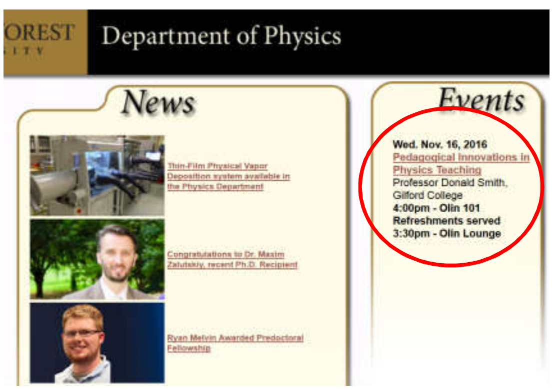
PHY 711 Classical Mechanics and Mathematical Methods
11-11:50 AM MWF Olin 107

Plan for Lecture 32:
Effects of viscosity in fluid motion – Chap.12 in Fetter & Walecka

1. Navier-Stokes equation
2. Terminal velocity of a sphere moving with constant applied force in a viscous medium
3. Stokes' viscosity relation

19	Fri, 10/14/2016	App. A	Laplace transforms and contour integrals	Take-home exam	
20	Mon, 10/17/2016	Chap. 5	Mechanics of rigid bodies	Exam due	
21	Wed, 10/19/2016	Chap. 5	Mechanics of rigid bodies	#16	10/24/2016
	Fri, 10/21/2016		Fall break -- no class		
22	Mon, 10/24/2016	Chap. 8	Mechanics of Elastic Membranes	#17	10/28/2016
23	Wed, 10/26/2016	Chap. 9	Introduction to hydrodynamics		
24	Fri, 10/28/2016	Chap. 9	Introduction to hydrodynamics	#18	10/31/2016
25	Mon, 10/31/2016	Chap. 9	Sound waves	#19	11/02/2016
26	Wed, 11/02/2016	Chap. 9	Sound waves	#20	11/04/2016
27	Fri, 11/04/2016	Chap. 9	Non-linear sound	#21	11/07/2016
28	Mon, 11/07/2016	Chap. 10	Surface waves in fluids		
29	Wed, 11/09/2016	Chap. 10	Surface waves in fluids	#22	11/11/2016
30	Fri, 11/11/2016	Chap. 11	Heat conductivity	#23	11/14/2016
31	Mon, 11/14/2016	Chap. 12	Viscous fluids	#24	11/16/2016
32	Wed, 11/16/2016	Chap. 12	Viscous fluids	#25	11/18/2016
33	Fri, 11/18/2016				
34	Mon, 11/21/2016				
	Wed, 11/23/2016		Thanksgiving Holiday -- no class		
	Fri, 11/25/2016		Thanksgiving Holiday -- no class		
	Wed, 12/07/2016		Presentations I		
	Fri, 12/09/2016		Presentations II		



WFU Physics Colloquium

TITLE: Recent Pedagogical Innovations in Teaching Physics to Students of the Life Sciences

SPEAKER: Professor Donald Smith,,

Department of Physics
Guilford College

TIME: Wednesday November 16, 2016 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

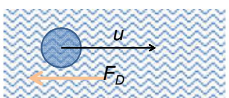
ABSTRACT

Over the last few years, a wave of innovation has been sweeping across the nations' physics departments. The tools of physics education research were brought to bear on the way physics faculty have traditionally taught the required introductory physics for biology and medical students. At the same time, the new MCAT 2015 shifted the emphasis of that exam away from rote memorization toward a more context-rich analysis paradigm. Furthermore, the improvement and ubiquity of computers and software have enabled introductory students to tackle more complicated problems than ever before. These three trends have created a perfect storm for innovation, which has led to a set of recommendations for how an IPLS class (Introductory Physics for the Life Sciences) can be more vital and enjoyable for both students and teachers. In this presentation, I will share how I came to redesign the "College Physics" algebra-based sequence at Guilford College into an IPLS course that premiered in the Fall semester of 2015. I will describe my successes and challenges, and make the case that not only are these improvements essential for teaching life sciences students, they can also benefit courses intended for physicists and engineers.

Brief introduction to viscous effects in incompressible fluids

Stokes' analysis of viscous drag on a sphere of radius R moving at speed u in medium with viscosity η:

$$F_D = -\eta(6\pi R u)$$



Plan:
1. Consider the general effects of viscosity on fluid equations
2. Consider the solution to the linearized equations for the case of steady-state flow of a sphere of radius R
3. Infer the drag force needed to maintain the steady-state flow

Newton-Euler equation for incompressible fluid, modified by viscous contribution (Navier-Stokes equation):

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \mathbf{f}_{applied} - \frac{\nabla p}{\rho} + \underbrace{\frac{\eta}{\rho}}_{\upsilon}\nabla^2\mathbf{v}$$

υ Kinematic viscosity

Typical kinematic viscosities at 20° C and 1 atm:

Fluid	ν (m^2/s)
Water	1.00 x 10^{-6}
Air	14.9 x 10^{-6}
Ethyl alcohol	1.52 x 10^{-6}
Glycerine	1183 x 10^{-6}

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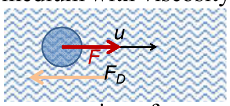

Effects of drag force on motion of particle of mass m with constant force F:

$$F - 6\pi R\eta u = m\frac{du}{dt} \qquad \text{with } u(0)=0$$

$$\Rightarrow u(t) = \frac{F}{6\pi R\eta}\left(1 - e^{-\frac{6\pi R\eta}{m}t}\right)$$

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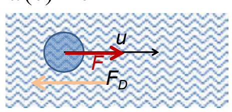
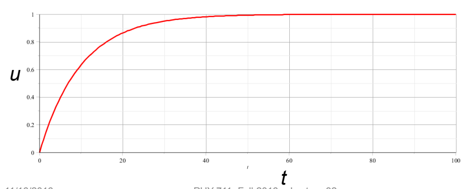
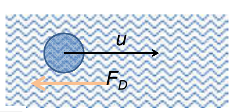


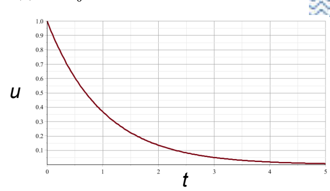


Effects of drag force on motion of particle of mass m with an initial velocity with $u(0)=U_0$ and no external force

$$-6\pi R\eta u = m\frac{du}{dt}$$

$$\Rightarrow u(t) = U_0 e^{-\frac{6\pi R\eta}{m}t}$$



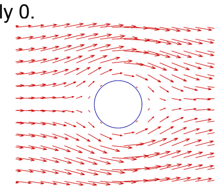


Recall: PHY 711 -- Assignment #24 Oct. 28, 2015

Determine the form of the velocity potential for an incompressible fluid representing uniform velocity in the **z** direction at large distances from a spherical obstruction of radius *a*. Find the form of the velocity potential and the velocity field for all *r* > *a*. Assume that for *r* = *a*, the velocity in the radial direction is 0 but the velocity in the azimuthal direction is not necessarily 0.

$$\nabla^2\Phi = 0$$

$$\Phi(r,\theta) = -v_0\left(r + \frac{a^3}{2r^2}\right)\cos\theta$$



Newton-Euler equation for incompressible fluid, modified by viscous contribution (Navier-Stokes equation):

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \mathbf{f}_{applied} - \frac{\nabla p}{\rho} + \frac{\eta}{\rho}\nabla^2\mathbf{v}$$

Continuity equation: $\nabla\cdot\mathbf{v} = 0$

Assume steady state: $\Rightarrow \frac{\partial \mathbf{v}}{\partial t} = 0$

Assume non-linear effects small

Initially set $\mathbf{f}_{applied} = 0$;

$$\Rightarrow \nabla p = \eta\nabla^2\mathbf{v}$$

$$\nabla p = \eta\nabla^2\mathbf{v}$$

Take curl of both sides of equation:

$$\nabla\times(\nabla p) = 0 = \eta\nabla^2(\nabla\times\mathbf{v})$$

Assume (with a little insight from Landau):

$$\mathbf{v} = \nabla\times(\nabla\times f(r)\mathbf{u}) + \mathbf{u}$$

where $f(r)\xrightarrow[r\to\infty]{} 0$

Note that:

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$

Digression

Some comment on assumption: $\mathbf{v} = \nabla \times \left(\nabla \times f(r)\mathbf{u} \right) + \mathbf{u}$

$\nabla \times \left(\nabla \times \mathbf{A} \right) = \nabla \left(\nabla \cdot \mathbf{A} \right) - \nabla^2 \mathbf{A}$

Here $\mathbf{A} = f(r)\mathbf{u}$

$\nabla \times \mathbf{v} = \nabla \times \left(\nabla \times \left(\nabla \times \mathbf{A} \right) \right) = -\nabla \times \left(\nabla^2 \mathbf{A} \right)$

Also note: $\nabla p = \eta \nabla^2 \mathbf{v}$

$\Rightarrow \nabla \times \nabla p = \nabla \times \eta \nabla^2 \mathbf{v}$ or $\nabla^2 \left(\nabla \times \mathbf{v} \right) = 0$

$\mathbf{v} = \nabla \times \left(\nabla \times f(r)\mathbf{u} \right) + \mathbf{u}$

$\mathbf{u} = u\hat{\mathbf{z}}$

$\nabla \times \left(\nabla \times f(r)\hat{\mathbf{z}} \right) = \nabla \left(\nabla \cdot f(r)\hat{\mathbf{z}} \right) - \nabla^2 f(r)\hat{\mathbf{z}}$

$\nabla \times \mathbf{v} = 0 \qquad \Rightarrow \nabla^2 \left(\nabla \times \mathbf{v} \right) = 0$

$\nabla^4 \left(\nabla \times f(r)\hat{\mathbf{z}} \right) = 0 \quad \Rightarrow \nabla^4 \left(\nabla f(r) \times \hat{\mathbf{z}} \right) = 0 \quad \Rightarrow \nabla^4 f(r) = 0$

$$f(r) = C_1 r^2 + C_2 r + C_3 + \frac{C_4}{r}$$

$$v_r = u\cos\theta\left(1 - \frac{2}{r}\frac{df}{dr}\right) = u\cos\theta\left(1 - 4C_1 - \frac{2C_2}{r} - \frac{2C_4}{r^3}\right)$$

$$v_\theta = -u\sin\theta\left(1 - \frac{d^2 f}{dr^2} - \frac{1}{r}\frac{df}{dr}\right) = -u\sin\theta\left(1 - 4C_1 - \frac{C_2}{r} + \frac{C_4}{r^3}\right)$$

Some details:

$$\nabla^4 f(r) = 0 \qquad \Rightarrow \left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} \right)^2 f(r) = 0$$

$$f(r) = C_1 r^2 + C_2 r + C_3 + \frac{C_4}{r}$$

$$\mathbf{v} = u\left(\nabla \times \left(\nabla \times f(r)\hat{\mathbf{z}} \right) + \hat{\mathbf{z}} \right)$$

$$= u\left(\nabla \left(\nabla \cdot \left(f(r)\hat{\mathbf{z}} \right) \right) - \nabla^2 f(r)\hat{\mathbf{z}} + \hat{\mathbf{z}} \right)$$

Note that: $\hat{\mathbf{z}} = \cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}}$

$$\mathbf{v} = u\left(\nabla\left(\frac{df}{dr}\cos\theta \right) - \left(\nabla^2 \left(f(r) \right) - 1 \right)\left(\cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}} \right) \right)$$

$$v_r = u\cos\theta\left(1-\frac{2}{r}\frac{df}{dr}\right) = u\cos\theta\left(1-4C_1-\frac{2C_2}{r}-\frac{2C_4}{r^3}\right)$$
$$v_\theta = -u\sin\theta\left(1-\frac{d^2 f}{dr^2}-\frac{1}{r}\frac{df}{dr}\right) = -u\sin\theta\left(1-4C_1-\frac{C_2}{r}+\frac{C_4}{r^3}\right)$$

To satisfy $\mathbf{v}(r\to\infty)=\mathbf{u}$: $\Rightarrow C_1 = 0$

To satisfy $\mathbf{v}(R)=0$ solve for C_2, C_4

$$v_r = u\cos\theta\left(1-\frac{3R}{2r}+\frac{R^3}{2r^3}\right)$$
$$v_\theta = -u\sin\theta\left(1-\frac{3R}{4r}-\frac{R^3}{4r^3}\right)$$

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$$v_r = u\cos\theta\left(1-\frac{3R}{2r}+\frac{R^3}{2r^3}\right)$$
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Determining pressure:

$$\nabla p = \eta\nabla^2\mathbf{v} = -\eta\nabla\left(u\cos\theta\left(\frac{3R}{2r^2}\right)\right)$$
$$\Rightarrow p = p_0 - \eta u\cos\theta\left(\frac{3R}{2r^2}\right)$$

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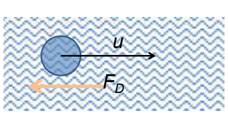
$$p = p_0 - \eta u\cos\theta\left(\frac{3R}{2r^2}\right)$$

Corresponds to:

$$F_D\cos\theta = \left(p(R)-p_0\right)4\pi R^2$$
$$\Rightarrow \mathrm{F}_D = -\eta u\left(6\pi R\right)$$



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