = v(0) + at = v_0 + at \]
\[ x(t) = x(0) + v(0)t + \frac{1}{2}at^2 = x_0 + v_0t + \frac{1}{2}at^2 \]

Special relationships between \( t, x, v, a \) for constant \( a \):

**Special relationship:**

\[ v(t) = v_0 + at \]
\[ x(t) = x_0 + v_0t + \frac{1}{2}at^2 \]
\[ t = \frac{v(t) - v_0}{a} \iff x(t) = x_0 + \frac{1}{2a} \left( (v(t))^2 - v_0^2 \right) \]
Special case: constant velocity due to earth's gravity

In this case, the “one” dimension is the vertical direction with “up” chosen as positive:

\[ a = -g = -9.8 \text{ m/s}^2 \]

\[ y(t) = y_0 + v_0t - \frac{1}{2} gt^2 \]

\[ y_0 = 0 \quad v_0 = 20 \text{ m/s} \]

At what time \( t_m \) will the ball hit the ground \( y_m = -50 \text{ m} \)?

Solve: \( y_m = y_0 + v_0 t_m - \frac{1}{2} g t_m^2 \)

\[ \text{quadratic equation} \]

\[ \text{physical solution: } t_m = 5.83 \text{ s} \]

Helpful mathematical relationships
(see Appendix B of your text)

**Quadratic Equations**

The general form of a quadratic equation is

\[ ax^2 + bx + c = 0 \]  \hspace{1cm} (B.7)

where \( x \) is the unknown quantity and \( a, b, \) and \( c \) are numerical factors referred to as coefficients of the equation. This equation has two roots, given by

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]  \hspace{1cm} (B.8)

If \( b^2 > 4ac \), the roots are real.

\[ y_m = y_0 + v_0 t_m - \frac{1}{2} g t_m^2 \]

**Quadratic equation for \( t_m \):**

\[ -\frac{1}{2} g t_m^2 + v_0 t_m + (y_0 - y_m) = 0 \]

\[ t_m = \frac{-v_0 \pm \sqrt{v_0^2 + 2g(y_0 - y_m)}}{-g} \]