# PHY 752 Solid State Physics
# 11-11:50 AM MWF Olin 103

## Plan for Lecture 33:

## Chapter 15 in GGGPP -- continued:

## Electron Gas in Magnetic Fields

1. **Landau diamagnetism and de Haas-van Alphen effect**
2. **Hall effect**

Lecture notes prepared with materials from GGGPP textbook



| | | | | |
|---|---|---|---|---|
| 22 | Wed, 10/21/2015 | Chap. 10 | Scattering of particles by crystals | #20 |
| 23 | Fri, 10/23/2015 | Chap. 11 | Optical and transport properties of metals | #21 |
| 24 | Mon, 10/26/2015 | Chap. 11 | Optical and transport properties of metals | #22 |
| 25 | Wed, 10/28/2015 | Chap. 11 | Transport in metals | #23 |
| 26 | Fri, 10/30/2015 | Chap. 12 | Optical properties of semiconductors and insulators | |
| 27 | Mon, 11/02/2015 | Chap. 7 & 12 | Excitons | #24 |
| 28 | Wed, 11/04/2015 | Chap. 9 | Lattice vibrations | #25 |
| 29 | Fri, 11/06/2015 | Chap. 9 | Lattice vibrations | #26 |
| 30 | Mon, 11/09/2015 | Chap. 13 | Defects in semiconductors | #27 |
| 31 | Wed, 11/11/2015 | Chap. 14 | Transport in semiconductors | #28 |
| 32 | Fri, 11/13/2015 | Chap. 15 | Electron gas in Magnetic fields | #29 |
| 33 | Mon, 11/16/2015 | Chap. 15 | Electron gas in Magnetic fields | Prepare presentation. |
| 34 | Wed, 11/18/2015 | Chap. 17 | Magnetic ordering in crystals | Prepare presentation. |
| 35 | Fri, 11/20/2015 | Chap. 18 | Superconductivity | Prepare presentation. |
| 36 | Mon, 11/23/2015 | Chap. 18 | Superconductivity | Prepare presentation. |
| | Wed, 11/25/2015 | | *Thanksgiving Holiday* | |
| | Fri, 11/27/2015 | | *Thanksgiving Holiday* | |
| 37 | Mon, 11/30/2015 | Chap. 18 | Superconductivity | Prepare presentation. |
| | Wed, 12/02/2015 | | Student presentations I | |
| | Fri, 12/04/2015 | | Student presentations II | |
| | Mon, 12/07/2015 | | Begin Take-home final | |



Results from last lecture --

Magnetic field effects on 3-dimensional electron gas

$$H = \frac{1}{2m}\left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2 = \frac{1}{2m}\left(p_x - \frac{eB}{c}y\right)^2 + \frac{1}{2m}p_y^2 + \frac{1}{2m}p_z^2;$$

$$E_{nk_z} = \left(n + \frac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m},$$

Note: the effects of electron spin have been ignored in this treatment; inclusion of the intrinsic electron spin changes some of the details, but the general phenomenon remains the same.

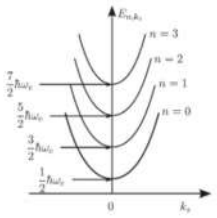



**Figure 15.2** Schematic representation of the energy levels for a three-dimensional electron gas in the presence of a magnetic field in the $z$-direction.



Density of states for 3-dimensional system

$$D_n(E,B) = \frac{L_z}{2\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} \cdot N_L(B) \cdot 2 \frac{1}{\sqrt{E-\left(n+\frac{1}{2}\right)\hbar\omega_c}} \quad \text{for} \quad E > \left(n+\frac{1}{2}\right)\hbar\omega_c.$$

$$D(E,B) = \frac{1}{2}\hbar\omega_c A \sum_{n=0}^{\infty} \frac{1}{\sqrt{E-(n+\frac{1}{2})\hbar\omega_c}} \Theta\left[E-\left(n+\frac{1}{2}\right)\hbar\omega_c\right],$$

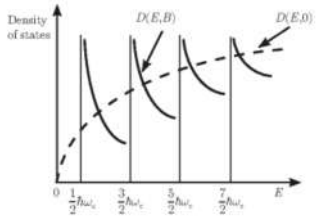
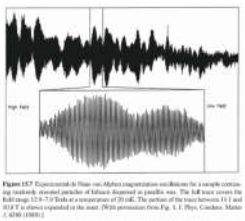



**Figure 15.3** Density-of-states of a three-dimensional electron gas in the absence of a magnetic field (dashed line) and in the presence of a magnetic field (continuous lines).



As a result of the Landau levels, there is an oscillatory behavior of free energy of free electron gas:

$$F(T,B) = F(T,0) + \frac{\hbar^2\omega_c^2}{24}D(\mu) + \frac{\hbar^2\omega_c^2}{4\pi^2}D(\mu)\left(\frac{\hbar\omega_c}{2\mu}\right)^{1/2} \cdot \sum_{s=1}^{\infty} \frac{(-1)^s}{s^{5/2}} \frac{2\pi^2 s k_B T}{\hbar\omega_c}$$
$$\times \frac{1}{\sinh\left(2\pi^2 s k_B T/\hbar\omega_c\right)} \cos\left(2\pi s \frac{B_F}{B} - \frac{\pi}{4}\right), \qquad (15.26)$$



Some steps in the evaluation of the free energy of the electron gas in a magnetic field; summation of the contributions of the Landau levels (Thanks to appendix of Chapter 15 of GGGPP.)

Poisson sum formula

$$f\left(n+\frac{1}{2}\right)=\sum_{s=-\infty}^{+\infty}(-1)^s\int_n^{n+1}f(x)e^{2\pi isx}\,dx.$$

$$\boxed{\sum_{n=0}^{+\infty}f\left(n+\frac{1}{2}\right)=\int_0^\infty f(x)\,dx+2\sum_{s=1}^{\infty}(-1)^s\int_0^\infty f(x)\cos(2\pi sx)\,dx}$$

For a well-behaved function *f(x)*



Integrating by parts

$$\begin{aligned}\int_0^\infty f(x)\cos(2\pi sx)\,dx&=\frac{1}{2\pi s}\int_0^\infty f(x)\,d\sin(2\pi sx)\\&=-\frac{1}{2\pi s}\int_0^\infty f'(x)\sin(2\pi sx)\,dx=\frac{1}{4\pi^2s^2}\int_0^\infty f'(x)\,d\cos(2\pi sx)\\&=-\frac{1}{4\pi^2s^2}f'(0)-\frac{1}{4\pi^2s^2}\int_0^\infty f''(x)\cos(2\pi sx)\,dx.\end{aligned}$$

Poisson sum becomes

$$\boxed{\begin{aligned}\sum_{n=0}^{+\infty}f\left(n+\frac{1}{2}\right)=\int_0^\infty f(x)\,dx+\frac{1}{24}f'(0)\\-\sum_{s=1}^{\infty}\frac{(-1)^s}{2\pi^2s^2}\int_0^\infty f''(x)\cos(2\pi sx)\,dx\end{aligned}}.$$



For convenience, assume that we have shifted the energy scale so that the range of energies of interest are $0<E<\mu$ where $\mu$ denotes the chemical potential. In terms of the density of states *D(E)*, we can define:

$$P_1(E)=\int_0^E D(E')\,dE' \quad \text{with} \quad P_1(0)=0,$$

where $P_1(\mu)=N,$ (the number of electrons)

Similarly, we can define

$$P_2(E)=\int_0^E P_1(E')\,dE'=\int_0^E dE'\int_0^{E'}D(E'')\,dE'' \quad \text{with} \quad P_2(0)=0.$$

The ground state energy of the system of *N* electrons is given by

$$E_0=\int_0^\mu E\,D(E)\,dE$$

$$E_0=\int_0^\mu E\,dP_1(E)=\mu P_1(\mu)-\int_0^\mu P_1(E)\,dE=N\mu-P_2(\mu);$$

Evaluating the free energy of our system

$$F(T) = N\mu - k_B T \sum_n \ln\left[1 + e^{(\mu - E_n)/k_B T}\right].$$

In terms of the density of states:

$$F(T) = N\mu - k_B T \int_0^\infty \ln\left[1 + e^{(\mu - E)/k_B T}\right] D(E)\, dE.$$

which can be integrated by parts:

$$F(T) = N\mu - \int_0^\infty P_2(E)\left(-\frac{\partial f}{\partial E}\right) dE.$$

$$\approx N\mu - P_2(\mu, B)$$



Using the results for the 3-dimensional free electron gas

For *B=0*

$$D(E) = AE^{1/2} \quad \text{and} \quad P_2(E) = \frac{4}{15} AE^{5/2}; \quad E > 0 \quad A = V\frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2};$$

For *B>0*

$$D(E, B) = \frac{1}{2}\hbar\omega_c A \sum_{n=0}^{\infty} \frac{1}{\sqrt{E - (n + 1/2)\hbar\omega_c}} \Theta\left[E - \left(n + \frac{1}{2}\right)\hbar\omega_c\right]$$

Evaluating the "second primitive function"

$$P_2(E, B) = \frac{2}{3}\hbar\omega_c A \sum_{n=0}^{\infty} \left[E - \left(n + \frac{1}{2}\right)\hbar\omega_c\right]^{3/2} \Theta\left[E - \left(n + \frac{1}{2}\right)\hbar\omega_c\right]$$



Using Poisson sum formula with

$$P_2(E, B) = \frac{2}{3}\hbar\omega_c A \sum_{n=0}^{\infty} f\left(n + \frac{1}{2}\right) \quad \text{with} \quad f(x) = (E - x\hbar\omega_c)^{3/2}\Theta(E - x\hbar\omega_c).$$

$$\sum_{n=0}^{+\infty} f\left(n + \frac{1}{2}\right) = \int_0^\infty f(x)\, dx + \frac{1}{24} f'(0) - \sum_{s=1}^{\infty} \frac{(-1)^s}{2\pi^2 s^2} \int_0^\infty f''(x) \cos(2\pi s x)\, dx.$$

$$\int_0^\infty f(x)\, dx = \int_0^{E/\hbar\omega_c} (E - x\hbar\omega_c)^{3/2}\, dx = \frac{2}{5}\frac{1}{\hbar\omega_c} E^{5/2}.$$

$$\frac{1}{24} f'(0) = \frac{1}{24}\frac{3}{2}(-\hbar\omega_c) E^{1/2} = -\frac{1}{16}\hbar\omega_c E^{1/2}.$$

$$\int_0^\infty f''(x) \cos(2\pi s x)\, dx = \frac{3}{4}\hbar^2\omega_c^2 \int_0^{E/\hbar\omega_c} \frac{1}{\sqrt{E - x\hbar\omega_c}} \cos(2\pi s x)\, dx$$

$$= \frac{3}{4}(\hbar\omega_c)^{3/2} \frac{1}{\sqrt{2s}} \cos\left(\frac{2\pi s}{\hbar\omega_c} E - \frac{\pi}{4}\right)$$

Collecting terms:

$$P_2(E,B) = \frac{4}{15}AE^{5/2} - \frac{\hbar^2\omega_c^2}{24}AE^{1/2} - \frac{A}{\pi^2}\left(\frac{\hbar\omega_c}{2}\right)^{5/2}\sum_{s=1}^{\infty}\frac{(-1)^s}{s^{5/2}}\cos\left(\frac{2\pi s}{\hbar\omega_c}E - \frac{\pi}{4}\right)$$

Free energy at *T=0 K*

$$E_0(B) = N\mu - P_2(\mu,B)$$

$$E_0(B) = E_0(0) + \frac{\hbar^2\omega_c^2}{24}D(\mu) + \frac{\hbar^2\omega_c^2}{4\pi^2}D(\mu)\left(\frac{\hbar\omega_c}{2\mu}\right)^{1/2}\sum_{s=1}^{\infty}\frac{(-1)^s}{s^{5/2}}\cos\left(\frac{2\pi s}{\hbar\omega_c}\mu - \frac{\pi}{4}\right)$$



Free energy at *T>0 K*

$$I_s = \int_0^{\infty}\cos\left(\frac{2\pi s}{\hbar\omega_c}E - \frac{\pi}{4}\right)\left(-\frac{\partial f}{\partial E}\right)dE,$$

Some tricks

$$\cos\left(\frac{2\pi s}{\hbar\omega_c}E - \frac{\pi}{4}\right) \equiv \cos\left[\left(\frac{2\pi s}{\hbar\omega_c}\mu - \frac{\pi}{4}\right) + \frac{2\pi s}{\hbar\omega_c}(E-\mu)\right]$$

$$I_s = \cos\left(\frac{2\pi s}{\hbar\omega_c}\mu - \frac{\pi}{4}\right)\int_{-\infty}^{+\infty}\exp\left(i\frac{2\pi s}{\hbar\omega_c}E\right)\left(-\frac{\partial f}{\partial E}\right)dE$$
$$= \frac{2\pi^2 s k_B T}{\hbar\omega_c}\frac{1}{\sinh(2\pi^2 s k_B T/\hbar\omega_c)}\cos\left(\frac{2\pi s}{\hbar\omega_c}\mu - \frac{\pi}{4}\right),$$



Free energy at finite temperature

$$F(T,B) = F(T,0) + \frac{\hbar^2\omega_c^2}{24}D(\mu) + \frac{\hbar^2\omega_c^2}{4\pi^2}D(\mu)\left(\frac{\hbar\omega_c}{2\mu}\right)^{1/2} \times \sum_{s=1}^{\infty}\frac{(-1)^s}{s^{5/2}}\frac{2\pi^2 s k_B T}{\hbar\omega_c}\frac{1}{\sinh(2\pi^2 s k_B T/\hbar\omega_c)}\cos\left(\frac{2\pi s}{\hbar\omega_c}\mu - \frac{\pi}{4}\right),$$

$$2\pi s\frac{B_F}{B}$$

$B_F$ is a measure of the area of an extremal orbit of an electron on the Fermi surface.

*J. Phys. Chem. Solids* Pergamon Press 1968. Vol. 29, pp. 1485–1502. Printed in Great Britain.

# THE DE HAAS-VAN ALPHEN EFFECT AND THE FERMI SURFACE OF TUNGSTEN*

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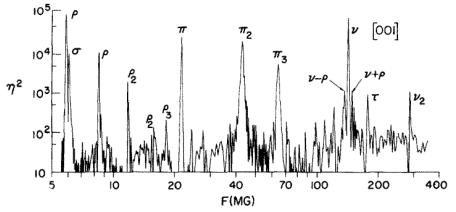
(*Received* 8 *April* 1968)

**Abstract** – The de Haas–van Alphen effect in tungsten has been studied in detail using large impulsive magnetic fields, and the frequencies have been determined to high accuracy with the aid of digital data recording, digital analysis and dynamic calibration of the entire apparatus. These improvements in experimental technique have resulted in the detection of several new frequency branches which have not been previously reported. A quantitative description of all sheets of the Fermi surface is given in terms of simple analytical functions, and this geometrical model is in excellent agreement with the observed angular variations of the de Haas–van Alphen frequencies. In most respects, the model surface is found to be quite similar to that deduced by Loucks from first-principles relativistic band calculations, and the predictions of the empirical model are compared with the results of other experiments relating to the Fermi surface.







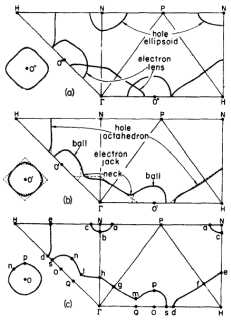
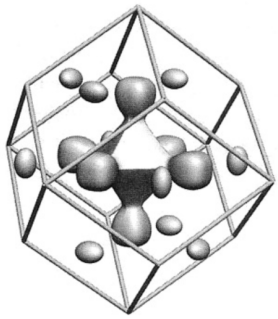




Fig. 6. Sections through the Fermi surface of tungsten: (a) Loucks' non-relativistic model[15]; (b) Loucks' relativistic model[16]; (c) empirical model from fit to the de Haas-van Alphen effect. The dimensions between



http://physics.oregonstate.edu/~minote/COURSES/ph671_2008/

## Hall effect

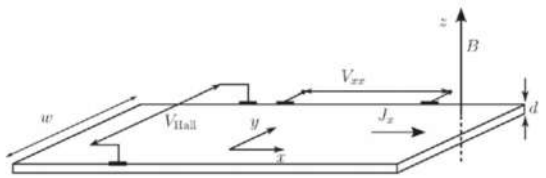



**Figure 15.9** Standard geometry for Hall effect and magnetoresistivity measurements. $V_{\text{Hall}}$ is the Hall potential, $J_x$ is the current density in the flow direction, and $w$ denotes the width of the strip.

$$R_{\text{Hall}}(B) = \frac{1}{B}\rho_{yx}(B) = \frac{1}{B}\frac{E_y}{J_x}. \qquad R_{\text{Hall}}(B) = -\frac{1}{nec}.$$



## Quantum Hall effect

$$\rho_{xy}(\nu) = \frac{1}{\nu}\frac{h}{e^2}.$$

$$\nu \equiv n_s hc/eB$$

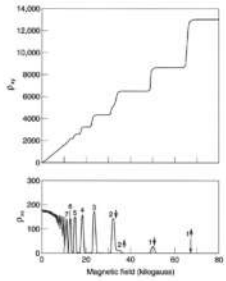



**Figure 15.10** Hall resistance $\rho_{xy}(B)$ and longitudinal resistance $\rho_{xx}(B)$ (in ohm) of a two-dimensional electron gas at the GaAs-$Al_xGa_{1-x}As$ interface, at 50 mK. The quantum Hall effect is already evident at about 10 kilogauss. The numbers and the arrows above $\rho_{xx}(B)$ maxima refer to the Landau quantum number and the spin polarization of the levels; for the Landau levels $n \geq 3$ the spin splitting is not resolved and plateaus are observed at the corresponding even filling factors [With permission from Fig. 1, Phys. Rev. B 25, 5566 (1982)].