PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 10:

Continue reading Chapter 3 & 6

1. Constants of the motion
2. Conserved quantities

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment
1	Wed, 8/26/2015	Chap. 1	Review of basic principles	#1
2	Fri, 8/28/2015	Chap. 1	Scattering theory	#2
3	Mon, 8/31/2015	Chap. 1	Scattering theory continued	#3
4	Wed, 9/02/2015	Chap. 2	Accelerated coordinate systems	#4
5	Fri, 9/04/2015	Chap. 3	Calculus of variations	#5
6	Mon, 9/07/2015	Chap. 3	Calculus of variations	#6
7	Wed, 9/09/2015	Chap. 3	Hamilton's principle	#7
8	Fri, 9/11/2015	Chap. 3 & 6	Hamilton's principle	#8
9	Mon, 9/14/2015	Chap. 3 & 6	Lagrangians with constraints	#9
10	Wed, 9/16/2015	Chap. 3 & 6	Lagrangians and constants of motion	#10
11	Fri, 9/18/2015	Chap. 3 & 6	Hamiltonian formalism	
12	Mon, 9/21/2015	Chap. 3 & 6	Hamiltonian formalism	
13	Wed, 9/23/2015	Chap. 3 & 6	Hamiltonian Jacobi transformations	
14	Fri, 9/25/2015	Chap. 4	Small oscillations	
15	Mon, 9/28/2015	Chap. 4	Normal modes of motion	
16	Wed, 9/30/2015	Chap. 4	Normal modes of motion	

News

Research Labs Tour Part I

Congratulations to Dr. Greg Smith, recent Ph.D. Recipient

Congratulations to Dr. Jie Liu, recent Ph.D. Recipient

Events

Wed. Sept. 16, 2015
Breaking down (and reintegrating) the role of cell and matrix mechanics in cell-matrix interactions
Professor Nicholas A. Kurniawan, Eindhoven University of Technology
Olin 101, 4:00 PM
Refreshments at 3:30 PM
Olin Lobby

Fri. Sept. 18, 2015
Thesis Presentation
Solid Electrolytes
Nicholas Lepley, WFU
Olin 101 at 2:30 PM

WFU Physics Colloquium

TITLE: Breaking down (and reintegrating) the role of cell and matrix mechanics in cell-matrix interactions

SPEAKER: Professor Nicholas A. Kurniawan,

Department of Biomedical Engineering,
Eindhoven University of Technology

TIME: Wednesday September 16, 2015 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

In recent years it has been increasingly realized that physical and mechanical interactions between the cell and the matrix are key determinants of a wide variety of physiological functions and pathological processes. In this talk, I will present our recent efforts to unravel (1) the underlying principles behind the fascinating nonlinear mechanical properties of protein networks such as the cytoskeleton and extracellular matrix like fibrin and collagen; and (2) the manifestations of the dynamic mechanical cell-matrix interactions during 3D cell

Summary of Lagrangian formalism (without constraints)

For independent generalized coordinates $q_\sigma(t)$:

$$L = L(\{q_\sigma(t)\}, \{\dot{q}_\sigma(t)\}, t)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

Note that if $\frac{\partial L}{\partial q_\sigma} = 0$, then $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} = 0$

$$\Rightarrow \frac{\partial L}{\partial \dot{q}_\sigma} = (\text{constant})$$

Examples of constants of the motion:

Example 1: one-dimensional potential:

$$L = \tfrac{1}{2} m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - V(z)$$

$$\Rightarrow \frac{d}{dt} m\dot{x} = 0 \qquad \Rightarrow m\dot{x} \equiv p_x \ \ (\text{constant})$$

$$\Rightarrow \frac{d}{dt} m\dot{y} = 0 \qquad \Rightarrow m\dot{y} \equiv p_y \ \ (\text{constant})$$

$$\Rightarrow \frac{d}{dt} m\dot{z} = -\frac{\partial V}{\partial z}$$

Examples of constants of the motion:

Example 2: Motion in a central potential

$$L = \tfrac{1}{2} m\left(\dot{r}^2 + r^2\dot{\varphi}^2\right) - V(r)$$

$$\Rightarrow \frac{d}{dt} mr^2\dot{\varphi} = 0 \qquad \Rightarrow mr^2\dot{\varphi} \equiv p_\varphi \ \text{ (constant)}$$

$$\Rightarrow \frac{d}{dt} m\dot{r} = mr\dot{\varphi}^2 - \frac{\partial V}{\partial r} = \frac{p_\varphi{}^2}{mr^3} - \frac{\partial V}{\partial r}$$

Recall alternative form of Euler-Lagrange equations:

Starting from:

$$L = L(\{q_\sigma(t)\}, \{\dot{q}_\sigma(t)\}, t)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

Also note that:
$$\frac{dL}{dt} = \sum_\sigma \frac{\partial L}{\partial q_\sigma}\dot{q}_\sigma + \sum_\sigma \frac{\partial L}{\partial \dot{q}_\sigma}\ddot{q}_\sigma + \frac{\partial L}{\partial t}$$

$$= \frac{d}{dt}\left(\sum_\sigma \frac{\partial L}{\partial \dot{q}_\sigma}\dot{q}_\sigma\right) + \frac{\partial L}{\partial t}$$

$$\Rightarrow \frac{d}{dt}\left(L - \sum_\sigma \frac{\partial L}{\partial \dot{q}_\sigma}\dot{q}_\sigma\right) = \frac{\partial L}{\partial t}$$

Additional constant of the motion:

If $\frac{\partial L}{\partial t} = 0;$

then: $\frac{d}{dt}\left(L - \sum_\sigma \frac{\partial L}{\partial \dot{q}_\sigma}\dot{q}_\sigma\right) = \frac{\partial L}{\partial t} = 0$

$$\Rightarrow L - \sum_\sigma \frac{\partial L}{\partial \dot{q}_\sigma}\dot{q}_\sigma = -E \quad \text{(constant)}$$

Example 1: one-dimensional potential:

$$L = \tfrac{1}{2} m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - V(z)$$

$$\Rightarrow \frac{d}{dt}\left(\tfrac{1}{2} m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - V(z) - m\dot{x}^2 - m\dot{y}^2 - m\dot{z}^2\right) = 0$$

$$\Rightarrow -\left(\tfrac{1}{2} m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + V(z)\right) = -E \quad \text{(constant)}$$

For this case, we also have $m\dot{x} \equiv p_x$ and $m\dot{y} \equiv p_y$

$$\Rightarrow E = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \tfrac{1}{2} m\dot{z}^2 + V(z)$$

Additional constant of the motion -- continued:

If $\frac{\partial L}{\partial t} = 0;$

then: $\frac{d}{dt}\left(L - \sum_\sigma \frac{\partial L}{\partial \dot{q}_\sigma}\dot{q}_\sigma \right) = \frac{\partial L}{\partial t} = 0$

$\Rightarrow L - \sum_\sigma \frac{\partial L}{\partial \dot{q}_\sigma}\dot{q}_\sigma = -E$ (constant)

Example 2: Motion in a central potential

$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2\right) - V(r)$

$\Rightarrow \frac{d}{dt}\left(\frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2\right) - V(r) - m\dot{r}^2 - mr^2\dot{\phi}^2\right) = 0$

$\Rightarrow -\left(\frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2\right) + V(r)\right) = -E$ (constant)

For this case, we also have $mr^2\dot{\varphi} \equiv p_\varphi$

$\Rightarrow E = \frac{p_\varphi^2}{2mr^2} + \frac{1}{2}m\dot{r}^2 + V(r)$

Other examples

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + \frac{q}{2c}B_0\left(-\dot{x}y + \dot{y}x\right)$$

$$\frac{\partial L}{\partial z} = 0 \qquad \Rightarrow m\dot{z} = p_z \quad \text{(constant)}$$

$$E = \sum_\sigma \frac{\partial L}{\partial \dot{q}_\sigma}\dot{q}_\sigma - L$$

$$= m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + \frac{q}{2c}B_0\left(-\dot{x}y + \dot{y}x\right)$$

$$-\frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \frac{q}{2c}B_0\left(-\dot{x}y + \dot{y}x\right)$$

$$= \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) + \frac{p_z^2}{2m}$$

Other examples

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \frac{q}{c}B_0\dot{x}y$$

$$\frac{\partial L}{\partial z} = 0 \qquad \Rightarrow m\dot{z} = p_z \quad \text{(constant)}$$

$$\frac{\partial L}{\partial x} = 0 \qquad \Rightarrow m\dot{x} = p_x \quad \text{(constant)}$$

$$E = \sum_\sigma \frac{\partial L}{\partial \dot{q}_\sigma}\dot{q}_\sigma - L$$

$$= m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \frac{q}{c}B_0\dot{x}y$$

$$-\frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + \frac{q}{c}B_0\dot{x}y$$

$$= \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) = \frac{1}{2}m\dot{y}^2 + \frac{p_x^2}{2m} + \frac{p_z^2}{2m}$$

Lagrangian picture

For independent generalized coordinates $q_\sigma(t)$:

$$L = L(\{q_\sigma(t)\},\{\dot{q}_\sigma(t)\},t)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

$\Rightarrow$ Second order differential equations for $q_\sigma(t)$

Switching variables – Legendre transformation

Mathematical transformations for continuous functions of several variables & Legendre transforms:

$$z(x,y) \quad \Leftrightarrow \quad x(y,z)???$$

$$z(x,y) \Rightarrow dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$x(y,z) \Rightarrow dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz$$

$$\text{But:} \quad \left(\frac{\partial x}{\partial y}\right)_z = -\frac{(\partial z/\partial y)_x}{(\partial z/\partial x)_y}$$

Mathematical transformations for continuous functions of several variables & Legendre transforms continued:

$$z(x,y) \Rightarrow dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

Let $u \equiv \left(\frac{\partial z}{\partial x}\right)_y$ and $v \equiv \left(\frac{\partial z}{\partial y}\right)_x$

Define new function

$$w(u,y) \Rightarrow dw = \left(\frac{\partial w}{\partial u}\right)_y du + \left(\frac{\partial w}{\partial y}\right)_u dy$$

For $w = z - ux$, $\quad dw = dz - udx - xdu = udx + vdy - udx - xdu$

$dw = -xdu + vdy$

$$\Rightarrow \left(\frac{\partial w}{\partial u}\right)_y = -x \qquad \left(\frac{\partial w}{\partial y}\right)_u = \left(\frac{\partial z}{\partial y}\right)_x = v$$

For thermodynamic functions:

Internal energy: $U = U(S,V)$

$$dU = TdS - PdV$$

$$dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV$$

$$\Rightarrow T = \left(\frac{\partial U}{\partial S}\right)_V \qquad P = -\left(\frac{\partial U}{\partial V}\right)_S$$

Enthalpy: $H = H(S,P) = U + PV$

$$dH = dU + PdV + VdP = TdS + VdP = \left(\frac{\partial H}{\partial S}\right)_P dS + \left(\frac{\partial H}{\partial P}\right)_S dP$$

$$\Rightarrow T = \left(\frac{\partial H}{\partial S}\right)_P \qquad V = \left(\frac{\partial H}{\partial P}\right)_S$$

Name	Potential	Differential Form
Internal energy	$E(S,V,N)$	$dE = TdS - PdV + \mu dN$
Entropy	$S(E,V,N)$	$dS = \frac{1}{T}dE + \frac{P}{T}dV - \frac{\mu}{T}dN$
Enthalpy	$H(S,P,N) = E + PV$	$dH = TdS + VdP + \mu dN$
Helmholtz free energy	$F(T,V,N) = E - TS$	$dF = -SdT - PdV + \mu dN$
Gibbs free energy	$G(T,P,N) = F + PV$	$dG = -SdT + VdP + \mu dN$
Landau potential	$\Omega(T,V,\mu) = F - \mu N$	$d\Omega = -SdT - PdV - Nd\mu$

Lagrangian picture

For independent generalized coordinates $q_\sigma(t)$:

$$L = L(\{q_\sigma(t)\},\{\dot{q}_\sigma(t)\},t)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

$\Rightarrow$ Second order differential equations for $q_\sigma(t)$

Switching variables – Legendre transformation

Define: $H = H(\{q_\sigma(t)\},\{p_\sigma(t)\},t)$

$$H = \sum_\sigma \dot{q}_\sigma p_\sigma - L \qquad \text{where } p_\sigma = \frac{\partial L}{\partial \dot{q}_\sigma}$$

$$dH = \sum_\sigma \left(\dot{q}_\sigma dp_\sigma + p_\sigma d\dot{q}_\sigma - \frac{\partial L}{\partial q_\sigma} dq_\sigma - \frac{\partial L}{\partial \dot{q}_\sigma} d\dot{q}_\sigma \right) - \frac{\partial L}{\partial t} dt$$

Hamiltonian picture – continued

$$H = H(\{q_\sigma(t)\},\{p_\sigma(t)\},t)$$

$$H = \sum_\sigma \dot{q}_\sigma p_\sigma - L \qquad \text{where} \quad p_\sigma = \frac{\partial L}{\partial \dot{q}_\sigma}$$

$$dH = \sum_\sigma \left(\dot{q}_\sigma dp_\sigma + p_\sigma d\dot{q}_\sigma - \frac{\partial L}{\partial q_\sigma} dq_\sigma - \frac{\partial L}{\partial \dot{q}_\sigma} d\dot{q}_\sigma \right) - \frac{\partial L}{\partial t} dt$$

$$= \sum_\sigma \left(\frac{\partial H}{\partial q_\sigma} dq_\sigma + \frac{\partial H}{\partial p_\sigma} dp_\sigma \right) + \frac{\partial H}{\partial t} dt$$

$$\Rightarrow \dot{q}_\sigma = \frac{\partial H}{\partial p_\sigma} \qquad \frac{\partial L}{\partial q_\sigma} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} \equiv \dot{p}_\sigma = -\frac{\partial H}{\partial q_\sigma} \qquad \frac{\partial L}{\partial t} = -\frac{\partial H}{\partial t}$$

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