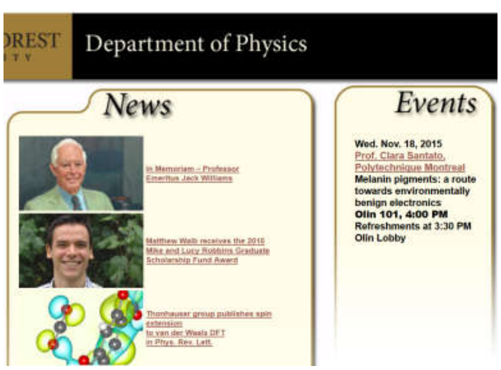
PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 34

Navier-Stokes equation – Chap. 12 in F&W

- **Effects of viscosity on sound**

22	Wed, 10/21/2015	Chap. 5	Motion of Rigid Bodies	#21
23	Fri, 10/23/2015	Chap. 8	Motion of Elastic membranes	#22
24	Mon, 10/26/2015	Chap. 9	Hydrodynamics	#23
25	Wed, 10/28/2015	Chap. 9	Hydrodynamics	#24
26	Fri, 10/30/2015	Chap. 9	Sound waves	#25
27	Mon, 11/02/2015	Chap. 9	Sound waves	#26
28	Wed, 11/04/2015	Chap. 9	Sound waves	
29	Fri, 11/06/2015	Chap. 10	Surface waves on fluids	#27
30	Mon, 11/09/2015	Chap. 10	Surface waves on fluids	#28
31	Wed, 11/11/2015	Chap. 11	Heat Conduction	#29
32	Fri, 11/13/2015	Chap. 12	Viscosity	#30
33	Mon, 11/16/2015	Chap. 12	Viscosity	Prepare presentation.
34	Wed, 11/18/2015	Chap. 12	Viscosity	Prepare presentation.
35	Fri, 11/20/2015	Chap. 12	More viscosity	Prepare presentation.
36	Mon, 11/23/2015	Chap. 13	Elastic Continua	Prepare presentation.
	Wed, 11/25/2015		*Thanksgiving Holiday*	
	Fri, 11/27/2015		*Thanksgiving Holiday*	
37	Mon, 11/30/2015	Chap. 13	Elastic Continua	Prepare presentation.
	Wed, 12/02/2015		Student presentations I	
	Fri, 12/04/2015		Student presentations II	
	Mon, 12/07/2015		Begin Take-home final	



WFU Physics Colloquium

TITLE: Melanin pigments: a route towards environmentally benign electronics

SPEAKER: Professor Clara Santato,

Department of Engineering Physics,
Polytechnique Montreal

TIME: Wednesday November 18, 2015 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

efreshments will be served at 3:30 PM in the Olin Lounge. All
iterested persons are cordially invited to attend.

ABSTRACT

Melanins are biomacromolecules responsible for the pigmentation of many plants and animals. The biological functions of melanins, also present in the inner ear and the substantia nigra of the human brain, go far beyond coloration and include photoprotection, anti-oxidant behavior, and metal chelation. Melanins are also intensively studied for their involvement in melanoma skin cancer and Parkinson's disease. In the class of melanins, eumelanins are the most studied by material scientists because of their photoconductivity and hybrid ionic-electronic transport properties.

We will discuss extended structure properties relationships in eumelanins. The film forming properties will be introduced considering their notoriously poorly solubility in all solvents. The interfaces of eumelanin with metals and electrolytes under electrical bias will be critically presented considering the metal binding properties of melanins. The charge transfer and transport properties in different experimental conditions (vacuum vs .wet.) and different electrolytes will be reported.

PHY 711 Presentation Schedule

Wednesday, December 2, 2015

	Presenter	Title of presentation
10:00-10:17		
10:17-10:34		
10:34-10:51		

Friday, December 4, 2015

	Presenter	Title of presentation
10:00-10:17		
10:17-10:34		
10:34-10:51		

Please sign up

Newton-Euler equations for viscous fluids

Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{f} - \frac{1}{\rho}\nabla p + \frac{\eta}{\rho}\nabla^2 \mathbf{v} + \frac{1}{\rho}\left(\zeta + \frac{1}{3}\eta\right)\nabla(\nabla \cdot \mathbf{v})$$

Continuity condition

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Newton-Euler equations for viscous fluids – effects on sound

Without viscosity terms:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \mathbf{f} - \frac{1}{\rho}\nabla p \qquad \frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$

Assume: $\mathbf{v} = 0 + \delta\mathbf{v} \qquad \mathbf{f}=0 \qquad \rho = \rho_0 + \delta\rho$

$$p = p_0 + \delta p = p_0 + \left(\frac{\partial p}{\partial \rho}\right)_s \delta\rho \equiv p_0 + c^2\delta\rho$$

Linearized equations: $\frac{\partial \delta\mathbf{v}}{\partial t} = -\frac{c^2}{\rho_0}\nabla\delta\rho \qquad \frac{\partial \delta\rho}{\partial t} + \rho_0\nabla\cdot(\delta\mathbf{v}) = 0$

Let $\delta\mathbf{v} \equiv \delta\mathbf{v}_0\, e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \qquad \delta\rho \equiv \delta\rho_0\, e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

Sound waves without viscosity -- continued

Linearized equations: $\frac{\partial \delta\mathbf{v}}{\partial t} = -\frac{c^2}{\rho_0}\nabla\delta\rho \qquad \frac{\partial \delta\rho}{\partial t} + \rho_0\nabla\cdot(\delta\mathbf{v}) = 0$

Let $\delta\mathbf{v} \equiv \delta\mathbf{v}_0\, e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \qquad \delta\rho \equiv \delta\rho_0\, e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

$$\frac{\partial \delta\mathbf{v}}{\partial t} = -\frac{c^2}{\rho_0}\nabla\delta\rho \qquad \Rightarrow \omega\delta\mathbf{v}_0 = \frac{c^2\delta\rho_0}{\rho_0}\mathbf{k}$$

$$\frac{\partial \delta\rho}{\partial t} + \rho_0\nabla\cdot(\delta\mathbf{v}) = 0 \qquad \Rightarrow -\omega\delta\rho_0 + \rho_0\mathbf{k}\cdot\delta\mathbf{v}_0 = 0$$

$$\Rightarrow k^2 = \frac{\omega^2}{c^2} \qquad \frac{\delta\rho_0}{\rho_0} = \frac{\hat{\mathbf{k}}\cdot\delta\mathbf{v}_0}{c}$$

➔Pure longitudinal harmonic wave solutions

Newton-Euler equations for viscous fluids – effects on sound

Recall full equations:

Navier-Stokes equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \mathbf{f} - \frac{1}{\rho}\nabla p + \frac{\eta}{\rho}\nabla^2\mathbf{v} + \frac{1}{\rho}\left(\zeta + \frac{1}{3}\eta\right)\nabla(\nabla\cdot\mathbf{v})$$

Continuity condition

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$

Assume: $\mathbf{v} = 0 + \delta\mathbf{v} \qquad \mathbf{f}=0 \qquad \rho = \rho_0 + \delta\rho$

$$p = p_0 + \delta p = p_0 + c^2\delta\rho + \left(\frac{\partial p}{\partial s}\right)_\rho \delta s$$

where $c^2 \equiv \left(\frac{\partial p}{\partial \rho}\right)_s$

viscosity causes heat transfer

Newton-Euler equations for viscous fluids – effects on sound

Note that pressure now depends both on density and entropy so that entropy must be coupled into the equations

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \mathbf{f} - \frac{1}{\rho}\nabla p + \frac{\eta}{\rho}\nabla^2\mathbf{v} + \frac{1}{\rho}\left(\zeta + \frac{1}{3}\eta\right)\nabla(\nabla\cdot\mathbf{v})$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0 \qquad \rho T\frac{\partial s}{\partial t} = k_{th}\nabla^2 T$$

Assume: $\mathbf{v} = 0 + \delta\mathbf{v}$ $\mathbf{f}=0$ $\rho = \rho_0 + \delta\rho$

$$p = p_0 + \delta p = p_0 + c^2\delta\rho + \left(\frac{\partial p}{\partial s}\right)_\rho \delta s \quad \text{where } c^2 \equiv \left(\frac{\partial p}{\partial \rho}\right)_s$$

$$T = T_0 + \delta T = T_0 + \left(\frac{\partial T}{\partial \rho}\right)_s \delta\rho + \left(\frac{\partial T}{\partial s}\right)_\rho \delta s$$

$$s = s_0 + \delta s$$

Newton-Euler equations for viscous fluids – linearized equations

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \mathbf{f} - \frac{1}{\rho}\nabla p + \frac{\eta}{\rho}\nabla^2\mathbf{v} + \frac{1}{\rho}\left(\zeta + \frac{1}{3}\eta\right)\nabla(\nabla\cdot\mathbf{v})$$

$$\Rightarrow \frac{\partial \delta\mathbf{v}}{\partial t} = \underbrace{-\frac{1}{\rho_0}\nabla\delta p}\, + \frac{\eta}{\rho}\nabla^2\delta\mathbf{v} + \frac{1}{\rho_0}\left(\zeta + \frac{1}{3}\eta\right)\nabla(\nabla\cdot\delta\mathbf{v})$$

$$-\frac{1}{\rho_0}\left\{\left(\frac{\partial p}{\partial \rho}\right)_s \nabla\delta\rho + \left(\frac{\partial p}{\partial s}\right)_\rho \nabla\delta s\right\} = -\frac{c^2}{\rho_0}\nabla\delta\rho - \rho_0\left(\frac{\partial T}{\partial \rho}\right)_s \nabla\delta s$$

Digression -- from the first law of thermodynamics:

$$d\epsilon = Tds + \frac{p}{\rho^2}d\rho$$

$$\left(\frac{\partial}{\partial\rho}\left(\frac{\partial\epsilon}{\partial s}\right)_\rho\right)_s = \left(\frac{\partial T}{\partial\rho}\right)_s \quad \Leftrightarrow \quad \left(\frac{\partial}{\partial s}\left(\frac{\partial\epsilon}{\partial\rho}\right)_s\right)_\rho = \left(\frac{\partial p/\rho^2}{\partial s}\right)_\rho \approx \frac{1}{\rho_0^2}\left(\frac{\partial p}{\partial s}\right)_\rho$$

Newton-Euler equations for viscous fluids – linearized equations

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$

$$\Rightarrow \frac{\partial \delta\rho}{\partial t} + \rho_0\nabla\cdot(\delta\mathbf{v}) = 0$$

$$\rho T\frac{\partial s}{\partial t} = k_{th}\nabla^2 T$$

$$\Rightarrow \frac{\partial \delta s}{\partial t} = \frac{k_{th}}{\rho_0 T_0}\left(\left(\frac{\partial T}{\partial s}\right)_\rho \nabla^2\delta s + \left(\frac{\partial T}{\partial \rho}\right)_s \nabla^2\delta\rho\right)$$

Further relationships:

$$\left(\frac{\partial T}{\partial s}\right)_\rho \approx \frac{T_0}{c_v} \qquad \kappa = \frac{k_{th}}{\rho c_p}$$

heat capacity at constant volume

Newton-Euler equations for viscous fluids – linearized equations

$$\Rightarrow \frac{\partial \delta s}{\partial t} = \left(\gamma\kappa\nabla^2 \delta s + \frac{c_p\kappa}{T_0}\left(\frac{\partial T}{\partial \rho}\right)_s \nabla^2 \delta\rho \right) \quad \text{where } \gamma \equiv \frac{c_p}{c_v}$$

Newton-Euler equations for viscous fluids – effects on sound
Linearized equations (with the help of various thermodynamic relationships):

$$\frac{\partial \delta \mathbf{v}}{\partial t} = -\frac{c^2}{\rho_0}\nabla\delta\rho - \rho_0\left(\frac{\partial T}{\partial \rho}\right)_s \nabla\delta s + \frac{\eta}{\rho_0}\nabla^2\delta\mathbf{v} + \frac{1}{\rho_0}\left(\zeta + \frac{1}{3}\eta\right)\nabla\left(\nabla\cdot\delta\mathbf{v}\right)$$

$$\frac{\partial \delta\rho}{\partial t} + \rho_0\nabla\cdot\left(\delta\mathbf{v}\right) = 0$$

$$\frac{\partial \delta s}{\partial t} = \gamma\kappa\nabla^2\delta s + \frac{c_p\kappa}{T_0}\left(\frac{\partial T}{\partial \rho}\right)_s \nabla^2\delta\rho$$

Here: $\gamma = \frac{c_p}{c_v}$ $\qquad \kappa = \frac{k_{th}}{c_p\rho_0}$

Linearized hydrodynamic equations

$$\frac{\partial \delta \mathbf{v}}{\partial t} = -\frac{c^2}{\rho_0}\nabla\delta\rho - \rho_0\left(\frac{\partial T}{\partial \rho}\right)_s \nabla\delta s + \frac{\eta}{\rho_0}\nabla^2\delta\mathbf{v} + \frac{1}{\rho_0}\left(\zeta + \frac{1}{3}\eta\right)\nabla\left(\nabla\cdot\delta\mathbf{v}\right)$$

$$\frac{\partial \delta\rho}{\partial t} + \rho_0\nabla\cdot\left(\delta\mathbf{v}\right) = 0$$

$$\frac{\partial \delta s}{\partial t} = \gamma\kappa\nabla^2\delta s + \frac{c_p\kappa}{T_0}\left(\frac{\partial T}{\partial \rho}\right)_s \nabla^2\delta\rho$$

It can be shown that

$$\left(\frac{\partial T}{\partial \rho}\right)_s = \frac{Tc^2\beta}{\rho c_p} \quad \text{where} \quad \beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \quad \text{(thermal expansion)}$$

Let $\delta\mathbf{v} \equiv \delta\mathbf{v}_0\, e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ $\quad \delta\rho \equiv \delta\rho_0\, e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ $\quad \delta s \equiv \delta s_0\, e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

Linearized hydrodynamic equations; plane wave solutions:

$$\omega\delta\mathbf{v}_0 = \frac{c^2\delta\rho_0}{\rho_0}\mathbf{k} + \frac{T_0\beta c^2}{c_p}\delta s_0\mathbf{k} - \frac{i\eta k^2}{\rho_0}\delta\mathbf{v}_0 - \frac{i}{\rho_0}\left(\zeta + \frac{1}{3}\eta\right)\mathbf{k}\left(\mathbf{k}\cdot\delta\mathbf{v}_0\right)$$
$$\omega\delta\rho_0 - \rho_0\ \mathbf{k}\cdot\delta\mathbf{v}_0 = 0$$
$$\omega\delta s_0 = -i\gamma\kappa k^2\delta s_0 - \frac{i\kappa\beta c^2}{\rho_0}k^2\delta\rho_0$$

In the absense of thermal expansion, $\beta = 0$

$$\omega\delta\mathbf{v}_0 = \frac{c^2\delta\rho_0}{\rho_0}\mathbf{k} - \frac{i\eta k^2}{\rho_0}\delta\mathbf{v}_0 - \frac{i}{\rho_0}\left(\zeta + \frac{1}{3}\eta\right)\mathbf{k}\left(\mathbf{k}\cdot\delta\mathbf{v}_0\right)$$
$$\omega\delta\rho_0 - \rho_0\ \mathbf{k}\cdot\delta\mathbf{v}_0 = 0$$
$$\omega\delta s_0 = -i\gamma\kappa k^2\delta s_0$$

➔Entropy and mechanical modes are independent

Linearized hydrodynamic equations; full plane wave solutions:

$$\omega\delta\mathbf{v}_0 = \frac{c^2\delta\rho_0}{\rho_0}\mathbf{k} + \frac{T_0\beta c^2}{c_p}\delta s_0\mathbf{k} - \frac{i\eta k^2}{\rho_0}\delta\mathbf{v}_0 - \frac{i}{\rho_0}\left(\zeta + \frac{1}{3}\eta\right)\mathbf{k}\left(\mathbf{k}\cdot\delta\mathbf{v}_0\right)$$
$$\omega\delta\rho_0 - \rho_0\ \mathbf{k}\cdot\delta\mathbf{v}_0 = 0$$
$$\omega\delta s_0 = -i\gamma\kappa k^2\delta s_0 - \frac{i\kappa\beta c^2}{\rho_0}k^2\delta\rho_0$$

Transverse modes $(\delta\mathbf{v}\cdot\mathbf{k} = 0)$:

$$\delta\rho_0 = 0 \quad \delta s_0 = 0$$
$$\left(\omega + \frac{i\eta k^2}{\rho_0}\right)\left(\delta\mathbf{v}\times\ \mathbf{k}\right) = 0 \qquad k = \pm\left(\frac{i\omega\rho_0}{\eta}\right)^{1/2}$$

Linearized hydrodynamic equations; full plane wave solutions:

$$\omega\delta\mathbf{v}_0 = \frac{c^2\delta\rho_0}{\rho_0}\mathbf{k} + \frac{T_0\beta c^2}{c_p}\delta s_0\mathbf{k} - \frac{i\eta k^2}{\rho_0}\delta\mathbf{v}_0 - \frac{i}{\rho_0}\left(\zeta + \frac{1}{3}\eta\right)\mathbf{k}\left(\mathbf{k}\cdot\delta\mathbf{v}_0\right)$$
$$\omega\delta\rho_0 - \rho_0\ \mathbf{k}\cdot\delta\mathbf{v}_0 = 0$$
$$\omega\delta s_0 = -i\gamma\kappa k^2\delta s_0 - \frac{i\kappa\beta c^2}{\rho_0}k^2\delta\rho_0$$

Longitudinal solutions: $(\delta\mathbf{v}\cdot\mathbf{k} \neq 0)$:

$$\left(\omega^2 - c^2k^2 + i\frac{\omega k^2}{\rho_0}\left(\frac{4}{3}\eta + \zeta\right)\right)\delta\rho_0 - \frac{\rho_0 T_0\beta c^2k^2}{c_p}\delta s_0 = 0$$
$$\frac{i\kappa\beta c^2}{\rho_0}k^2\delta\rho_0 + \left(\omega + i\gamma\kappa k^2\right)\delta s_0 = 0$$

Linearized hydrodynamic equations; full plane wave solutions:

Longitudinal solutions: $(\delta \mathbf{v} \cdot \mathbf{k} \neq 0)$:

$$\left(\omega^2 - c^2k^2 + i\frac{\omega k^2}{\rho_0}\left(\frac{4}{3}\eta + \zeta\right)\right)\delta\rho_0 - \frac{\rho_0 T_0 \beta c^2 k^2}{c_p}\delta s_0 = 0$$

$$\frac{i\kappa\beta c^2}{\rho_0}k^2\delta\rho_0 + \left(\omega + i\gamma\kappa k^2\right)\delta s_0 = 0$$

Approximate solution: $k = \frac{\omega}{c} + i\alpha$

where $\alpha \approx \frac{\omega^2}{2c^3\rho_0}\left(\frac{4}{3}\eta + \zeta\right) + \frac{\kappa T_0 \beta^2 \omega^2}{2c_p c}$

$$\delta\rho = \delta\rho_0 e^{-\alpha \hat{\mathbf{k}}\cdot\mathbf{r}} e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$