PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 7:

Continue reading Chapter 3

1. **Lagrange's equations**
2. **D'Alembert's principle**

PHY 711 Classical Mechanics and Mathematical Methods

MWF 10 AM-10:50 PM	OPL 103	http://www.wfu.edu/~natalie/f15phy711/

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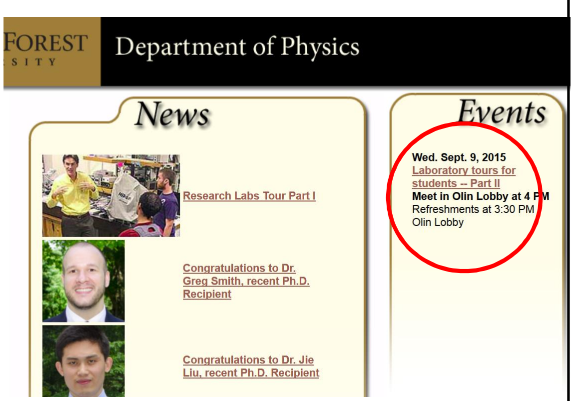
Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment
1	Wed, 8/26/2015	Chap. 1	Review of basic principles	#1
2	Fri, 8/28/2015	Chap. 1	Scattering theory	#2
3	Mon, 8/31/2015	Chap. 1	Scattering theory continued	#3
4	Wed, 9/02/2015	Chap. 2	Accelerated coordinate systems	#4
5	Fri, 9/04/2015	Chap. 3	Calculus of variations	#5
6	Mon, 9/07/2015	Chap. 3	Calculus of variations	#6
7	Wed, 9/09/2015	Chap. 3	Hamilton's principle	#7
8	Fri, 9/11/2015	Chap. 3 & 6	Hamilton's principle	
9	Mon, 9/14/2015	Chap. 3 & 6	Lagrangians with constraints	
10	Wed, 9/16/2015	Chap. 3 & 6	Lagrangians and constants of motion	



WFU Physics Colloquium

TITLE: "Laboratory tours for students -- Part II"
TIME: Wednesday Sept. 9, 2015 at 4:00 PM
PLACE: Olin Lounge

Refreshments will be served for everyone at 3:30 PM in the lounge. All interested persons are cordially invited to attend.

PROGRAM

This is the second of three opportunities that students will have to learn about the research which is being done in the Physics Department. Students will meet in the Olin lobby and divide into three groups. Each group will tour the labs of Professors Guthold, Kim-Shapiro, and Macosko.

Summary of results from the calculus of variation

For the class of problems where we need to perform an extremization on an integral form:

$$I = \int_{x_i}^{x_f} f\left(\left\{ y(x), \frac{dy}{dx} \right\}, x\right) dx$$

A necessary condition is the Euler - Lagrange equations:

$$\left(\frac{\partial f}{\partial y}\right) - \frac{d}{dx}\left[\left(\frac{\partial f}{\partial (dy/dx)}\right)\right] = 0$$

$$\frac{d}{dx}\left(f - \frac{\partial f}{\partial (dy/dx)} \frac{dy}{dx}\right) = \left(\frac{\partial f}{\partial x}\right)$$

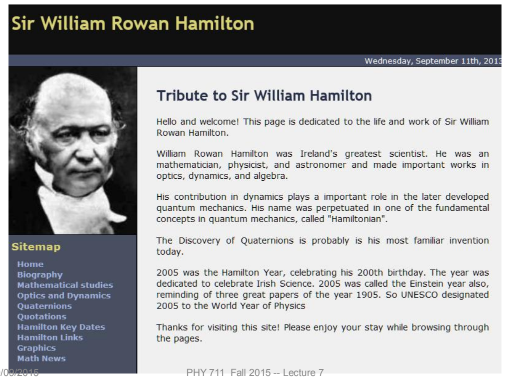
Application to particle dynamics

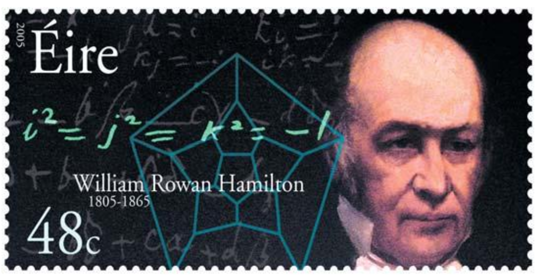
Simple example: vertical trajectory of particle of mass *m* subject to constant downward acceleration *a=-g.*

$$m\frac{d^2 y}{dt^2} = -mg$$

$$y(t) = y_i + v_i t - \tfrac{1}{2} g t^2$$

http://www.hamilton2005.ie/





http://rjlipton.wordpress.com

Now consider the Lagrangian defined to be:

$$L\left(\left\{y(t), \frac{dy}{dt}\right\}, t\right) \equiv T - U$$

Kinetic energy (T) — Potential energy (U)

In our example:

$$L\left(\left\{y(t), \frac{dy}{dt}\right\}, t\right) \equiv T - U = \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 - mgy$$

Hamilton's principle states:

$$S \equiv \int_{t_i}^{t_f} \left(\frac{1}{2} m \left(\frac{dy}{dt}\right)^2 - mgy \right) dt \quad \text{is minimized for physical } y(t):$$

Condition for minimizing the action:

$$S \equiv \int_{t_i}^{t_f} \left(\frac{1}{2} m \left(\frac{dy}{dt} \right)^2 - mgy \right) dt$$

Euler - Lagrange relations:

$$\frac{\partial L}{\partial y} - \frac{d}{dt} \frac{\partial L}{\partial \dot{y}} = 0$$

$$\Rightarrow -mg - \frac{d}{dt} m\dot{y} = 0$$

$$\Rightarrow \frac{d}{dt} \frac{dy}{dt} = -g$$

$$y(t) = y_i + v_i t - \tfrac{1}{2} g t^2$$

Check:

$$S \equiv \int_{t_i}^{t_f} \left(\frac{1}{2} m \left(\frac{dy}{dt} \right)^2 - mgy \right) dt$$

Assume $t_i = 0, \; y_i = h \equiv \tfrac{1}{2} g T^2; \quad t_f = T, \; y_f = 0$

Trial trajectories: $y_1(t) = \tfrac{1}{2} g T^2 \left(1 - t/T\right) = h - \tfrac{1}{2} gTt$

$$y_2(t) = \tfrac{1}{2} g T^2 \left(1 - t^2/T^2\right) = h - \tfrac{1}{2} g t^2$$

$$y_3(t) = \tfrac{1}{2} g T^2 \left(1 - t^3/T^3\right) = h - \tfrac{1}{2} g t^3 / T$$

Maple says:

$$S_1 = -0.125 m g^2 T^3$$

$$S_2 = -0.167 m g^2 T^3$$

$$S_3 = -0.150 m g^2 T^3$$

Jean d'Alembert 1717-1783

D'Alembert's principle:

Generalized coordinates: $q_\sigma(\{x_i\})$

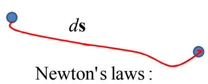
Newton's laws:

$$\mathbf{F}\text{-}m\mathbf{a}=0 \qquad \Rightarrow (\mathbf{F}\text{-}m\mathbf{a})\cdot d\mathbf{s}=0$$

$$\mathbf{F}\cdot d\mathbf{s}=\sum_\sigma\sum_i F_i\frac{\partial x_i}{\partial q_\sigma}\delta q_\sigma$$

For a conservative force: $F_i=-\dfrac{\partial U}{\partial x_i}$

$$\mathbf{F}\cdot d\mathbf{s}=-\sum_\sigma\sum_i\frac{\partial U}{\partial x_i}\frac{\partial x_i}{\partial q_\sigma}\delta q_\sigma=-\sum_\sigma\frac{\partial U}{\partial q_\sigma}\delta q_\sigma$$



Generalized coordinates: $q_\sigma(\{x_i\})$

Newton's laws:

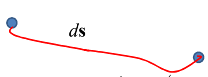
$$\mathbf{F}\text{-}m\mathbf{a}=0 \qquad \Rightarrow (\mathbf{F}\text{-}m\mathbf{a})\cdot d\mathbf{s}=0$$

$$m\mathbf{a}\cdot d\mathbf{s}=\sum_\sigma\sum_i m\ddot{x}_i\frac{\partial x_i}{\partial q_\sigma}\delta q_\sigma$$

$$=\sum_\sigma\sum_i\left(\frac{d}{dt}\left(m\dot{x}_i\frac{\partial x_i}{\partial q_\sigma}\right)-m\dot{x}_i\frac{d}{dt}\frac{\partial x_i}{\partial q_\sigma}\right)\delta q_\sigma$$

Claim: $\dfrac{\partial x_i}{\partial q_\sigma}=\dfrac{\partial \dot{x}_i}{\partial \dot{q}_\sigma}$ and $\dfrac{d}{dt}\dfrac{\partial x_i}{\partial q_\sigma}=\dfrac{\partial}{\partial q_\sigma}\dfrac{dx_i}{dt}\equiv\dfrac{\partial \dot{x}_i}{\partial q_\sigma}$

$$m\mathbf{a}\cdot d\mathbf{s}=\sum_\sigma\sum_i\left(\frac{d}{dt}\left(\frac{\partial\left(\frac{1}{2}m\dot{x}_i^{\,2}\right)}{\partial\dot{q}_\sigma}\right)-\frac{\partial\left(\frac{1}{2}m\dot{x}_i^{\,2}\right)}{\partial q_\sigma}\right)\delta q_\sigma$$



Generalized coordinates: $q_\sigma(\{x_i\})$

$$m\mathbf{a}\cdot d\mathbf{s}=\sum_\sigma\sum_i\left(\frac{d}{dt}\left(\frac{\partial\left(\frac{1}{2}m\dot{x}_i^{\,2}\right)}{\partial\dot{q}_\sigma}\right)-\frac{\partial\left(\frac{1}{2}m\dot{x}_i^{\,2}\right)}{\partial q_\sigma}\right)\delta q_\sigma$$

Define -- kinetic energy: $T\equiv\sum_i\frac{1}{2}m\dot{x}_i^{\,2}$

$$m\mathbf{a}\cdot d\mathbf{s}=\sum_\sigma\left(\frac{d}{dt}\frac{\partial T}{\partial\dot{q}_\sigma}-\frac{\partial T}{\partial q_\sigma}\right)\delta q_\sigma$$

Recall:

$$\mathbf{F}\cdot d\mathbf{s}=\sum_\sigma\sum_i\frac{\partial U}{\partial x_i}\frac{\partial x_i}{\partial q_\sigma}\delta q_\sigma=\sum_\sigma\frac{\partial U}{\partial q_\sigma}\delta q_\sigma$$

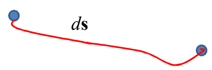
$$(\mathbf{F}\text{-}m\mathbf{a})\cdot d\mathbf{s}=\sum_\sigma\frac{\partial U}{\partial q_\sigma}\delta q_\sigma-\sum_\sigma\left(\frac{d}{dt}\frac{\partial T}{\partial\dot{q}_\sigma}-\frac{\partial T}{\partial q_\sigma}\right)\delta q_\sigma=0$$

Generalized coordinates:
$q_\sigma(\{x_i\})$

$$(\mathbf{F}\text{-}m\mathbf{a})\cdot d\mathbf{s} = -\sum_\sigma \frac{\partial U}{\partial q_\sigma}\delta q_\sigma - \sum_\sigma\left(\frac{d}{dt}\frac{\partial T}{\partial \dot q_\sigma} - \frac{\partial T}{\partial q_\sigma}\right)\delta q_\sigma = 0$$
$$= -\sum_\sigma \left(\frac{d}{dt}\frac{\partial(T-U)}{\partial\dot q_\sigma} - \frac{\partial (T-U)}{\partial q_\sigma}\right)\delta q_\sigma = 0$$

$$L(q_\sigma, \dot q_\sigma; t) = T - U$$

Note: This is only true if $\frac{\partial U}{\partial \dot q_\sigma} = 0$



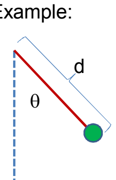
Generalized coordinates:
$q_\sigma(\{x_i\})$

Define -- Lagrangian: $L \equiv T - U$
$$L = L(\{q_\sigma\}, \{\dot q_\sigma\}, t)$$
$$(\mathbf{F}\text{-}m\mathbf{a})\cdot d\mathbf{s} = -\sum_\sigma \left(\frac{d}{dt}\frac{\partial L}{\partial \dot q_\sigma} - \frac{\partial L}{\partial q_\sigma}\right)\delta q_\sigma = 0$$

$\Rightarrow$ Minimization integral: $S = \int_{t_i}^{t_f} L(\{q_\sigma\}, \{\dot q_\sigma\}, t)\,dt$

Euler – Lagrange equations: $L = L(\{q_\sigma\}, \{\dot q_\sigma\}, t) = T - U$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot q_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

Example:



$$L = L(\theta, \dot\theta) = \tfrac{1}{2} m d^2 \dot\theta^2 - mg(d - d\cos\theta)$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot q_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0 \quad \Rightarrow \frac{d}{dt} m d^2 \dot\theta - mgd\sin\theta = 0$$
$$\frac{d^2\theta}{dt^2} = \frac{g}{d}\sin\theta$$

Another example : $L = L(\{q_\sigma\}, \{\dot{q}_\sigma\}, t) = T - U$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

$$L = L(\alpha, \beta, \gamma, \dot{\alpha}, \dot{\beta}, \dot{\gamma}) = \tfrac{1}{2} I_1\left(\dot{\alpha}^2 \sin^2\beta + \dot{\beta}^2\right) + \tfrac{1}{2} I_3\left(\dot{\alpha}\cos\beta + \dot{\gamma}\right)^2 - Mgd\cos\beta$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\alpha}} = \frac{d}{dt}\left(I_1\dot{\alpha}\sin^2\beta + I_3\left(\dot{\alpha}\cos\beta + \dot{\gamma}\right)\cos\beta\right) = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\beta}} = \frac{d}{dt}\left(I_1\dot{\beta}\right) = \text{mess}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\gamma}} = \frac{d}{dt}\left(I_3\left(\dot{\alpha}\cos\beta + \dot{\gamma}\right)\right) = 0$$