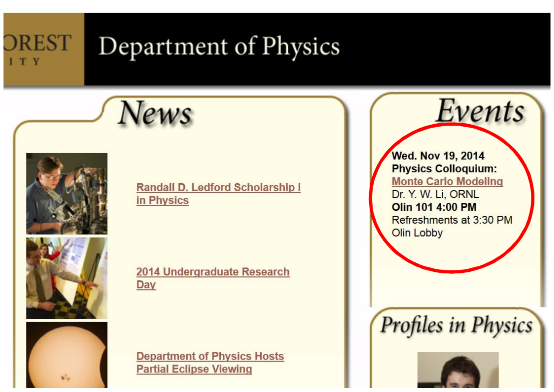
PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 36:

1. **Chapter 12: Effects of viscosity in fluid motion**
2. **Stokes' viscosity relation**

24	Wed, 10/22/2014	Chap. 5	Motion of Rigid Bodies	#17
25	Fri, 10/24/2014	Chap. 5	Motion of Rigid Bodies	#18
26	Mon, 10/27/2014	Chap. 5	Symmetric top in gravitational field	#18
27	Wed, 10/29/2014	Chap. 8	Vibrations of membranes	#19
28	Fri, 10/31/2014	Chap. 9	Physics of fluids	#20
29	Mon, 11/03/2014	Chap. 9	Physics of fluids	#21
30	Wed, 11/05/2014	Chap. 9	Sound waves	
31	Fri, 11/07/2014	Chap. 9	Sound waves	Begin Take-Home
32	Mon, 11/10/2014	Chap. 9	Non-linear effects	Continue Take-Home
33	Wed, 11/12/2014	Chap. 10	Surface waves in fluids	Continue Take-Home
34	Fri, 11/14/2014	Chap. 10	Surface waves in fluids	Continue Take-Home
35	Mon, 11/17/2014	Chap. 11	Heat Conduction	Take-Home due #22
36	Wed, 11/19/2014	Chap. 12	Viscosity	#23
37	Fri, 11/21/2014			
38	Mon, 11/24/2014			
	Wed, 11/26/2014		*Thanksgiving Holiday*	
	Fri, 11/28/2014		*Thanksgiving Holiday*	
39	Mon, 12/01/2014			
	Wed, 12/03/2014		Student presentations I	
	Fri, 12/05/2014		Student presentations II	
	Mon, 12/08/2014		Begin Take-home final	



Signup for PHY 711 Presentations

Wed. December 3, 2014

Time	Name	Title
10:00 AM	Lauren Nelson	
10:25 AM		

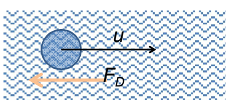
Fri. December 5, 2014

Time	Name	Title
10:00 AM		
10:25 AM		

Brief introduction to viscous effects in incompressible fluids

Stokes' analysis of viscous drag on a sphere of radius R moving at speed u in medium with viscosity η:

$$F_D = -\eta(6\pi R u)$$



Plan:
1. Consider the general effects of viscosity on fluid equations
2. Consider the solution to the linearized equations for the case of steady-state flow of a sphere of radius R
3. Infer the drag force needed to maintain the steady-state flow

Newton-Euler equation for incompressible fluid, modified by viscous contribution (Navier-Stokes equation):

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \mathbf{f}_{applied} - \frac{\nabla p}{\rho} + \underbrace{\frac{\eta}{\rho}}_{\upsilon}\nabla^2\mathbf{v}$$

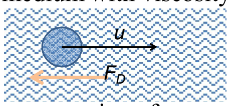
υ Kinematic viscosity

Typical kinematic viscosities at 20° C and 1 atm:

Fluid	ν (m^2/s)
Water	1.00×10^{-6}
Air	14.9×10^{-6}
Ethyl alcohol	1.52×10^{-6}
Glycerine	1183×10^{-6}

Stokes' analysis of viscous drag on a sphere of radius R moving at speed u in medium with viscosity η:

$$F_D = -\eta(6\pi R u)$$



Effects of drag force on motion of particle of mass m with constant force F:

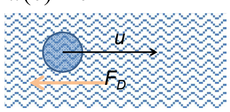
$$F - 6\pi R\eta u = m\frac{du}{dt} \qquad \text{with } u(0) = 0$$

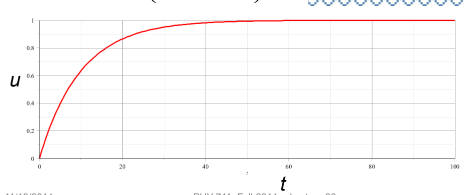
$$\Rightarrow u(t) = \frac{F}{6\pi R\eta}\left(1 - e^{-\frac{6\pi R\eta}{m}t}\right)$$

Effects of drag force on motion of particle of mass m with constant force F:

$$F - 6\pi R\eta u = m\frac{du}{dt} \qquad \text{with } u(0) = 0$$

$$\Rightarrow u(t) = \frac{F}{6\pi R\eta}\left(1 - e^{-\frac{6\pi R\eta}{m}t}\right)$$



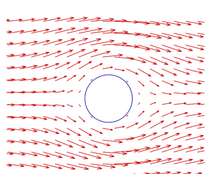


Recall: PHY 711 -- Assignment #21 Nov. 03, 2014

Determine the form of the velocity potential for an incompressible fluid representing uniform velocity in the **z** direction at large distances from a spherical obstruction of radius *a*. Find the form of the velocity potential and the velocity field for all $r > a$. Assume that the velocity in the radial direction is 0 for $r = a$ and assume that the velocity is uniform in the azimuthal direction.

$$\nabla^2 \Phi = 0$$

$$\Phi(r,\theta) = \left(-v_0 r + \frac{v_0 R^3}{2r^2}\right)\cos\theta$$



Newton - Euler equation for incompressible fluid,
modified by viscous contribution (Navier - Stokes equation):

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \mathbf{f}_{applied} - \frac{\nabla p}{\rho} + \frac{\eta}{\rho}\nabla^2\mathbf{v}$$

Continuity equation: $\nabla\cdot\mathbf{v} = 0$ Irrotational flow: $\nabla\times\mathbf{v} = 0$

Assume steady state: $\Rightarrow \frac{\partial \mathbf{v}}{\partial t} = 0$

Assume non - linear effects small

Initially set $\mathbf{f}_{applied} = 0$; $\quad \nabla p = \eta\nabla^2\mathbf{v}$

Assume $\mathbf{v} = \nabla\times(\nabla\times f(r)\mathbf{u}) + \mathbf{u}$

where $f(r) \xrightarrow[r\to\infty]{} 0$

Digression

Some comment on assumption: $\mathbf{v} = \nabla\times(\nabla\times f(r)\mathbf{u}) + \mathbf{u}$

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$

Here $\mathbf{A} = \nabla\times f(r)\mathbf{u}$

$$\nabla\times\mathbf{v} = 0 = \nabla\times(\nabla\times\mathbf{A}) = -\nabla^2\mathbf{A}$$

Also note: $\nabla p = \eta\nabla^2\mathbf{v}$

$$\Rightarrow \nabla\times\nabla p = \nabla\times\eta\nabla^2\mathbf{v} \qquad \text{or} \qquad \nabla^2(\nabla\times\mathbf{v}) = 0$$

$$\mathbf{v} = \nabla\times(\nabla\times f(r)\mathbf{u}) + \mathbf{u}$$

$$\mathbf{u} = u\hat{\mathbf{z}}$$

$$\nabla\times(\nabla\times f(r)\hat{\mathbf{z}}) = \nabla(\nabla\cdot f(r)\hat{\mathbf{z}}) - \nabla^2 f(r)\hat{\mathbf{z}}$$

$$\nabla\times\mathbf{v} = 0 \qquad \Rightarrow \nabla^2(\nabla\times\mathbf{v}) = 0$$

$$\nabla^4(\nabla\times f(r)\hat{\mathbf{z}}) = 0 \quad \Rightarrow \nabla^4(\nabla f(r)\times\hat{\mathbf{z}}) = 0 \quad \Rightarrow \nabla^4 f(r) = 0$$

$$f(r) = C_1 r^2 + C_2 r + C_3 + \frac{C_4}{r}$$

$$v_r = u\cos\theta\left(1 - \frac{2}{r}\frac{df}{dr}\right) = u\cos\theta\left(1 - 4C_1 - \frac{2C_2}{r} - \frac{2C_4}{r^3}\right)$$

$$v_\theta = -u\sin\theta\left(1 - \frac{d^2 f}{dr^2} - \frac{1}{r}\frac{df}{dr}\right) = -u\sin\theta\left(1 - 4C_1 - \frac{C_2}{r} + \frac{C_4}{r^3}\right)$$

$$v_r = u\cos\theta\left(1-\frac{2}{r}\frac{df}{dr}\right) = u\cos\theta\left(1-4C_1-\frac{2C_2}{r}-\frac{2C_4}{r^3}\right)$$

$$v_r = u\cos\theta\left(1-\frac{2}{r}\frac{df}{dr}\right) = -u\sin\theta\left(1-4C_1-\frac{C_2}{r}+\frac{C_4}{r^3}\right)$$

To satisfy $\mathbf{v}(r\rightarrow\infty)=\mathbf{u}$: $\quad \Rightarrow C_1 = 0$

To satisfy $\mathbf{v}(R)=0$ solve for C_2, C_4

$$v_r = u\cos\theta\left(1-\frac{3R}{2r}+\frac{R^3}{2r^3}\right)$$

$$v_\theta = -u\sin\theta\left(1-\frac{3R}{4r}-\frac{R^3}{4r^3}\right)$$

$$v_r = u\cos\theta\left(1-\frac{3R}{2r}+\frac{R^3}{2r^3}\right)$$

$$v_\theta = -u\sin\theta\left(1-\frac{3R}{4r}-\frac{R^3}{4r^3}\right)$$

Determining pressure:

$$\nabla p = \eta\nabla^2\mathbf{v} = -\eta\nabla\left(u\cos\theta\left(\frac{3R}{2r^2}\right)\right)$$

$$\Rightarrow p = p_0 - \eta u\cos\theta\left(\frac{3R}{2r^2}\right)$$

$$p = p_0 - \eta u\cos\theta\left(\frac{3R}{2r^2}\right)$$

Corresponds to:

$$F_D = \left(p(R)-p_0\right)\frac{4\pi R^2}{\cos\theta}$$

$$= -\eta u\left(6\pi R\right)$$