PHY 752 Solid State Physics
11-11:50 AM MWF Olin 103

Plan for Lecture 23:

- Optical and transport properties of metals (Chap. 11 in GGGPP)
  - Macroscopic theory
  - Drude model

Note: Debye-Waller discussion postponed to consideration of Chapter 9.

9	Mon, 9/14/2015	Chap. 2.4-2.7	Densities of states	#8`
10	Wed, 9/16/2015	Chap. 3	Free electron model	#9
11	Fri, 9/18/2015	Chap. 4	One electron approximations to the many electron problem	#10
12	Mon, 9/21/2015	Chap. 4	One electron approximations to the many electron problem	#11
13	Wed, 9/23/2015	Chap. 4	Density functional theory	#12
14	Fri, 9/25/2015	Chap. 5	Implementation of density functional theory	#13
15	Mon, 9/28/2015	Chap. 5	Implementation of density functional theory	#14
16	Wed, 9/30/2015	Chap. 5	First principles pseudopotential methods	#15
17	Fri, 10/02/2015	Chap. 6	Example electronic structures	#16
18	Mon, 10/05/2015	Chap. 6	Ionic and covalent crystals	#17
19	Wed, 10/07/2015	Chap. 6	More examples of electronic structures	#18
20	Fri, 10/09/2015	Chap. 1-6	Review	Start exam
	Mon, 10/12/2015		No class	Take-home exam
	Wed, 10/14/2015		No class	Exam due before 10/19/2015
	Fri, 10/16/2015		Fall break -- no class	
21	Mon, 10/19/2015	Chap. 10	X-ray and neutron diffraction	#HW19
22	Wed, 10/21/2015	Chap. 10	Scattering of particles by crystals	#HW20
23	Fri, 10/23/2015	Chap. 11	Optical and transport properties of metals	#HW21
	Wed, 12/02/2015		Student presentations I	
	Fri, 12/04/2015		Student presentations II	

Maxwell's Equations (cgs Gaussian units)

$$\operatorname{div}\mathbf{E} = 4\pi\rho, \quad \operatorname{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\operatorname{div}\mathbf{B} = 0, \quad \operatorname{curl}\mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J}$$

Assume pure time-harmonic frequency ω and fixed geometry:

$$\mathbf{E}(\mathbf{r},t) = E(z)e^{-i\omega t}\,\widehat{\mathbf{e}}_x$$
$$\mathbf{B}(\mathbf{r},t) = B(z)e^{-i\omega t}\,\widehat{\mathbf{e}}_y$$
$$\mathbf{J}(\mathbf{r},t) = J(z)e^{-i\omega t}\,\widehat{\mathbf{e}}_x$$

$\rho = 0 \qquad \nabla\cdot\mathbf{J} = 0$

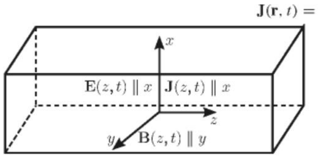
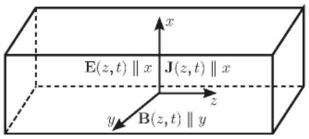


Figure 11.1 Geometry chosen for the description of transverse electromagnetic fields in isotropic materials. The electric field and the internal density current are in the x-direction, the magnetic field is in the y-direction, the wave propagation is along the z-direction.



$$\frac{dE(z)}{dz} = \frac{i\omega}{c}B(z),$$
$$-\frac{dB(z)}{dz} = -\frac{i\omega}{c}E(z) + \frac{4\pi}{c}J(z).$$

$$\boxed{\frac{d^2E(z)}{dz^2} = -\frac{\omega^2}{c^2}E(z) - \frac{4\pi i\omega}{c^2}J(z)}$$

Assume linear response of the electric field to produce the conductivity in terms of conductivity:

$$J_\alpha(\mathbf{r},t) = \sum_\beta \int d\mathbf{r}' \int_{-\infty}^{t} \sigma_{\alpha\beta}(\mathbf{r},\mathbf{r}',t,t')E_\beta(\mathbf{r}',t')\,dt' \qquad (\alpha,\beta = x,y,z),$$

For our geometry: $\mathbf{J}(\mathbf{r},t) = \int_{-\infty}^{+\infty} d\mathbf{r}' \int_{-\infty}^{+\infty} dt'\sigma(\mathbf{r}-\mathbf{r}',t-t')\,\mathbf{E}(\mathbf{r}',t')$

with: $\sigma(\mathbf{r}-\mathbf{r}',t-t') \equiv 0 \quad \text{for } t' > t.$

Taking Fourier transform in space and time:

$$\sigma(\mathbf{q},\omega) = \int_{-\infty}^{+\infty} e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}\sigma(\mathbf{r},t)\,d\mathbf{r}\,dt\,;$$

➡ $\boxed{\mathbf{J}(\mathbf{q},\omega) = \sigma(\mathbf{q},\omega)\,\mathbf{E}(\mathbf{q},\omega)}\,.$

For spatially uniform conductivity response, the **q**-dependence is trivial: $\mathbf{J}(\mathbf{r}) = \sigma(\omega)\mathbf{E}(\mathbf{r}),$

$$\frac{d^2E(z)}{dz^2} = -\frac{\omega^2}{c^2}E(z) - \frac{4\pi i\omega}{c^2}J(z) = -\frac{\omega^2}{c^2}\left[1 + \frac{4\pi i\sigma(\omega)}{\omega}\right]E(z).$$

For $E(z) = E_0 e^{i(\omega/c)Nz}$. with $q = N\frac{\omega}{c}$ $N \equiv$ complex refractive index

$$N^2 = 1 + \frac{4\pi i\sigma(\omega)}{\omega}$$

Writing complex refractive index in terms of real functions

N(ω)=n(ω)+ik(ω)

$$E(z) = E_0 e^{i(\omega/c)Nz} = E_0 e^{i(\omega/c)nz} e^{-(\omega/c)kz} \equiv E_0 e^{i(\omega/c)nz} e^{-z/\delta}$$

skin depth: $\delta = \frac{c}{\omega k}$

In terms of dielectric function:

$$D(z) = \epsilon E(z) = (\epsilon_1 + i\epsilon_2)E(z)$$

$$\varepsilon_1 = n^2 - k^2 \quad \text{and} \quad \varepsilon_2 = 2nk;$$

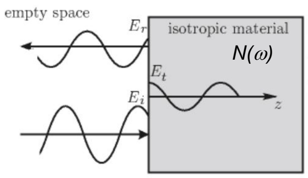
$$n^2 = \frac{1}{2}\left(\varepsilon_1 + \sqrt{\varepsilon_1^2+\varepsilon_2^2}\right) \quad \text{and} \quad k^2 = \frac{1}{2}\left(-\varepsilon_1 + \sqrt{\varepsilon_1^2+\varepsilon_2^2}\right).$$

In terms of complex conductivity:

$$\varepsilon(\omega) = 1 + \frac{4\pi i \sigma(\omega)}{\omega}$$

$$\varepsilon_1(\omega) = 1 - \frac{4\pi\sigma_2(\omega)}{\omega} \quad \text{and} \quad \varepsilon_2(\omega) = \frac{4\pi\sigma_1(\omega)}{\omega}$$

Reflection of electromagnetic wave at a planar interface at normal interface



$$E(z) = E_t e^{i(\omega/c)Nz} \qquad z > 0,$$
$$E(z) = E_i e^{i(\omega/c)z} + E_r e^{-i(\omega/c)z} \qquad z < 0.$$

Reflectivity: $R = \left|\frac{E_r}{E_i}\right|^2 = \left|\frac{1-N}{1+N}\right|^2 = \frac{(n-1)^2+k^2}{(n+1)^2+k^2}.$

Summary of relationships

Conductivity	$\sigma = \sigma_1 + i\sigma_2$
Dielectric constant	$\varepsilon = \varepsilon_1 + i\varepsilon_2, \quad \varepsilon = 1 + \frac{4\pi i\sigma}{\omega}, \quad \varepsilon_1 = 1 - \frac{4\pi\sigma_2}{\omega}, \quad \varepsilon_2 = \frac{4\pi\sigma_1}{\omega}$
Refractive index	$N = n + ik, \quad \varepsilon = N^2, \quad \varepsilon_1 = n^2 - k^2, \quad \varepsilon_2 = 2nk$
	$n^2 = \frac{1}{2}\left(\varepsilon_1 + \sqrt{\varepsilon_1^2+\varepsilon_2^2}\right), \quad k^2 = \frac{1}{2}\left(-\varepsilon_1 + \sqrt{\varepsilon_1^2+\varepsilon_2^2}\right)$
Absorption coefficient	$\alpha(\omega) = 2\omega k/c = \omega\varepsilon_2/nc$
Classical skin depth	$\delta = c/\omega k$
Surface impedance	$Z = 4\pi/cN$
Reflectivity	$R = \frac{(n-1)^2+k^2}{(n+1)^2+k^2}$ (at normal incidence)

Analytic properties of dielectric function

$$\epsilon(t) = 0 \text{ for } t < 0.$$

$$\epsilon(\omega) = \int_0^\infty dt\, e^{i\omega t}\epsilon(t).$$

$$\epsilon(\omega) = \oint \frac{d\omega'}{2\pi i}\frac{\epsilon(\omega')}{\omega' - \omega - i\eta}.$$

$$\epsilon(\omega) - \epsilon^\infty = \oint \frac{d\omega'}{2\pi i}\frac{\epsilon(\omega') - \epsilon^\infty}{\omega' - \omega - i\eta}.$$

Here η represents a small infinitesimal imaginary contribution

$$\epsilon(\omega) - \epsilon^{\infty} = \oint \frac{d\omega'}{2\pi i} \frac{\epsilon(\omega') - \epsilon^{\infty}}{\omega' - \omega - i\eta}.$$

Evaluation using Cauchy integral theorem

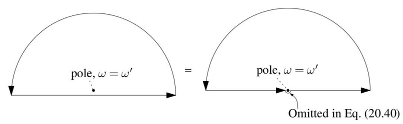


Figure 20.3: The integral at distance η above the real axis can be deformed into a contour integral on the real axis, with a contribution from a half-circuit around the pole at $\omega = \omega'$.

In terms of principle parts integral over negative and positive frequencies

$$\epsilon(\omega) - \epsilon^{\infty} = \mathcal{P} \int \frac{d\omega'}{\pi i} \frac{\epsilon(\omega') - \epsilon^{\infty}}{\omega' - \omega}$$

$$\begin{aligned} \mathrm{Re}[\epsilon(\omega) - \epsilon^{\infty}] &= \mathcal{P} \int \frac{d\omega'}{\pi} \frac{\mathrm{Im}[\epsilon(\omega') - \epsilon^{\infty}]}{\omega' - \omega} \\ \mathrm{Im}[\epsilon(\omega) - \epsilon^{\infty}] &= -\mathcal{P} \int \frac{d\omega'}{\pi} \frac{\mathrm{Re}[\epsilon(\omega') - \epsilon^{\infty}]}{\omega' - \omega}. \end{aligned}$$

In terms of positive frequencies only

$$\begin{aligned} \epsilon_1(\omega) - \epsilon^{\infty} &= \mathcal{P} \int_0^{\infty} \frac{2\omega' d\omega'}{\pi} \frac{\epsilon_2(\omega')}{\omega'^2 - \omega^2} \\ \epsilon_2(\omega) &= -\mathcal{P} \int_0^{\infty} \frac{2\omega d\omega'}{\pi} \frac{\epsilon_1(\omega') - \epsilon^{\infty}}{\omega'^2 - \omega^2}. \end{aligned}$$

Analysis of reflectivity data

$$\tilde{r} = \frac{\tilde{n}-1}{\tilde{n}+1} \equiv \rho e^{i\theta}.$$

$$\ln\left(\frac{\tilde{r}(\omega)}{\tilde{r}(0)}\right) = \ln(\rho(\omega)/\rho(0)) + i(\theta(\omega)-\theta(0)),$$

$$\theta(\omega)-\theta(0) = -\frac{1}{\pi}\mathcal{P}\int d\omega' \ln\left[\frac{\rho(\omega')}{\rho(0)}\right]\left[\frac{1}{\omega'-\omega}-\frac{1}{\omega'}\right]$$

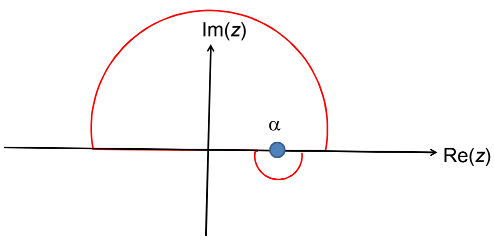
$$\Rightarrow \theta(\omega) = -\frac{2\omega}{\pi}\mathcal{P}\int_0^\infty d\omega' \frac{\ln\rho(\omega')}{\omega'^2-\omega^2}.$$

Some more detailed notes:

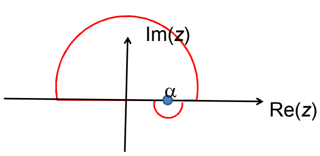
Analytic properties of the dielectric function (in the Drude model or from "first principles" -- Kramers-Kronig transform

Consider Cauchy's integral formula for an analytic function $f(z)$:

$$\oint dz\, f(z) = 0 \qquad f(\alpha) = \frac{1}{2\pi i}\oint_{\text{includes }\alpha} dz \frac{f(z)}{z-\alpha}$$



Kramers-Kronig transform -- continued



$$f(\alpha) = \frac{1}{2\pi i}\oint_{\text{includes }\alpha} dz \frac{f(z)}{z-\alpha} = \frac{1}{2\pi i}\left(\int_{-\infty}^{\infty} dz_R \frac{f(z_R)}{z_R-\alpha} + \int dz \frac{f(z)}{z-\alpha}\right)$$

=0

$$f(\alpha) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} dz_R \frac{f(z_R)}{z_R-\alpha} = \frac{1}{2\pi i}P\int_{-\infty}^{\infty} dz_R \frac{f(z_R)}{z_R-\alpha} + \frac{1}{2}f(\alpha)$$

Kramers-Kronig transform -- continued

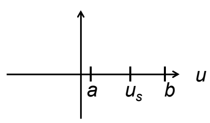
$$f(\alpha)=\frac{1}{2\pi i}P\int_{-\infty}^{\infty}dz_R\,\frac{f(z_R)}{z_R-\alpha}+\frac{1}{2}f(\alpha)$$

Suppose $f(z_R)=f_R(z_R)+if_I(z_R)$:

$$\Rightarrow \frac{1}{2}\left(f_R(\alpha)+if_I(\alpha)\right)=\frac{1}{2\pi i}P\int_{-\infty}^{\infty}dz_R\,\frac{f_R(z_R)+if_I(z_R)}{z_R-\alpha}$$

$$\Rightarrow f_R(\alpha)=\frac{1}{\pi}P\int_{-\infty}^{\infty}dz_R\,\frac{f_I(z_R)}{z_R-\alpha}$$

$$f_I(\alpha)=-\frac{1}{\pi}P\int_{-\infty}^{\infty}dz_R\,\frac{f_R(z_R)}{z_R-\alpha}$$



Some practical considerations

Principal parts integration:

$$P\int_a^b du\, g(u)=\lim_{\upsilon\to 0}\left(\int_a^{u_s-\upsilon}du\, g(u)+\int_{u_s+\upsilon}^{b}du\, g(u)\right)$$

Example:

$$P\int_a^b du\,\frac{1}{u-u_s}=\lim_{\upsilon\to 0}\left(\int_a^{u_s-\upsilon}du\,\frac{1}{u-u_s}+\int_{u_s+\upsilon}^{b}du\,\frac{1}{u-u_s}\right)$$

$$=\lim_{\upsilon\to 0}\left(\ln\left(\frac{\upsilon}{u_s-a}\right)+\ln\left(\frac{b-u_s}{\upsilon}\right)\right)=\ln\left(\frac{b-u_s}{u_s-a}\right)$$

Kramers-Kronig transform -- continued

$$f_R(\alpha)=\frac{1}{\pi}P\int_{-\infty}^{\infty}dz_R\,\frac{f_I(z_R)}{z_R-\alpha}$$

$$f_I(\alpha)=-\frac{1}{\pi}P\int_{-\infty}^{\infty}dz_R\,\frac{f_R(z_R)}{z_R-\alpha}$$

This Kramers-Kronig transform is useful for the dielectric function

when $f(z_R)\Rightarrow\frac{\varepsilon(\omega)}{\varepsilon_0}-1$

Must show that: 1. $f(z)$ is analytic for $z_I\geq 0$

2. $f(z)$ vanishes for $z\to\infty$

To be justified from Drude model --

Kramers-Kronig transform – for dielectric function:

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0}-1=\frac{1}{\pi}P\int_{-\infty}^{\infty}d\omega'\frac{\varepsilon_I(\omega')}{\varepsilon_0}\frac{1}{\omega'-\omega}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0}=-\frac{1}{\pi}P\int_{-\infty}^{\infty}d\omega'\left(\frac{\varepsilon_R(\omega')}{\varepsilon_0}-1\right)\frac{1}{\omega'-\omega}$$

$$\text{with}\quad \varepsilon_R(-\omega)=\varepsilon_R(\omega);\quad \varepsilon_I(-\omega)=-\varepsilon_I(\omega)$$

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0}-1=\frac{2}{\pi}P\int_{0}^{\infty}d\omega'\frac{\varepsilon_I(\omega')}{\varepsilon_0}\frac{\omega'}{\omega'^2-\omega^2}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0}=-\frac{2}{\pi}P\int_{-\infty}^{\infty}d\omega'\left(\frac{\varepsilon_R(\omega')}{\varepsilon_0}-1\right)\frac{1}{\omega'^2-\omega^2}$$

Paul Karl Ludwig Drude 1863-1906

http://photos.aip.org/history/Thumbnails/drude_paul_a1.jpg

Drude model:

Vibrations of charged particles near equilibrium:

$$m\delta\ddot{\mathbf{r}}=q\mathbf{E}_0e^{-i\omega t}-m\omega_0^2\delta\mathbf{r}-m\gamma\delta\dot{\mathbf{r}}$$

$$\text{For}\quad \delta\mathbf{r}\equiv\delta\mathbf{r}_0e^{-i\omega t},\quad \delta\mathbf{r}_0=\frac{q\mathbf{E}_0}{m}\frac{1}{\omega_0^2-\omega^2-i\omega\gamma}$$

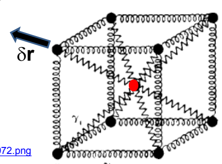
Induced dipole:

$$\mathbf{p}=q\,\delta\mathbf{r}=\frac{q^2\mathbf{E}_0}{m}\frac{1}{\omega_0^2-\omega^2-i\omega\gamma}e^{-i\omega t}$$

Displacement field:

$$\mathbf{D}=\varepsilon\mathbf{E}=\varepsilon_0\mathbf{E}+\mathbf{P}$$

$$\mathbf{P}=\sum_i\mathbf{p}_i\delta^3(\mathbf{r}-\mathbf{r}_i)$$



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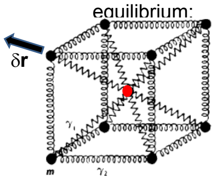
Drude model:

Vibration of particle of charge q and mass m near equilibrium:



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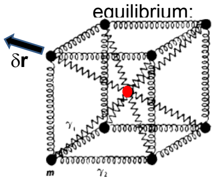
$$m\delta\,\ddot{\mathbf{r}} = q\mathbf{E}_0 e^{-i\omega t} - m\omega_0^2\delta\,\mathbf{r} - m\gamma\delta\,\dot{\mathbf{r}}$$

Note that:
- $\gamma > 0$ represents dissipation of energy.
- ω_0 represents the natural frequency of the vibration; $\omega_0=0$ would represent a free (unbound) particle

Drude model:

Vibration of particle of charge q and mass m near equilibrium:



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$$m\delta\,\ddot{\mathbf{r}} = q\mathbf{E}_0 e^{-i\omega t} - m\omega_0^2\delta\,\mathbf{r} - m\gamma\delta\,\dot{\mathbf{r}}$$

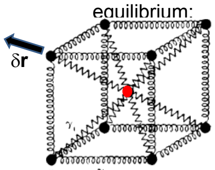
For $\delta\mathbf{r} \equiv \delta\mathbf{r}_0 e^{-i\omega t}, \quad \delta\mathbf{r}_0 = \frac{q\mathbf{E}_0}{m}\frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}$

Induced dipole:

$$\mathbf{p} = q\;\delta\mathbf{r} = \frac{q^2\mathbf{E}_0}{m}\frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}e^{-i\omega t}$$

Drude model:

Vibration of particle of charge q and mass m near equilibrium:



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$$m\delta\,\ddot{\mathbf{r}} = q\mathbf{E}_0 e^{-i\omega t} - m\omega_0^2\delta\,\mathbf{r} - m\gamma\delta\,\dot{\mathbf{r}}$$

Displacement field:

$$\mathbf{D} = \varepsilon\mathbf{E} \quad = \varepsilon_0\mathbf{E} + \mathbf{P}$$

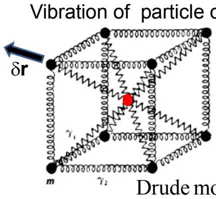
$$\mathbf{P} = \sum_i \mathbf{p}_i\delta^3(\mathbf{r}-\mathbf{r}_i) \approx N\sum_i f_i\,\mathbf{p}_i$$

$N \equiv$ number dipole/volume

$f_i \equiv$ fraction of type i dipoles

Drude model:

Vibration of particle of charge *q* and mass *m* near equilibrium:



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Drude model expression for permittivity :

$$\mathbf{D} = \varepsilon\mathbf{E} \quad = \varepsilon_0\mathbf{E} + \mathbf{P} = \varepsilon_0\mathbf{E} + N\sum_i f_i\,\mathbf{p}_i$$

$$\mathbf{p}_i \;= q_i\;\delta\mathbf{r} = \frac{q_i^{\;2}\mathbf{E}_0}{m_i}\frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}e^{-i\omega t}$$

$$\varepsilon\mathbf{E} \;\; = \varepsilon_0\mathbf{E}_0 e^{-i\omega t}\left(1 + N\sum_i f_i\frac{q_i^{\;2}}{\varepsilon_0 m_i}\frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}\right)$$

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Drude model dielectric function:

$$\frac{\varepsilon(\omega)}{\varepsilon_0} \;= 1 + N\sum_i f_i\frac{q_i^{\;2}}{\varepsilon_0 m_i}\frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

$$= \frac{\varepsilon_R(\omega)}{\varepsilon_0} + i\frac{\varepsilon_I(\omega)}{\varepsilon_0}$$

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i\frac{q_i^{\;2}}{\varepsilon_0 m_i}\frac{\omega_i^2 - \omega^2}{\left(\omega_i^2 - \omega^2\right)^2 + \omega^2\gamma_i^{\;2}}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0} = N\sum_i f_i\frac{q_i^{\;2}}{\varepsilon_0 m_i}\frac{\omega\gamma_i}{\left(\omega_i^2 - \omega^2\right)^2 + \omega^2\gamma_i^{\;2}}$$

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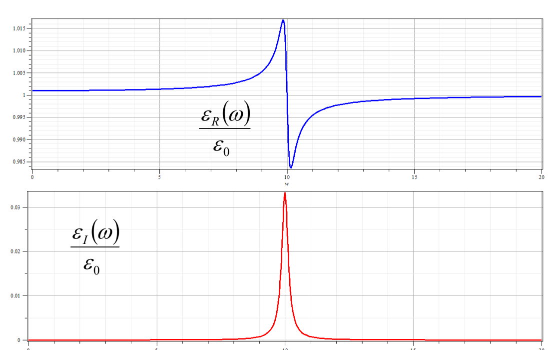
Drude model dielectric function:



Drude model dielectric function – some analytic properties:

$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

$$\text{For } \omega >> \omega_i \quad \frac{\varepsilon(\omega)}{\varepsilon_0} \approx 1 - \frac{1}{\omega^2}\left(N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\right)$$

$$\equiv 1 - \frac{\omega_P^2}{\omega^2}$$

Drude model dielectric function – some analytic properties:

$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

For $\omega_0 = 0$ (representing a free particle of charge q_0, mass m_0

$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + N\sum_{i>0} f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i} + iNf_0\frac{q_0^2}{\varepsilon_0 m_0}\frac{1}{\omega(\gamma_0 - i\omega)}$$

$$\equiv \frac{\varepsilon_b(\omega)}{\varepsilon_0} + i\frac{\sigma(\omega)}{\varepsilon_0\omega}$$

Some details:

$$\mathbf{D} = \varepsilon_b \mathbf{E} \qquad \mathbf{J} = \sigma\mathbf{E}$$

$$\nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t} = (\sigma - i\omega\varepsilon_b)\mathbf{E} = \varepsilon\frac{\partial\mathbf{E}}{\partial t} = -i\omega\left(\varepsilon_b + \frac{i\sigma}{\omega}\right)\mathbf{E}$$

$$\Rightarrow \sigma(\omega) = Nf_0\frac{q_0^2}{m_0}\frac{1}{(\gamma_0 - i\omega)}$$

Analysis for Drude model dielectric function:

$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

$$\text{Let} \quad f(z) = \frac{\varepsilon(z)}{\varepsilon_0} - 1 = N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{1}{\omega_i^2 - z^2 - iz\gamma_i}$$

For $|z| >> \omega_i$

$$f(z) \approx -\frac{1}{z^2}\left(N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\right) \Rightarrow \text{vanishes at large } z$$

Analysis for Drude model dielectric function – continued --
Analytic properties:

$$f(z) = \frac{\varepsilon(z)}{\varepsilon_0} - 1 = N \sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i} \frac{1}{\omega_i^2 - z^2 - iz\gamma_i}$$

$f(z)$ has poles z_P at $\omega_i^2 - {z_P}^2 - iz_P\gamma_i = 0$

$$z_P = -i\frac{\gamma_i}{2} \pm \sqrt{\omega_i^2 - \left(\frac{\gamma_i}{2}\right)^2}$$

Note that $\Im(z_P) \leq 0 \;\Rightarrow$ $f(z)$ is analytic for $\Im(z_P) > 0$

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