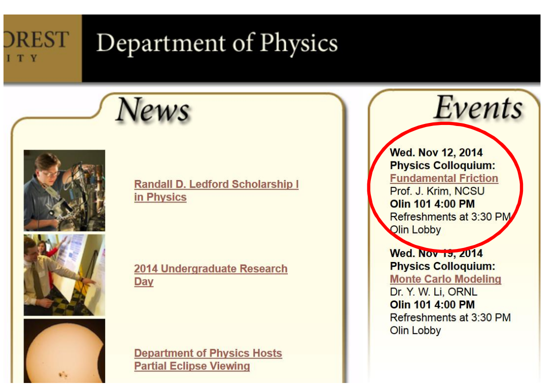
PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 33:

Chapter 10 in F & W: Surface waves

1. **Water waves in a channel**
2. **Wave-like solutions; wave speed**

16	Wed, 10/01/2014	Chap. 4	Normal modes of motion	Continue Take-Home
17	Fri, 10/03/2014	Chap. 4	Normal modes of motion	Take-Home due
18	Mon, 10/06/2014	Chap. 7	Wave motion	#12
19	Wed, 10/08/2014	Chap. 7	Sturm-Liouville Equations	#13
20	Fri, 10/10/2014	Chap. 7	Sturm-Liouville Equations	#13
21	Mon, 10/13/2014	Chap. 7	Sturm-Liouville Equations	#14
22	Wed, 10/15/2014	Appendix A	Contour integration methods	#15
	Fri, 10/17/2014		Fall break -- no class	
23	Mon, 10/20/2014	Appendix A	Fourier transforms	#16
24	Wed, 10/22/2014	Chap. 5	Motion of Rigid Bodies	#17
25	Fri, 10/24/2014	Chap. 5	Motion of Rigid Bodies	#18
26	Mon, 10/27/2014	Chap. 5	Symmetric top in gravitational field	#18
27	Wed, 10/29/2014	Chap. 8	Vibrations of membranes	#19
28	Fri, 10/31/2014	Chap. 9	Physics of fluids	#20
29	Mon, 11/03/2014	Chap. 9	Physics of fluids	#21
30	Wed, 11/05/2014	Chap. 9	Sound waves	
31	Fri, 11/07/2014	Chap. 9	Sound waves	Begin Take-Home
32	Mon, 11/10/2014	Chap. 9	Non-linear effects	Continue Take-Home
33	Wed, 11/12/2014	Chap. 10	Surface waves in fluids	Continue Take-Home
34	Fri, 11/14/2014	Chap. 10	Surface waves in fluids	Continue Take-Home
35	Mon, 11/17/2014	Chap. 10	Surface waves in fluids	Take-Home due
36	Wed, 11/19/2014			



TITLE: Friction, magnetism and superconductivity: Are they interrelated?

SPEAKER: Professor Jacqueline Krim,

Physics Department,
North Carolina State University

TIME: Wednesday November 12, 2014 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

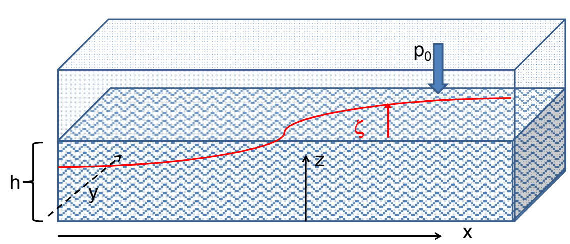
Studies of the fundamental origins of friction have undergone rapid progress in recent years with the development of new experimental and computational techniques for measuring and simulating friction at atomic length and time scales. [1] The increased interest has sparked a variety of discussions and debates concerning the nature of the atomic-scale and quantum mechanisms that dominate the dissipative process by which mechanical energy is transformed into heat. Measurements of the sliding friction of physisorbed monolayers and bilayers can provide information on the relative contributions of electronic to phononic dissipative mechanisms, since phonon dissipation is present at all film coverages, while electronic dissipation primarily impacts the monolayer. Lead is of particular interest on account the observation of changes in friction levels recently reported for helium, nitrogen and water films sliding on lead substrates that were driven in and out of the superconducting state by means of an applied magnetic field. AFM measurements of sliding friction of magnetic tips on YBCO films meanwhile reveal major drops in friction in advance of the superconducting transition itself. The experiments will be discussed within the context of current theories of electronic mechanisms for friction.

[1] J. Krim, *Advances in Physics*, **61**, (2012) pp155-323

Physics of incompressible fluids and their surfaces

Reference: Chapter 10 of Fetter and Walecka

Consider a container of water with average height h and surface h+ζ(x,y,t); (h ←→ z_0 on some of the slides)



Euler's equation for incompressible fluid:

$$\frac{d\mathbf{v}}{dt} = f_{applied} - \frac{\nabla p}{\rho} = -g\hat{\mathbf{z}} - \frac{\nabla p}{\rho}$$

Assume that $v_z << v_x, v_y$ $\quad \Rightarrow -g - \frac{1}{\rho}\frac{\partial p}{\partial z} \approx 0$

$$\Rightarrow p(x,y,z,t) = p_0 + \rho g\left(\zeta(x,y,t) + h - z\right)$$

Horizontal fluid motions:

$$\frac{dv_x}{dt} \approx \frac{\partial v_x}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} = -g\frac{\partial \zeta}{\partial x}$$

$$\frac{dv_y}{dt} \approx \frac{\partial v_y}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} = -g\frac{\partial \zeta}{\partial y}$$

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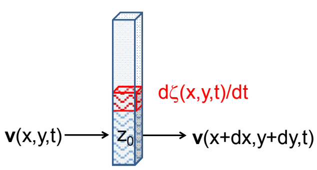

Continuity condition for flow of incompressible fluid:

$$\frac{\partial \zeta}{\partial t} + z_0 \nabla \cdot \mathbf{v} = 0$$

From horizontal flow relations: $\frac{\partial \mathbf{v}}{\partial t} = -g\nabla\zeta$

Equation for surface function: $\frac{\partial^2 \zeta}{\partial t^2} - g z_0 \nabla^2 \zeta = 0$

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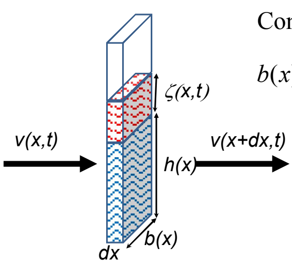
Digression:

The form of the continuity equation given on previous (and subsequent) sides assumes that the transverse cross section of the channel is either very large or at least uniform. Your text also considers the case where there is a channel of varying width:



Continuity condition:

$$b(x)\frac{\partial \zeta}{\partial t} = -\frac{\partial}{\partial x}\left(h(x)b(x)v(x,t)\right)$$

For constant b and h:

$$\frac{\partial \zeta}{\partial t} = -h\frac{\partial}{\partial x}\left(v(x,t)\right)$$

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For uniform channel:

Surface wave equation:

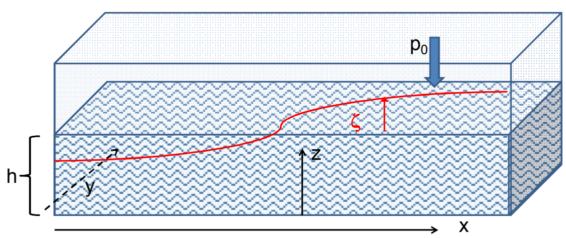
$$\frac{\partial^2 \zeta}{\partial t^2} - c^2 \nabla^2 \zeta = 0 \qquad c^2 = gh$$

More complete analysis finds:

$$c^2 = \frac{g}{k}\tanh(kh) \qquad \text{where } k = \frac{2\pi}{\lambda}$$

More details: -- recall setup --

Consider a container of water with average height h and surface h+ζ(x,y,t)



Equations describing fluid itself (without boundaries)

Euler's equation for incompressible fluid:

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = \frac{\partial \mathbf{v}}{\partial t} + \nabla\left(\tfrac{1}{2}v^2\right) + \mathbf{v}\times\left(\nabla\times\mathbf{v}\right) = -\nabla U - \frac{\nabla p}{\rho}$$

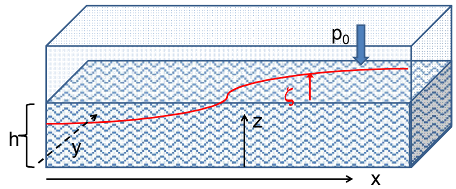
Assume that $\nabla\times\mathbf{v} = 0$ (irrotational flow) $\Rightarrow \mathbf{v} = -\nabla\Phi$

$$\Rightarrow \nabla\left(-\frac{\partial\Phi}{\partial t} + \tfrac{1}{2}v^2 + U + \frac{p}{\rho}\right) = 0$$

$$\Rightarrow -\frac{\partial\Phi}{\partial t} + \tfrac{1}{2}v^2 + U + \frac{p}{\rho} = \text{constant (within the fluid)}$$

For the same system, the continuity condition becomes

$$\nabla\cdot\mathbf{v} = -\nabla^2\Phi = 0$$



Within fluid: $0 \le z \le h+\zeta$

$$-\frac{\partial \Phi}{\partial t} + \tfrac{1}{2}v^2 + g(z-h) = \text{constant}$$ (We have absorbed p_0 in "constant")

$$-\nabla^2\Phi = 0$$

At surface: $z = h+\zeta$ with $\zeta = \zeta(x,y,t)$

$$\frac{d\zeta}{dt} = \frac{\partial \zeta}{\partial t} + v_x\frac{\partial \zeta}{\partial x} + v_y\frac{\partial \zeta}{\partial y} \quad \text{where } v_{x,y} = v_{x,y}(x,y,h+\zeta,t)$$

Full equations:

Within fluid: $0 \le z \le h+\zeta$

$$-\frac{\partial \Phi}{\partial t} + \tfrac{1}{2}v^2 + g(z-h) = \text{constant}$$ (We have absorbed p_0 in "constant")

$$-\nabla^2\Phi = 0$$

At surface: $z = h+\zeta$ with $\zeta = \zeta(x,y,t)$

$$\frac{d\zeta}{dt} = \frac{\partial \zeta}{\partial t} + v_x\frac{\partial \zeta}{\partial x} + v_y\frac{\partial \zeta}{\partial y} \quad \text{where } v_{x,y} = v_{x,y}(x,y,h+\zeta,t)$$

Linearized equations:

$$\text{For } 0 \le z \le h+\zeta: \quad -\frac{\partial \Phi}{\partial t} + g(z-h) = 0 \qquad -\nabla^2\Phi = 0$$

$$\text{At surface}: \quad z = h+\zeta \qquad \frac{d\zeta}{dt} = \frac{\partial \zeta}{\partial t} = v_z(x,y,h+\zeta,t)$$

$$-\frac{\partial \Phi(x,y,h+\zeta,t)}{\partial t} + g\zeta = 0$$

For simplicity, keep only linear terms and assume that horizontal variation is only along *x*:

$$\text{For } 0 \le z \le h+\zeta: \quad \nabla^2\Phi = \left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2}\right)\Phi(x,z,t) = 0$$

Consider and periodic waveform: $\Phi(x,z,t) = Z(z)\cos\big(k(x-ct)\big)$

$$\Rightarrow \left(\frac{d^2}{dz^2} - k^2\right)Z(z) = 0$$

Boundary condition at bottom of tank: $v_z(x,0,t) = 0$

$$\Rightarrow \frac{dZ}{dz}(0) = 0 \qquad Z(z) = A\cosh(kz)$$

For simplicity, keep only linear terms and assume that horizontal variation is only along *x* – continued:

$$\text{At surface: } \quad z = h + \zeta \qquad \frac{\partial \zeta}{\partial t} = v_z(x, h+\zeta, t) = -\frac{\partial \Phi(x, h+\zeta, t)}{\partial z}$$

$$-\frac{\partial \Phi(x, h+\zeta, t)}{\partial t} + g\zeta = 0$$

$$-\frac{\partial^2 \Phi(x, h+\zeta, t)}{\partial t^2} + g\frac{\partial \zeta}{\partial t} = -\frac{\partial^2 \Phi(x, h+\zeta, t)}{\partial t^2} - g\frac{\partial \Phi(x, h+\zeta, t)}{\partial z} = 0$$

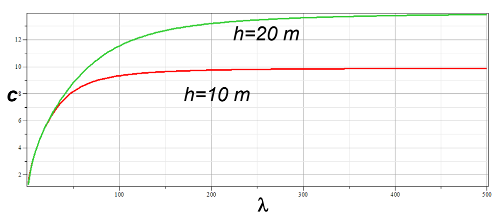
$$\text{For} \quad \Phi(x, (h+\zeta), t) = A\cosh(k(h+\zeta))\cos(k(x-ct))$$

$$A\cosh(k(h+\zeta))\cos(k(x-ct))\left(k^2c^2 - gk\frac{\sinh(k(h+\zeta))}{\cosh(k(h+\zeta))}\right) = 0$$

For simplicity, keep only linear terms and assume that horizontal variation is only along *x* – continued:

$$c^2 = \frac{g}{k}\frac{\sinh(k(h+\zeta))}{\cosh(k(h+\zeta))} = \frac{g}{k}\tanh(k(h+\zeta))$$

$$\text{Assuming } \zeta << h: \qquad c^2 = \frac{g}{k}\tanh(kh)$$



For simplicity, keep only linear terms and assume that horizontal variation is only along *x* – continued:

$$c^2 \approx \frac{g}{k}\tanh(kh) \qquad \text{For } \lambda >> h, \; c^2 \approx gh$$

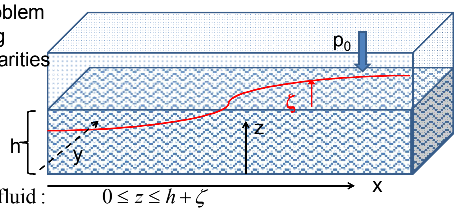
$$\Phi(x, z, t) = A\cosh(kz)\cos(k(x-ct))$$

$$\zeta(x,t) = \frac{1}{g}\frac{\partial \Phi(x, h+\zeta, t)}{\partial t} \approx \frac{kc}{g}A\cosh(kh)\sin(k(x-ct))$$

Note that for $\lambda >> h$, $\quad c^2 \approx gh$

(solutions are consistent with previous analysis)

General problem including non-linearities



Within fluid: $0 \le z \le h + \zeta$

$$-\frac{\partial \Phi}{\partial t} + \tfrac{1}{2} v^2 + g(z-h) = \text{constant}$$ (We have absorbed p_0 in our constant.)

$$-\nabla^2 \Phi = 0$$

At surface: $z = h + \zeta$ with $\zeta = \zeta(x, y, t)$

$$\frac{d\zeta}{dt} = \frac{\partial \zeta}{\partial t} + v_x \frac{\partial \zeta}{\partial x} + v_y \frac{\partial \zeta}{\partial y}$$ where $v_{x,y} = v_{x,y}(x, y, h+\zeta, t)$