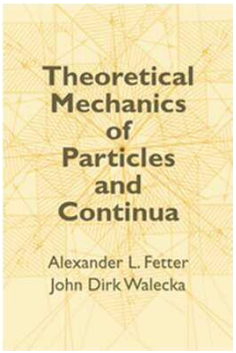
# PHY 711 Classical Mechanics and Mathematical Methods
# 10-10:50 AM MWF Olin 103

## Plan for Lecture 1:

1. Welcome & overview
2. Class structure & announcements
3. Introduction to Maple software
4. Chapter 1 – scattering theory



Textbook:




SIGNIFICANT NAMES IN MECHANICS AND MATHEMATICAL PHYSICS'

Isaac Newton (1642–1727)
Daniel Bernoulli (1700–1782)
Leonhard Euler (1707–1783)
Jean Le Rond d'Alembert (1717–1783)
Joseph Louis Lagrange (1736–1813)
Pierre Simon de Laplace (1749–1827)
Adrien Marie Legendre (1752–1833)
Jean Baptiste Joseph Fourier (1768–1830)
Karl Friedrich Gauss (1777–1855)
Simèon-Denis Poisson (1781–1840)
Friedrich Wilhelm Bessel (1784–1846)
Augustin-Louis Cauchy (1789–1857)
George Green (1793–1841)
Carl Gustav Jacob Jacobi (1804–1851)
William Rowan Hamilton (1805–1865)
Joseph Liouville (1809–1882)
George Gabriel Stokes (1819–1903)
Hermann Ludwig Ferdinand Helmholtz (1821–1894)
Gustav Robert Kirchhoff (1824–1887)
William Thomson (Lord Kelvin) (1824–1907)
Georg Friedrich Bernhard Riemann (1826–1866)
John William Strutt (Lord Rayleigh) (1842–1919)



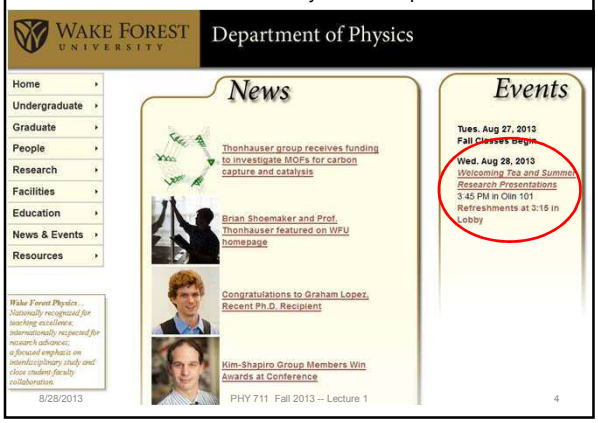
Fall 2013 Schedule
for N. A. W. Holzwarth

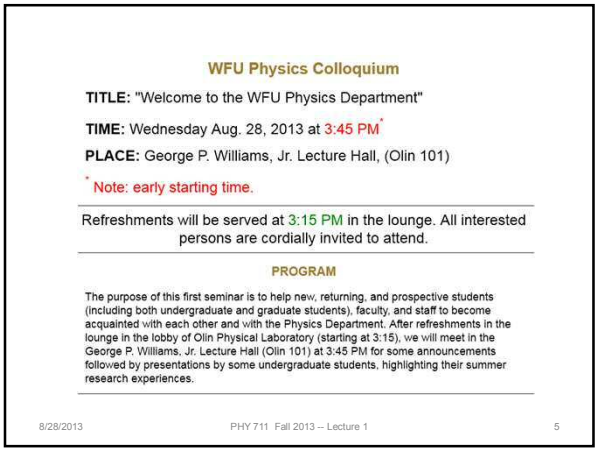
| | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| 8:00-10:00 | Lecture Preparation/ Office Hours | Lecture Preparation/ Office Hours | Lecture Preparation/ Office Hours | Lecture Preparation/ Office Hours | Lecture Preparation/ Office Hours |
| 10:00-11:00 | Classical Mech PHY711 | Office Hours | Classical Mech PHY711 | Office Hours | Classical Mech PHY711 |
| 11:00-12:30 | Office Hours | General Physics I PHY113 | Office Hours | General Physics I PHY113 | Office Hours |
| 12:30-2:00 | Condensed Matter Theory Journal Club | Office Hours | Physics Research | Office Hours | Physics Research |
| 2:00-3:30 | Condensed Matter Monthly Meeting | Physics Research | | Physics Research | |
| 3:30-5:00 | Physics Research | | Physics Colloquium | | |

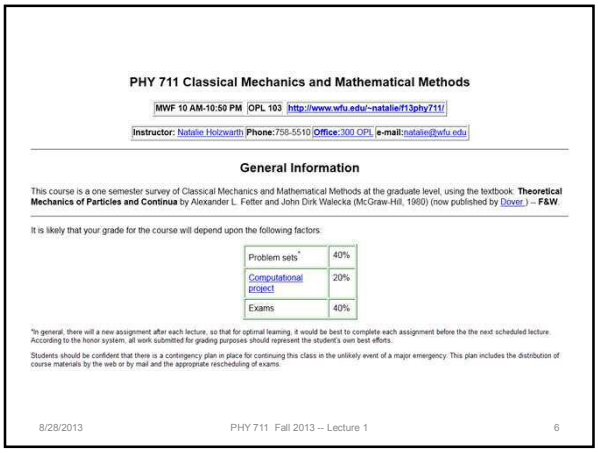
Travel dates:

- Oct. 27 -- Nov. 1, 2013 Electrochemical Society Meeting

## Comment about Physics Colloquia



Introduction to algebraic manipulation software

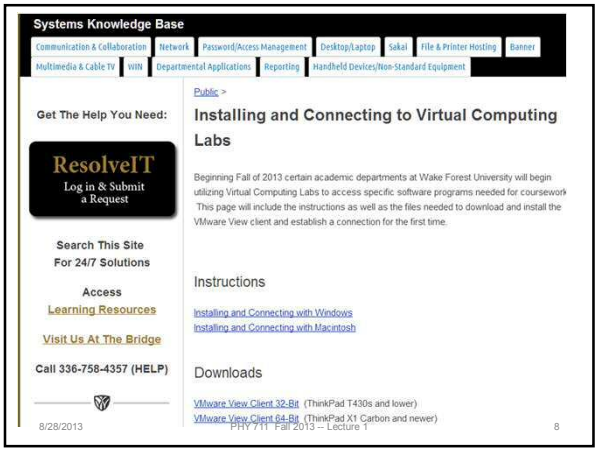
Maple 17

Available from the webpage:
http://help.wfu.edu/public/vcl

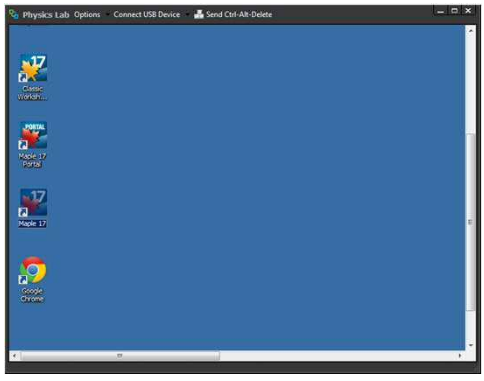
If you have any trouble with this installation and setup, please contact Ching-Wan Yip at yipcw@wfu.edu

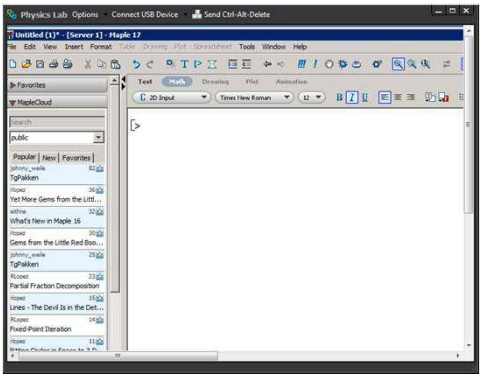
Note: This is a remote server that the university is testing. If you are off campus you will need to use VPN to gain access.









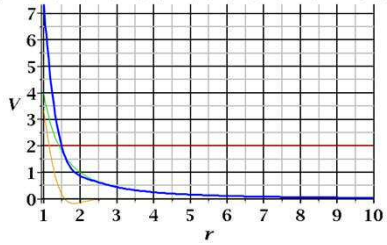



Example plot from Maple

```
> V := r -> 10*sin(2*r)*exp(-r^2);
> VL := r -> 4/r^2;
> plot([V(r), VL(r), V(r)+VL(r), 2], r=1..10, axesfont=["Times","bold",20], labels=["r","V"], labelfont=["Times","bolditalic",20]);
```

$$V := r \mapsto 10\sin(2r)e^{-r^2}$$

$$VL := r \mapsto \frac{4}{r^2}$$






Maple example:
Evaluation of algebraic and integral expressions

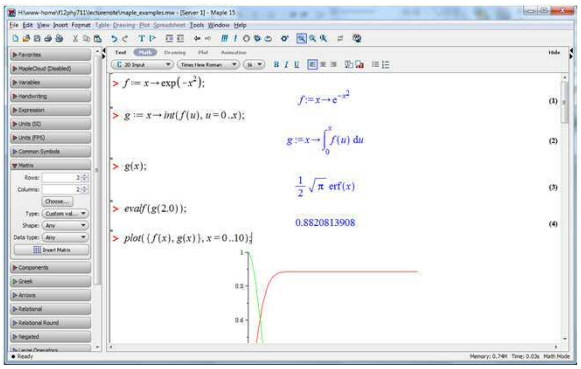
```
> assume(k > 0); assume(b > 0);
> solve(1 - b^2*u^2 - k*u = 0, u);
```

$$\frac{1}{2}\,\frac{-k\sim+\sqrt{4\,b\sim^2+k\sim^2}}{b\sim^2},\ -\frac{1}{2}\,\frac{k\sim+\sqrt{4\,b\sim^2+k\sim^2}}{b\sim^2}$$

```
> int(1/sqrt(1 - b^2*u^2 - k*u), u = 0..(1/b)*(-k/(2*b) + sqrt((k/(2*b))^2 + 1)));
```

$$\frac{1}{2}\,\frac{-2\arcsin\left(\frac{k\sim}{\sqrt{k\sim^2+4\,b\sim^2}}\right)+\pi}{b\sim}$$

http://www.wfu.edu/~natalie/f13phy711/lecturenote/maple_example.mw






Scattering theory:

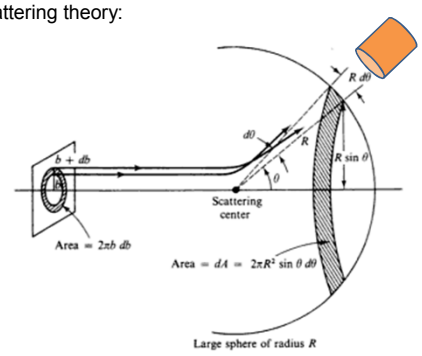


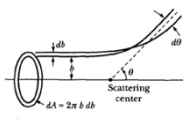

**Figure 5.5** The scattering problem and relation of cross section to impact parameter.



Differential cross section

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\text{Number of detected particles at } \theta \text{ per target particle}}{\text{Number of incident particles per unit area}}$$

$$= \text{Area of incident beam that is scattered into detector at angle } \theta$$



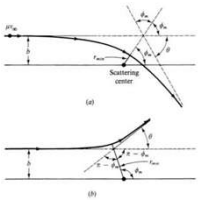

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{2\pi\, b\, db}{2\pi \sin\theta\, d\theta} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Figure from Marion & Thorton, Classical Dynamics

Differential cross section

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{2\pi\, b\, db}{2\pi \sin\theta\, d\theta} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

How can we find $b(\theta)$?



Note that:

$\ell = \mu v_\infty b$

$\mu$ = reduced mass

$v_\infty$ = velocity at large separation



Conservation of energy in the center of mass frame:

$$E = \frac{1}{2}\mu\left(\frac{d\mathbf{r}}{dt}\right)^2 + V(r)$$

$$= \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

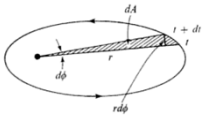
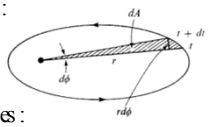


**Figure 3.2** The areal velocity in a central field.



Conservation of angular momentum:

$$\ell = \mu r^2\left(\frac{d\phi}{dt}\right)$$



Transformation of trajectory variables:

$r(t) \Leftrightarrow r(\phi)$

$$\frac{dr}{dt} = \frac{dr}{d\phi}\frac{d\phi}{dt} = \frac{dr}{d\phi}\frac{\ell}{\mu r^2}$$

$$\Rightarrow E = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

$$= \frac{1}{2}\mu\left(\frac{dr}{d\phi}\frac{\ell}{\mu r^2}\right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

$$\Rightarrow E = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

$$= \frac{1}{2}\mu\left(\frac{dr}{d\phi}\frac{\ell}{\mu r^2}\right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

Solving for $r(\phi) \Leftrightarrow \phi(r)$

$$\left(\frac{dr}{d\phi}\right)^2 = \left(\frac{2\mu r^4}{\ell^2}\right)\left(E - \frac{\ell^2}{2\mu r^2} - V(r)\right)$$

$$d\phi = dr\left(\frac{\ell / r^2}{\sqrt{2\mu\left(E - \frac{\ell^2}{2\mu r^2} - V(r)\right)}}\right)$$



$$d\phi = dr\left(\frac{\ell / r^2}{\sqrt{2\mu\left(E - \frac{\ell^2}{2\mu r^2} - V(r)\right)}}\right)$$

Further simplification at large separation:

$$\ell = \mu v_\infty b$$

$$E = \frac{1}{2}\mu v_\infty^2$$

$$\Rightarrow \ell = \sqrt{2\mu E}\, b$$

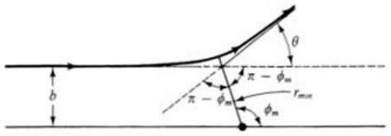


When the dust clears :

$$d\phi = dr\left(\frac{\ell / r^2}{\sqrt{2\mu\left(E - \frac{\ell^2}{2\mu r^2} - V(r)\right)}}\right)$$

$$d\phi = dr\left(\frac{b / r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}}\right)$$

$$\Rightarrow \phi(b, E)$$



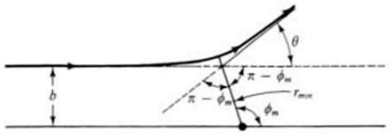

$$d\phi = dr\left(\frac{b/r^2}{\sqrt{1-\frac{b^2}{r^2}-\frac{V(r)}{E}}}\right)$$

$$\phi_m(b,E) = \int_{\infty}^{r_{\min}}\left(\frac{b/r^2}{\sqrt{1-\frac{b^2}{r^2}-\frac{V(r)}{E}}}\right)dr = b\int_0^{1/r_{\min}}\frac{du}{\sqrt{1-b^2u^2-V(1/u)/E}}$$



Evaluation of scattering expression:

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{2\pi\, b\, db}{2\pi \sin\theta\, d\theta} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$




$$\theta + 2(\pi - \phi_m) = \pi$$