PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 35:

1. Chapter 12: Effects of viscosity in fluid motion
2. Class evaluation

24	Wed, 10/23/2013	Chap. 5	Rigid rotations	#21
25	Fri, 10/25/2013	Chap. 5	Rigid rotations	
	Mon, 10/28/2013	No class	Take-home exam	
	Wed, 10/30/2013	No class	Take-home exam	
	Fri, 11/01/2013	No class	Take-home exam	
26	Mon, 11/04/2013	Chap. 8	Oscillations in two-dimensional membranes	Take-home exam due
27	Wed, 11/06/2013	Chap. 9	Physics of fluids	#22
28	Fri, 11/08/2013	Chap. 9	Physics of fluids	#23
29	Mon, 11/11/2013	Chap. 9	Sound Waves	#24
30	Wed, 11/13/2013	Chap. 9	Sound Waves	#25
31	Fri, 11/15/2013	Chap. 9	Non linear effects in Sound	#26
32	Mon, 11/18/2013	Chap. 10	Surface waves	
33	Wed, 11/20/2013	Chap. 10	Surface waves	
34	Fri, 11/22/2013	Chap. 11	Heat conduction	
35	Mon, 11/25/2013	Chap. 12	Viscous fluids	
	Wed, 11/27/2013		*Thanksgiving Holiday*	
	Fri, 11/29/2013		*Thanksgiving Holiday*	
36	Mon, 12/02/2013		Student presentations I	
37	Wed, 12/04/2013		Student presentations II	
38	Fri, 12/06/2013		Student presentations III	
	Mon, 12/09/2013		Begin Take-home final	

preparation for presentations

Monday -- 12/2/2013

Time	Name	Title
10-10:20	Sam Flynn	Scattering Theory: Classical vs. Quantum
10:20-10:40	Hyunsu Lee	
10:40-11	Drew Onken	Foucault Pendulum

Wednesday -- 12/4/2013

Time	Name	Title
10-10:20	Calvin Arter	
10:20-10:40	Junwei Xu	
10:40-11	Ahmad Al-qawasmeh	Greens function

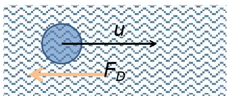
Friday -- 12/6/2013

Time	Name	Title
10-10:20	Ryan Melvin	Scattering of plane waves by isotropic spheres and cylinders in fluids
10:20-10:40	Bradley Hicks	
10:40-11	Wenxiao Huang	inverted pendulum

Brief introduction to viscous effects in incompressible fluids

Stokes' analysis of viscous drag on a sphere of radius R moving at speed u in medium with viscosity η :

$$F_D = -\eta(6\pi R u)$$



Plan:
1. Consider the general effects of viscosity on fluid equations
2. Consider the solution to the linearized equations for the case of steady-state flow of a sphere of radius R
3. Infer the drag force needed to maintain the steady-state flow

Stokes' analysis of viscous drag on a sphere of radius R moving at speed u in medium with viscosity η :

$$F_D = -\eta(6\pi R u)$$

Effects of drag force on motion of particle of mass m with constant force F :

$$F - 6\pi R\eta u = m\frac{du}{dt} \qquad \text{with } u(0) = 0$$

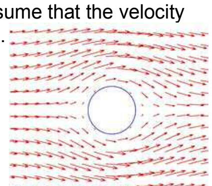
$$\Rightarrow u(t) = \frac{F}{6\pi R\eta}\left(1 - e^{-\frac{6\pi R\eta}{m}t}\right)$$

PHY 711 -- Assignment #22 Nov. 06, 2013
Continue reading Chapter 9 in **Fetter & Walecka**.

Determine the form of the velocity potential for an incompressible fluid representing uniform velocity in the **z** direction at large distances from a spherical obstruction of radius *a*. Find the form of the velocity potential and the velocity field for all *r* > *a*. Assume that the velocity in the radial direction is 0 for *r* = *a* and assume that the velocity is uniform in the azimuthal direction.

$$\nabla^2\Phi = 0$$

$$\Phi(r,\theta) = \left(-v_0 r + \frac{v_0 R^3}{2r^2}\right)\cos\theta$$



Newton - Euler equation for incompressible fluid, modified by viscous contribution (Navier - Stokes equation) :

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}=\mathbf{f}_{applied}-\frac{\nabla p}{\rho}+\frac{\eta}{\rho}\nabla^2\mathbf{v}$$

Continuity equation : $\nabla\cdot\mathbf{v}=0$ Irrotational flow : $\nabla\times\mathbf{v}=0$

Assume steady state : $\Rightarrow \frac{\partial \mathbf{v}}{\partial t}=0$

Assume non - linear effects small

Initially set $\mathbf{f}_{applied}=0;\qquad \nabla p=\eta\nabla^2\mathbf{v}$

Assume $\mathbf{v}=\nabla\times(\nabla\times f(r)\mathbf{u})+\mathbf{u}$

where $f(r)\xrightarrow[r\to\infty]{}0$

Digression

Some comment on assumption : $\mathbf{v}=\nabla\times(\nabla\times f(r)\mathbf{u})+\mathbf{u}$

$\nabla\times(\nabla\times\mathbf{A})=\nabla(\nabla\cdot\mathbf{A})-\nabla^2\mathbf{A}$

Here $\mathbf{A}=\nabla\times f(r)\mathbf{u}$

$\nabla\times\mathbf{v}=0=\nabla\times(\nabla\times\mathbf{A})=-\nabla^2\mathbf{A}$

Also note : $\nabla p=\eta\nabla^2\mathbf{v}$

$\Rightarrow \nabla\times\nabla p=\nabla\times\eta\nabla^2\mathbf{v}\qquad$ or $\quad\nabla^2(\nabla\times\mathbf{v})=0$

$\mathbf{v}=\nabla\times(\nabla\times f(r)\mathbf{u})+\mathbf{u}$

$\mathbf{u}=u\hat{\mathbf{z}}$

$\nabla\times(\nabla\times f(r)\hat{\mathbf{z}})=\nabla(\nabla\cdot f(r)\hat{\mathbf{z}})-\nabla^2 f(r)\hat{\mathbf{z}}$

$\nabla\times\mathbf{v}=0\qquad\Rightarrow\nabla^2(\nabla\times\mathbf{v})=0$

$\nabla^4(\nabla\times f(r)\hat{\mathbf{z}})=0\quad\Rightarrow\nabla^4(\nabla f(r)\times\hat{\mathbf{z}})=0\quad\Rightarrow\nabla^4 f(r)=0$

$$f(r)=C_1r^2+C_2r+C_3+\frac{C_4}{r}$$

$$v_r=u\cos\theta\left(1-\frac{2}{r}\frac{df}{dr}\right)=u\cos\theta\left(1-4C_1-\frac{2C_2}{r}-\frac{2C_4}{r^3}\right)$$

$$v_\theta=-u\sin\theta\left(1-\frac{d^2f}{dr^2}-\frac{1}{r}\frac{df}{dr}\right)=-u\sin\theta\left(1-4C_1-\frac{C_2}{r}+\frac{C_4}{r^3}\right)$$

$$v_r = u\cos\theta\left(1 - \frac{2}{r}\frac{df}{dr}\right) = u\cos\theta\left(1 - 4C_1 - \frac{2C_2}{r} - \frac{2C_4}{r^3}\right)$$

$$v_r = u\cos\theta\left(1 - \frac{2}{r}\frac{df}{dr}\right) = -u\sin\theta\left(1 - 4C_1 - \frac{C_2}{r} + \frac{C_4}{r^3}\right)$$

To satisfy $\mathbf{v}(r \to \infty) = \mathbf{u}$: $\Rightarrow C_1 = 0$

To satisfy $\mathbf{v}(R) = 0$ solve for C_2, C_4

$$v_r = u\cos\theta\left(1 - \frac{3R}{2r} + \frac{R^3}{2r^3}\right)$$

$$v_\theta = -u\sin\theta\left(1 - \frac{3R}{4r} - \frac{R^3}{4r^3}\right)$$

$$v_r = u\cos\theta\left(1 - \frac{3R}{2r} + \frac{R^3}{2r^3}\right)$$

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Determining pressure:

$$\nabla p = \eta\nabla^2\mathbf{v} = -\eta\nabla\left(u\cos\theta\left(\frac{3R}{2r^2}\right)\right)$$

$$\Rightarrow p = p_0 - \eta u\cos\theta\left(\frac{3R}{2r^2}\right)$$

$$p = p_0 - \eta u\cos\theta\left(\frac{3R}{2r^2}\right)$$

Corresponds to:

$$F_D = \left(p(R) - p_0\right)\frac{4\pi R^2}{\cos\theta} = -\eta u(6\pi R)$$