PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 7:

Continue reading Chapter 3

1. **Lagrange's equations**
2. **D'Alembert's principle**

PHY 711 Classical Mechanics and Mathematical Methods

MWF 10 AM-10:50 PM | OPL 103 | http://www.wfu.edu/~natalie/f14phy711/

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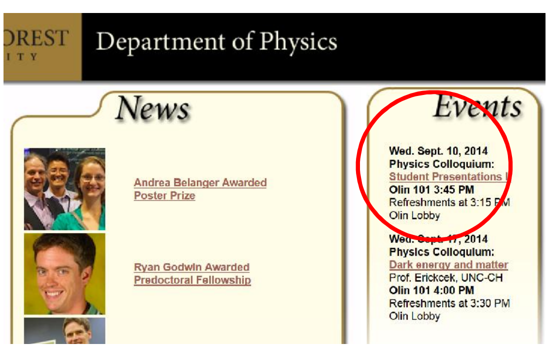
Course schedule

(Preliminary schedule -- subject to frequent adjustment.) rrr>

	Date	F&W Reading	Topic	Assignment
1	Wed, 8/27/2014	Chap. 1	Review of basic principles	#1
2	Fri, 8/29/2014	Chap. 1	Scattering theory	#2
3	Mon, 9/01/2014	Chap. 1	Scattering theory continued	#3
4	Wed, 9/03/2014	Chap. 2	Accelerated coordinate systems	#4
5	Fri, 9/05/2014	Chap. 3	Calculus of variations	#5
6	Mon, 9/08/2014	Chap. 3	Calculus of variations	#6
7	Wed, 9/10/2014	Chap. 3	Hamilton's principle	#7
8	Fri, 9/12/2014			
9	Mon, 9/15/2014			
10	Wed, 9/17/2014			
11	Fri, 9/19/2014			
12	Mon, 9/22/2014			



WFU Physics Colloquium

TITLE: "Welcome to the WFU Physics Department -- Part 2"

TIME: Wednesday Sept. 10, 2014 at 3:45 PM*

PLACE: George P. Williams, Jr. Lecture Hall, (Olin 101)

* Note: early starting time.

Refreshments will be served at 3:15 PM in the lounge. All interested persons are cordially invited to attend.

PROGRAM

This colloquium continues the welcoming program of last week. After refreshments in the lounge in the lobby of Olin Physical Laboratory (starting at 3:15), we will meet in the George P. Williams, Jr. Lecture Hall (Olin 101) at 3:45 PM for some presentations by undergraduate students, highlighting their summer research experiences, and senior graduate students presenting overviews of their research areas.

Summary of results from the calculus of variation

For the class of problems where we need to perform an extremization on an integral form:

$$I = \int_{x_i}^{x_f} f\left(\left\{y(x), \frac{dy}{dx}\right\}, x\right) dx$$

A necessary condition is the Euler - Lagrange equations:

$$\left(\frac{\partial f}{\partial y}\right) - \frac{d}{dx}\left[\left(\frac{\partial f}{\partial (dy/dx)}\right)\right] = 0$$

$$\frac{d}{dx}\left(f - \frac{\partial f}{\partial (dy/dx)}\frac{dy}{dx}\right) = \left(\frac{\partial f}{\partial x}\right)$$

Application to particle dynamics

Simple example: vertical trajectory of particle of mass *m* subject to constant downward acceleration *a=-g*.

$$m\frac{d^2y}{dt^2} = -mg$$

$$y(t) = y_i + v_i t - \tfrac{1}{2} g t^2$$

Now consider the Lagrangian defined to be :

$$L\left(\left\{y(t), \frac{dy}{dt}\right\}, t\right) \equiv T - U$$

Kinetic energy

Potential energy

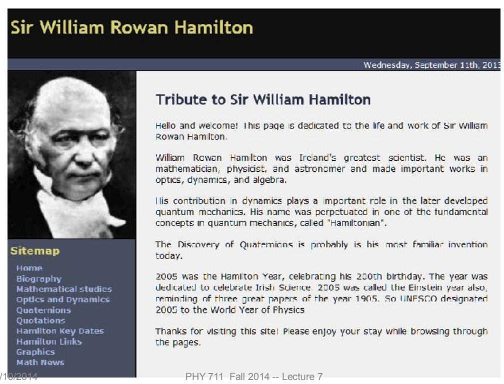
In our example :

$$L\left(\left\{y(t), \frac{dy}{dt}\right\}, t\right) \equiv T - U = \frac{1}{2}m\left(\frac{dy}{dt}\right)^2 - mgy$$

Hamilton's principle states :

$$S \equiv \int_{t_i}^{t_f} \left(\frac{1}{2}m\left(\frac{dy}{dt}\right)^2 - mgy\right)dt \quad \text{is minimized for physical } y(t):$$

http://www.hamilton2005.ie/


Sir William Rowan Hamilton

Wednesday, September 11th, 2013

Sitemap

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Tribute to Sir William Hamilton

Hello and welcome! This page is dedicated to the life and work of Sir William Rowan Hamilton.

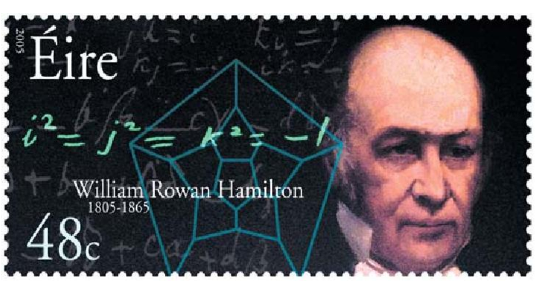
William Rowan Hamilton was Ireland's greatest scientist. He was an mathematician, physicist, and astronomer and made important works in optics, dynamics, and algebra.

His contribution in dynamics plays a important role in the later developed quantum mechanics. His name was perpetuated in one of the fundamental concepts in quantum mechanics, called "Hamiltonian".

The Discovery of Quaternions is probably is his most familiar invention today.

2005 was the Hamilton Year, celebrating his 200th birthday. The year was dedicated to celebrate Irish Science. 2005 was called the Einstein year also, reminding of three great papers of the year 1905. So UNESCO designated 2005 to the World Year of Physics

Thanks for visiting this site! Please enjoy your stay while browsing through the pages.



http://rjlipton.wordpress.com

Condition for minimizing the action :

$$S \equiv \int_{t_i}^{t_f} \left(\frac{1}{2} m \left(\frac{dy}{dt} \right)^2 - mgy \right) dt$$

Euler - Lagrange relations :

$$\frac{\partial L}{\partial y} - \frac{d}{dt}\frac{\partial L}{\partial \dot{y}} = 0$$

$$\Rightarrow -mg - \frac{d}{dt} m\dot{y} = 0$$

$$\Rightarrow \frac{d}{dt}\frac{dy}{dt} = -g$$

$$y(t) = y_i + v_i t - \tfrac{1}{2} g t^2$$

Check :

$$S \equiv \int_{t_i}^{t_f} \left(\frac{1}{2} m \left(\frac{dy}{dt} \right)^2 - mgy \right) dt$$

Assume $t_i = 0,\ y_i = H \equiv \tfrac{1}{2} g T^2;\ \ t_f = T,\ y_f = 0$

Trial trajectories : $y_1(t) = \tfrac{1}{2} g T^2 (1 - t/T) = H - \tfrac{1}{2} g T t$

$y_2(t) = \tfrac{1}{2} g T^2 (1 - t^2/T^2) = H - \tfrac{1}{2} g t^2$

$y_3(t) = \tfrac{1}{2} g T^2 (1 - t^3/T^3) = H - \tfrac{1}{2} g t^3 / T$

Maple says :

$$S_1 = -0.125 mg^2 T^3$$
$$S_2 = -0.167 mg^2 T^3$$
$$S_3 = -0.150 mg^2 T^3$$

Jean d'Alembert 1717-1783

D'Alembert's principle:

Generalized coordinates: $q_\sigma(\{x_i\})$

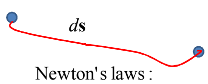
Newton's laws:

$\mathbf{F}\text{-}m\mathbf{a} = 0 \qquad \Rightarrow (\mathbf{F}\text{-}m\mathbf{a}) \cdot d\mathbf{s} = 0$

$$\mathbf{F} \cdot d\mathbf{s} = \sum_\sigma \sum_i F_i \frac{\partial x_i}{\partial q_\sigma} \delta q_\sigma$$

For a conservative force: $F_i = -\frac{\partial U}{\partial x_i}$

$$\mathbf{F} \cdot d\mathbf{s} = -\sum_\sigma \sum_i \frac{\partial U}{\partial x_i} \frac{\partial x_i}{\partial q_\sigma} \delta q_\sigma = -\sum_\sigma \frac{\partial U}{\partial q_\sigma} \delta q_\sigma$$



Generalized coordinates: $q_\sigma(\{x_i\})$

Newton's laws:

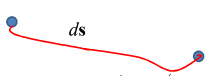
$\mathbf{F}\text{-}m\mathbf{a} = 0 \qquad \Rightarrow (\mathbf{F}\text{-}m\mathbf{a}) \cdot d\mathbf{s} = 0$

$$m\mathbf{a} \cdot d\mathbf{s} = \sum_\sigma \sum_i m\ddot{x}_i \frac{\partial x_i}{\partial q_\sigma} \delta q_\sigma$$

$$= \sum_\sigma \sum_i \left(\frac{d}{dt}\left(m\dot{x}_i \frac{\partial x_i}{\partial q_\sigma} \right) - m\dot{x}_i \frac{d}{dt} \frac{\partial x_i}{\partial q_\sigma} \right) \delta q_\sigma$$

Claim: $\frac{\partial x_i}{\partial q_\sigma} = \frac{\partial \dot{x}_i}{\partial \dot{q}_\sigma}$ and $\frac{d}{dt}\frac{\partial x_i}{\partial q_\sigma} = \frac{\partial}{\partial q_\sigma}\frac{dx_i}{dt} \equiv \frac{\partial \dot{x}_i}{\partial q_\sigma}$

$$m\mathbf{a} \cdot d\mathbf{s} = \sum_\sigma \sum_i \left(\frac{d}{dt}\left(\frac{\partial\left(\frac{1}{2} m\dot{x}_i^2\right)}{\partial \dot{q}_\sigma} \right) - \frac{\partial\left(\frac{1}{2} m\dot{x}_i^2\right)}{\partial q_\sigma} \right) \delta q_\sigma$$



Generalized coordinates: $q_\sigma(\{x_i\})$

$$m\mathbf{a} \cdot d\mathbf{s} = \sum_\sigma \sum_i \left(\frac{d}{dt}\left(\frac{\partial\left(\frac{1}{2} m\dot{x}_i^2\right)}{\partial \dot{q}_\sigma} \right) - \frac{\partial\left(\frac{1}{2} m\dot{x}_i^2\right)}{\partial q_\sigma} \right) \delta q_\sigma$$

Define -- kinetic energy: $T \equiv \sum_i \frac{1}{2} m\dot{x}_i^2$

$$m\mathbf{a} \cdot d\mathbf{s} = \sum_\sigma \left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_\sigma} - \frac{\partial T}{\partial q_\sigma} \right) \delta q_\sigma$$

Recall:

$$\mathbf{F} \cdot d\mathbf{s} = \sum_\sigma \sum_i \frac{\partial U}{\partial x_i} \frac{\partial x_i}{\partial q_\sigma} \delta q_\sigma = \sum_\sigma \frac{\partial U}{\partial q_\sigma} \delta q_\sigma$$

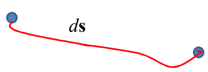
$$(\mathbf{F}\text{-}m\mathbf{a}) \cdot d\mathbf{s} = \sum_\sigma \frac{\partial U}{\partial q_\sigma} \delta q_\sigma - \sum_\sigma \left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_\sigma} - \frac{\partial T}{\partial q_\sigma} \right) \delta q_\sigma = 0$$

Generalized coordinates:
$q_\sigma(\{x_i\})$

$$(\mathbf{F}\text{-}m\mathbf{a})\cdot d\mathbf{s} = -\sum_\sigma \frac{\partial U}{\partial q_\sigma}\delta q_\sigma - \sum_\sigma \left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_\sigma} - \frac{\partial T}{\partial q_\sigma}\right)\delta q_\sigma = 0$$
$$= -\sum_\sigma \left(\frac{d}{dt}\frac{\partial (T-U)}{\partial \dot{q}_\sigma} - \frac{\partial (T-U)}{\partial q_\sigma}\right)\delta q_\sigma = 0$$

$$L(q_\sigma, \dot{q}_\sigma; t) = T - U$$

Note: This is only true if
$\frac{\partial U}{\partial \dot{q}_\sigma} = 0$



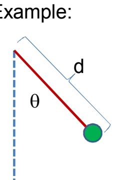
Generalized coordinates:
$q_\sigma(\{x_i\})$

Define -- Lagrangian: $L \equiv T - U$
$$L = L(\{q_\sigma\}, \{\dot{q}_\sigma\}, t)$$
$$(\mathbf{F}\text{-}m\mathbf{a})\cdot d\mathbf{s} = -\sum_\sigma \left(\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma}\right)\delta q_\sigma = 0$$

$\Rightarrow$ Minimization integral: $S = \int_{t_i}^{t_f} L(\{q_\sigma\}, \{\dot{q}_\sigma\}, t)\,dt$

Euler – Lagrange equations: $L = L(\{q_\sigma\}, \{\dot{q}_\sigma\}, t) = T - U$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

Example:



$$L = L(\theta, \dot{\theta}) = \tfrac{1}{2} m d^2 \dot{\theta}^2 - mg(d - d\cos\theta)$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0 \quad \Rightarrow \frac{d}{dt} m d^2 \dot{\theta} - mgd \sin\theta = 0$$
$$\frac{d^2\theta}{dt^2} = \frac{g}{d}\sin\theta$$

Another example : $L = L(\{q_\sigma\},\{\dot{q}_\sigma\},t) = T - U$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

$$L = L(\alpha,\beta,\gamma,\dot{\alpha},\dot{\beta},\dot{\gamma}) = \tfrac{1}{2}I_1\left(\dot{\alpha}^2\sin^2\beta + \dot{\beta}^2\right) + \tfrac{1}{2}I_3\left(\dot{\alpha}\cos\beta + \dot{\gamma}\right)^2 - Mgd\cos\beta$$