PHY 113 C General Physics I
11 AM-12:15 PM MWF Olin 101

Plan for Lecture 8:

Chapter 8 -- Conservation of energy

1. Potential and kinetic energy for conservative forces
2. Energy and non-conservative forces
3. Power

Comments on preparation for next Thursday’s exam
Comments on preparation for next Thursday’s exam – continued

What you should bring to the exam (in addition to your well-rested brain):
- A pencil or pen
- Your calculator
- An 8.5”x11” sheet of paper with your favorite equations (to be turned in together with the exam)

What you should NOT use during the exam
- Electronic devices (cell phone, laptop, etc.)
- Your textbook

Advice:
1. Keep basic concepts and equations at the top of your head.
2. Practice problem solving and math skills
3. Develop an equation sheet that you can consult.
iclicker question
Which of the following best describes your opinion about the equation sheet:

A. I have not prepared my equation sheet yet but hope to do so soon
B. I have a preliminary equation sheet but have not used it for doing homework
C. I have a preliminary equation sheet and have used it for doing homework
D. I am dreaming about preparing the equation sheet, but have not done it yet
E. In my opinion, the equation sheet is not really necessary

Review of energy concepts:
Definition of work: \( W_{i\to f} = \int F \cdot dr \)
Definition of kinetic energy: \( K = \frac{1}{2} mv^2 \)
Work - kinetic energy theorem:
\[
W_{i\to f}^{\text{total}} = \int F_{\text{total}} \cdot dr = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2
\]

Some questions from webassign Assignment #7
In 1990 Walter Arfeuille of Belgium lifted a 281.5-kg object through a distance of 17.1 cm using only his teeth.
(a) How much work was done on the object by Arfeuille in this lift, assuming the object was lifted at constant velocity?

\[
W_{i\to f} = \int F \cdot dr
\]
\[
W_{i\to f} = T(y_f - y_i)
\]

\[
T - mg = 0 \quad \Rightarrow T = mg
\]
Some questions from webassign Assignment #7

In 1990 Walter Arfeuille of Belgium lifted a 281.5-kg object through a distance of 17.1 cm using only his teeth. How much work was done on the object by gravity?

\[ W_{\text{grav}} = \int_{y_i}^{y_f} mg \, dy \quad \Rightarrow \quad W_{\text{grav}} = -mg(y_f - y_i) \]

Some questions from webassign Assignment #7

What is the work done as the particle moves from \( x=0 \) to \( x=3 \) m?

\[ W = \int_{0}^{3} F \cdot dx = \int_{0}^{3} F_x(x) \, dx = \frac{1}{2} (4.5 \text{N})(3 \text{m}) = 6.75 \text{N} \]

Some questions from webassign Assignment #7 -- continued

What is the work done as the particle moves from \( x=8 \) to \( x=10 \) m?

\[ W = \int_{8}^{10} F \cdot dx = \int_{8}^{10} F_x(x) \, dx = \frac{1}{2} (-3 \text{N})(2 \text{m}) = -2.5 \text{N} \]
Some questions from webassign Assignment #7

A 4.34-kg particle is subject to a net force that varies with position as shown in the figure. The particle starts from rest at $x = 0$. What is its speed at $x = 5.00$ m?

\[
W_{i \rightarrow f}^{\text{total}} = \int_{x_i}^{x_f} F_{\text{total}} \cdot dx = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2
\]

\[
W_{i \rightarrow f}^{\text{total}} = \int_{0}^{5} F(x)dx = \frac{1}{2} mv_f^2 - 0
\]

Work and potential energy

Definition of work: $W_{i \rightarrow f} = \int_{r_i}^{r_f} F \cdot dr$

Definition of potential energy:

\[
U(r) = -W_{\text{ref} \rightarrow r} = -\int_{r_{\text{ref}}}^{r} F \cdot dr
\]

Note: It is assumed that $F$ is conservative

Work and potential energy -- continued

Definition of potential energy: \[U(r) = -W_{\text{ref} \rightarrow r} = -\int_{r_{\text{ref}}}^{r} F \cdot dr\]

Example – gravity near the surface of the Earth:

\[
U(y) = -\int_{y_{\text{ref}}}^{y} (-mg)dy = mg(y - y_{\text{ref}})
\]
Work and potential energy for gravity (near Earth’s surface)

\[ U(r) = -W_{\text{ref-far}} = -\int_{r_{\text{ref}}}^r \mathbf{F} \cdot d\mathbf{r} \]

\[ U(y) = -\int_{y_{\text{ref}}}^y (-mg) dy' = mg(y - y_{\text{ref}}) \]

\[ W_{i\rightarrow f} = \int_{r_i}^{r_f} \mathbf{F} \cdot d\mathbf{r} = -W_{\text{ref-far}} = mg(y_f - y_i) \]

Alternatively: \[ W_{i\rightarrow f} = \int_{r_i}^{r_f} \mathbf{F} \cdot d\mathbf{r} = -\left[U(r_f) - U(r_i)\right] \]

Work and potential energy continued

Potential energy: \[ U(r) = -W_{\text{ref-far}} = -\int_{r_{\text{ref}}}^r \mathbf{F} \cdot d\mathbf{r} \]

\[ W_{i\rightarrow f} = \int_{r_i}^{r_f} \mathbf{F} \cdot d\mathbf{r} = -U(r_f) + U(r_i) \]

iclicker question
Why is there a strange “−” sign in the definition of potential energy?
A. Physicists like to be annoying
B. No reason at all
C. There is a somewhat good reason

Work and potential energy continued

Work-kinetic energy theorem:

\[ W_{\text{total}} = \int_{r_i}^{r_f} \mathbf{F}_{\text{total}} \cdot d\mathbf{r} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \]

\[ W_{i\rightarrow f} = \int_{r_i}^{r_f} \mathbf{F} \cdot d\mathbf{r} = -\left[U(r_f) - U(r_i)\right] \]

For the case that the total force acting on the system is conservative, we can use the definition of potential energy with the work-kinetic energy theorem

\[ W_{i\rightarrow f} = -\left[U(r_f) - U(r_i)\right] = K_f - K_i \]

Rearranging: \[ K_f + U_f = K_i + U_i = E \] (constant)
Summary of work, potential energy, kinetic energy relationships

Work - kinetic energy theorem:

\[ W_{\text{total}} = \int \mathbf{F}_{\text{total}} \cdot d\mathbf{r} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \]

\[ W_{\text{total}} = W_{\text{conservative}} + W_{\text{dissipative}} \]

\[ = -(U_f - U_i) + W_{\text{dissipative}} \]

\[ W_{\text{total}} = -(U(r_f) - U(r_i)) + W_{\text{dissipative}} = K_f - K_i \]

Rearranging:

\[ K_f + U_f = K_i + U_i + W_{\text{dissipative}} \]

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Example:

A block, initially at rest at a height \( h \), slides down a frictionless incline. What is its final velocity?

- Initial: \( v_i = 0 \)
- Initial potential energy: \( U(r_i) = mgh \)
- Initial kinetic energy: \( E = mgh \)
- Initial total energy: \( \frac{1}{2} m v_i^2 = mgh \)

- Final: \( v_f > 0 \)
- Final potential energy: \( U(r_f) = 0 \)
- Final kinetic energy: \( E = \frac{1}{2} m v_f^2 \)

\[ v_f = \sqrt{2gh} \]

\[ = \sqrt{2(9.8)(0.5)} \]

\[ = 3.13 \text{ m/s} \]

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Energy diagram:

- Kinetic energy (\( K \))
- Potential energy (\( U \))
- Total energy (\( E \))
Example of spring force:

\[ W_{x-x} = \int F \cdot dx = -\int k x \, dx = -\left(\frac{1}{2} k x^2 - \frac{1}{2} k x^2\right) \]

\[ \Rightarrow U(r_f) = \frac{1}{2} k x_f^2 \quad \text{and} \quad U(r_i) = \frac{1}{2} k x_i^2 \]

Energy diagrams

\[ E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 \]

Note: when \( x = x_{\text{max}} \):

\( v = 0, \quad E = \frac{1}{2} k x_{\text{max}}^2 \)

Note: when \( x = 0 \):

\( U(0) = 0, \quad \frac{1}{2} m v^2 = \frac{1}{2} k x_{\text{max}}^2 \)

Example: Model potential energy function \( U(x) \) representing the attraction of two atoms
Comment on relationship between potential energy and (conservative) force:

Potential energy: \( U(r) = -W_{ref \to r} = - \int_{ref}^{r} F \cdot dr \)

In one dimension: \( U(x) = -W_{ref \to x} = - \int_{ref}^{x} F(x) dx \)

Antiderivative: \( F(x) = -\frac{dU}{dx} \)

Example: Mass sliding on frictionless looping track

*iclicker exercise:* In order for the ball to complete the loop at A, what must the value of h be?
A. \( h=R \)
B. \( h=2R \)
C. \( h>2R \)
D. Not enough information.

Example: Mass sliding on frictionless looping track

\[
E = \frac{1}{2}mv_i^2 + mgh \\
E = \frac{1}{2}mv_f^2 + mg(2R)
\]

Condition for staying on track at A:

\[ m = mg = -mv_i^2 \\
\Rightarrow \frac{1}{2}mv_i^2 = \frac{1}{2}mgR \\
E = mgh = \frac{1}{2}mv_i^2 + mg(2R) = \frac{1}{2}mgR + 2mgR = \frac{5}{2}mgR \\
\Rightarrow h = \frac{5}{2}R \]
Another example; first without friction

Mass \( m_1 (=0.2\text{kg}) \) slides horizontally on a frictionless table and is initially at rest. What is its velocity when it moves a distance \( \Delta x=0.1\text{m} \) (and \( m_2 (=0.3\text{kg}) \) falls \( \Delta y=0.1\text{m} \))?

**iclicker exercise:** What is the relationship of the final velocity of \( m_1 \) and \( m_2 \)?
A. They are equal
B. \( m_2 \) is faster than \( m_1 \).
C. \( m_1 \) is faster than \( m_2 \).

Another example; now with friction

Mass \( m_1 (=0.2\text{kg}) \) slides horizontally on a table with kinetic friction and is initially at rest. What is its velocity when it moves a distance \( \Delta x=0.1\text{m} \) (and \( m_2 (=0.3\text{kg}) \) falls \( \Delta y=0.1\text{m} \))?
\[ K_f + U_f = K_i + U_i + W_{\text{dissipative}} \]
\[ \frac{1}{2}(m_1 + m_2)v_f^2 + m_2gy_f = m_2gy_i + (-\mu_k m_1 g\Delta x) \]
\[ \frac{1}{2}(m_1 + m_2)v_f^2 = m_2g\Delta x + (-\mu_k m_1 g\Delta x) \]
\[ v_f = \sqrt{\frac{2(m_2 g - \mu_k m_1 g)\Delta x}{m_1 + m_2}} \]

**iclicker exercise:**
Assume a mass \( m \) starts at rest at \( A \) and moves on the frictionless surface as shown. At what position is the speed the largest?

A. A  
B. B  
C. C  
D. none of these.
Power exerted by motor:

\[ P = \frac{dW}{dt} = F \cdot v = T \cdot v \]

For \( T = 2 \times 10^4 \) N, \( v = 3 \text{ m/s} \)

\[ P = \left( 2 \times 10^4 \right) \left( 3 \text{ m/s} \right) = 6 \times 10^4 \text{ Watts} \]