PHY 752 Solid State Physics
11-11:50 AM MWF Olin 103

Plan for Lecture 32:

Chapter 15 in GGGPP:

Electron Gas in Magnetic Fields

1. **Energy levels for 2-d electron gas**
2. **Energy levels for 3-d electron gas**

Lecture notes prepared with materials from GGGPP textbook

22	Wed, 10/21/2015	Chap. 10	Scattering of particles by crystals	#20
23	Fri, 10/23/2015	Chap. 11	Optical and transport properties of metals	#21
24	Mon, 10/26/2015	Chap. 11	Optical and transport properties of metals	#22
25	Wed, 10/28/2015	Chap. 11	Transport in metals	#23
26	Fri, 10/30/2015	Chap. 12	Optical properties of semiconductors and insulators	
27	Mon, 11/02/2015	Chap. 7 & 12	Excitons	#24
28	Wed, 11/04/2015	Chap. 9	Lattice vibrations	#25
29	Fri, 11/06/2015	Chap. 9	Lattice vibrations	#26
30	Mon, 11/09/2015	Chap. 13	Defects in semiconductors	#27
31	Wed, 11/11/2015	Chap. 14	Transport in semiconductors	#28
32	Fri, 11/13/2015	Chap. 15	Electron gas in Magnetic fields	#29
33	Mon, 11/16/2015	Chap. 15	Electron gas in Magnetic fields	
34	Wed, 11/18/2015	Chap. 17	Magnetic ordering in crystals	
35	Fri, 11/20/2015	Chap. 18	Superconductivity	
36	Mon, 11/23/2015	Chap. 18	Superconductivity	
	Wed, 11/25/2015		*Thanksgiving Holiday*	
	Fri, 11/27/2015		*Thanksgiving Holiday*	
37	Mon, 11/30/2015	Chap. 18	Superconductivity	
	Wed, 12/02/2015		Student presentations I	
	Fri, 12/04/2015		Student presentations II	
	Mon, 12/07/2015		Begin Take-home final	

Digression – densities of states for one, two, or three dimensional free electron gas

$$E(k_z) = E_0 + \frac{\hbar^2 k_z^2}{2m} \qquad D^{(1d)}(E) = \frac{L_z}{2\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{1}{\sqrt{E-E_0}}\Theta(E-E_0),$$

$$E(k_x,k_y) = E_0 + \frac{\hbar^2}{2m}\left(k_x^2+k_y^2\right) \qquad D^{(2d)}(E) = \frac{S}{4\pi}\frac{2m}{\hbar^2}\Theta(E-E_0),$$

$$E(\mathbf{k}) = E_0 + \frac{\hbar^2 k^2}{2m} \qquad D^{(3d)}(E) = \frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E-E_0}\,\Theta(E-E_0).$$

Consider a system of electrons confined to move in the x-y plane. In the absence of an external magnetic field, the energy levels are given by:

$$E(k_x, k_y) = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2\right)$$

Consider the effects of a uniform magnetic field along the z-axis

$$\mathbf{A}(\mathbf{r}) = (-By, 0, 0) \qquad \Longrightarrow \qquad \operatorname{curl}\mathbf{A}(\mathbf{r}) = B(0, 0, 1).$$

Effective Hamiltonian:

$$H = \frac{1}{2m}\left(p_x - \frac{eB}{c}y\right)^2 + \frac{1}{2m}p_y^2.$$

Effective potential -- continued

$$H = \frac{1}{2m}\left(p_x - \frac{eB}{c}y\right)^2 + \frac{1}{2m}p_y^2.$$

Taking into account constants of the motion

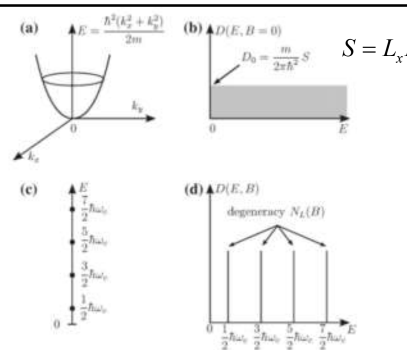
$$H = \frac{1}{2m}p_y^2 + \frac{1}{2m}\left(\hbar k_x - \frac{eB}{c}y\right)^2 = \frac{1}{2m}p_y^2 + \frac{1}{2}m\omega_c^2(y - y_0)^2,$$

where

$$\omega_c = \frac{eB}{mc}, \quad y_0 = \frac{\hbar c}{eB}k_x, \quad \text{and} \quad l = \sqrt{\frac{\hbar c}{eB}}$$

Eigenstates: $E_{nk_x} = \left(n + \frac{1}{2}\right)\hbar\omega_c \quad n = 0, 1, 2, \ldots,$

$$\phi_{nk_x}(\mathbf{r}) = \frac{1}{\sqrt{L_x}}e^{ik_x x}\left(\frac{m\omega_c}{\pi\hbar}\right)\frac{1}{2^{n/2}\sqrt{n!}}e^{-m\omega_c(y-y_0)^2/2\hbar}H_n\left[(y - y_0)\sqrt{\frac{m\omega_c}{\hbar}}\right]$$



$S = L_x L_y$

Figure 15.1 Energy levels (a) and density-of-states excluding spin degeneracy (b) of a two-dimensional electron gas in the absence of a magnetic field. The energy levels and the density-of-states in the presence of a magnetic field are indicated in (c) and (d), respectively. The degeneracy of the Landau levels is $N_L(B) = (e/hc)BS = D_0\hbar\omega_c$.

Degeneracy of Landau levels

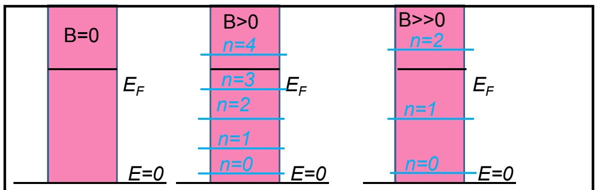
$$0 \le y_0 \equiv \frac{\hbar c}{eB} k_x < L_y \quad \text{or equivalently} \quad 0 \le k_x < \frac{eB}{\hbar c} L_y .$$

$$\boxed{N_L(B) = \frac{L_x}{2\pi} \frac{eB}{\hbar c} L_y \equiv \frac{e}{hc} BS}.$$

$$N_L(B) = \frac{\Phi(B)}{\Phi_0}, \qquad \Phi_0 = \frac{hc}{e} = 4.136 \times 10^{-7} \text{ gauss} \cdot \text{cm}^2$$

$$N_L(B) = D_0 \hbar \omega_c \quad \text{where} \quad D_0 = \frac{m}{2\pi \hbar^2} S$$

$$\nu = \frac{N}{\Phi/\Phi_0} = n_s \frac{hc}{eB}.$$ # occupied Landau levels



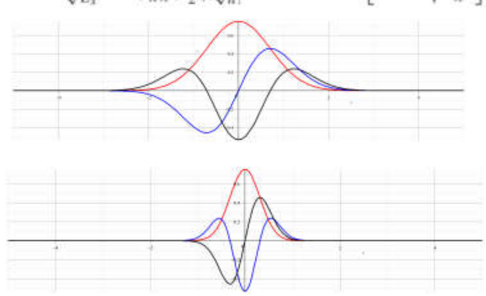
Degeneracy of each Landau level

$$N_L(B) = \underbrace{\frac{e}{hc}}_{\frac{1}{\Phi_0}} \underbrace{BL_xL_y}_{\Phi}$$

$$\Phi_0 = \frac{hc}{e} = 4.136 \times 10^{-7} \text{ gauss} \cdot \text{cm}^2$$

Eigenstates:

$$\phi_{nk_x}(\mathbf{r}) = \frac{1}{\sqrt{L_x}} e^{ik_x x} \left(\frac{m\omega_c}{\pi\hbar}\right) \frac{1}{2^{n/2}\sqrt{n!}} e^{-m\omega_c(y-y_0)^2/2\hbar} H_n\left[(y-y_0)\sqrt{\frac{m\omega_c}{\hbar}}\right]$$



Magnetic field effects on 3-dimensional electron gas

$$H = \frac{1}{2m}\left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2 = \frac{1}{2m}\left(p_x - \frac{eB}{c}y\right)^2 + \frac{1}{2m}p_y^2 + \frac{1}{2m}p_z^2\,;$$

$$E_{nk_z} = \left(n + \frac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m}\,,$$

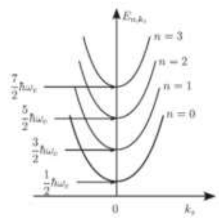


Figure 15.2 Schematic representation of the energy levels for a three-dimensional electron gas in the presence of a magnetic field in the z-direction.

Density of states for 3-dimensional system

$$D_n(E,B) = \frac{L_z}{2\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} \cdot N_L(B) \cdot 2\frac{1}{\sqrt{E - \left(n + \frac{1}{2}\right)\hbar\omega_c}} \quad \text{for} \quad E > \left(n + \frac{1}{2}\right)\hbar\omega_c.$$

$$D(E,B) = \frac{1}{2}\hbar\omega_c A \sum_{n=0}^{\infty} \frac{1}{\sqrt{E - (n + \frac{1}{2})\hbar\omega_c}}\,\Theta\left[E - \left(n + \frac{1}{2}\right)\hbar\omega_c\right].$$

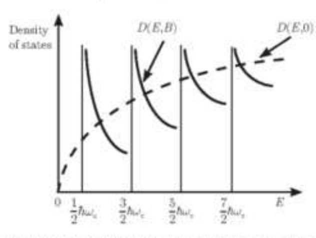


Figure 15.3 Density-of-states of a three-dimensional electron gas in the absence of a magnetic field (dashed line) and in the presence of a magnetic field (continuous lines).

Optical absorption in presence of Magnetic field; transitions between conduction and valence bands

$$E_{nk_z} = E_c + \left(n + \frac{1}{2}\right)\hbar\omega_c^* + \frac{\hbar^2 k_z^2}{2m_c^*}, \qquad E_{nk_z} = E_v - \left(n + \frac{1}{2}\right)\hbar\omega_v^* - \frac{\hbar^2 k_z^2}{2m_v^*}$$

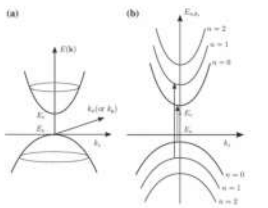


Figure 15.4 (a) Schematic representation of an isotropic two-band model semiconductor. (b) Schematic representation of quantization of electron states in the conduction band and in the valence band in the presence of a uniform magnetic field in the z direction. Arrows indicate examples of optical transitions between valence and conduction states with the same values of n and k_z.

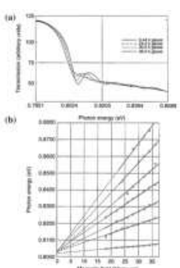


Figure 15.5 (a) Oscillatory magneto-absorption in germanium. (b) Energies of consecutive transmission minima as a function of the magnetic field. The lowest curve corresponds to transitions between Landau levels with $n = 0$, the successive curves correspond to transitions with higher quantum states of both conduction and valence bands [With permission from Figs. 1, 3, Phys. Rev. 108, 54 (1957)].

Next lecture – de Haas-van Alphen effect

Oscillator 1/B behavior of free energy of free electron gas

$$F(T,B) = F(T,0) + \frac{\hbar^2\omega_c^2}{24} D(\mu) + \frac{\hbar^2\omega_c^2}{4\pi^2} D(\mu)\left(\frac{\hbar\omega_c}{2\mu}\right)^{1/2} \cdot \sum_{s=1}^{\infty} \frac{(-1)^s}{s^{5/2}} \frac{2\pi^2 s k_B T}{\hbar\omega_c} \times \frac{1}{\sinh(2\pi^2 s k_B T/\hbar\omega_c)} \cos\left(2\pi s \frac{B_F}{B} - \frac{\pi}{4}\right), \tag{15.26}$$

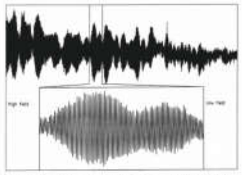


Figure 15.7 Experimental de Haas-van Alphen magnetization oscillations for a sample containing randomly oriented particles of lithium dispersed in paraffin wax. The full trace covers the field range 12.9–7.9 Tesla at a temperature of 20 mK. The portion of the trace between 11.1 and 10.8 T is shown expanded in the inset. [With permission from Fig. 1, J. Phys. Condens. Matter 1, 6749 (1989).]