11/17/2014

PHY 711 – Homework # 22

Start reading Chap. 11 in **Fetter and Walecka**.
In class, we showed how the equations for a non-linear wave on the surface of an incompressible fluid could be approximated by the famous Kortewegde Vries equation

$$\left(1 - \frac{gh}{c^2}\right)\eta(u) - \frac{h^2}{3}\frac{d^2\eta(u)}{du^2} - \frac{3}{2h}\left(\eta(u)\right)^2 = 0.$$

Here g is the gravitational acceleration, h is the average height of the water, and c is the constant velocity of the wave to be determined and in terms of position in time $u \equiv x - ct$.

1. For an assumed amplitude η_0, show that the following function satisfies the Kortewebde Vries equation:

$$\eta(u) = \frac{\eta_0}{\cosh^2\left(\sqrt{\frac{3\eta_0}{h}}\frac{u}{2h}\right)}.$$

   The consistent value of the wave speed is given by

$$c^2 = \frac{gh}{1 - \eta_0/h}.$$

2. In class, we also showed that to the lowest order approximation, the velocity potential function was related to the wave form function $\eta(u)$ is given by

$$\frac{d\chi(u)}{du} = -\frac{g}{c}\eta(u).$$

   From this relationship, find $\chi(u)$.