PHY 711 Classical Mechanics and Mathematical Methods
9-9:50 AM MWF Olin 107

Plan for Lecture 19:

Welcome back from fall break

1. Comments on mid-term exam
2. Comments on "computational projects"
3. Mechanics of continuous media (Chapters 4 & 7)

Three events this week --

Special Seminar: Oct. 16, 2017 at 3 PM
Special WFU Physics Seminar
TITLE: "Quantum Information Science"
SPEAKER: Jaewan Kim Korea Institute for Advanced Study TIME: Monday, October 16, 2017, at 3:00pm PLACE: Olin 101 There will be a reception ...

Career Event: Oct. 18 at 12:00 pm + pizza
WFU Physics Career Advising Event TITLE: "Opportunities for Undergraduate, Graduate, and Postdoctoral Research at Oak Ridge National Laboratory (ORNL)"
SPEAKER: Zachary D. Hood School of Chemistry and Biochemistry, Georgia Institute of ...

Colloquium: Oct. 18, 2017 at 4 PM
WFU Physics and Chemistry Colloquium
TITLE: "Solid Electrolytes and Their Interfaces: Bridging Mechanistic Understanding to Their Performance"
SPEAKER: Zachary D. Hood School of Chemistry and Biochemistry, Georgia Institute of Technology, ...

9	Fri, 9/15/2017	Chap. 3 and 6	Lagrangian mechanics and constraints	#8	9/20/2017
10	Mon, 9/18/2017	Chap. 3 and 6	Constants of the motion		
11	Wed, 9/20/2017	Chap. 3 and 6	Hamiltonian formalism	#9	9/27/2017
12	Fri, 9/22/2017	Chap. 3 and 6	Liouville equation		
13	Mon, 9/25/2017	Chap. 6	Canonical transformations		
14	Wed, 9/27/2017	Chap. 4	Motion from Small oscillations about equilibrium		
15	Fri, 9/29/2017	Chap. 1-4,6	Review		
	Mon, 10/02/2017		*Take-home exam -- No class*		
	Wed, 10/04/2017		*Take-home exam -- No class*		
16	Fri, 10/06/2017	Chap. 4	Vibrational modes		
17	Mon, 10/09/2017	Chap. 5	Rigid body motion	#10	10/16/2017
18	Wed, 10/11/2017	Chap. 5	Rigid body motion		
	Fri, 10/13/2017		*Fall break -- No class*		
19	Mon, 10/16/2017		Discuss exam questions and topics for presentations	Topic	10/18/2017
20	Wed, 10/18/2017				
21	Fri, 10/20/2017				
22	Mon, 10/23/2017				

1. Suppose that you want to find an extremum of an integral of a function between and initial and final time of the form

$$U = \int_{t_i}^{t_f} f(q(t), \dot{q}(t), t)dt,$$

where $q(t)$ represents a generalized coordinate as a function of t. Find the differential equation for $q(t)$ which results from optimization of U for the case that

$$f(q(t), \dot{q}(t), t) = Aq^2 + B\dot{q}^2 qt^2.$$

Here A and B are fixed constants, and $q(t_i) \equiv q_i$ and $q(t_f) \equiv q_f$ are also fixed constants. It is not necessary to solve for $q(t)$.

Euler-Lagrange equations:

$$\frac{d}{dt}\frac{\partial f}{\partial \dot{q}} - \frac{\partial f}{\partial q} = 0$$

$$\frac{\partial f}{\partial q} = 2Aq + B\dot{q}^2 t^2$$

$$\frac{\partial f}{\partial \dot{q}} = 2B\dot{q}qt^2$$

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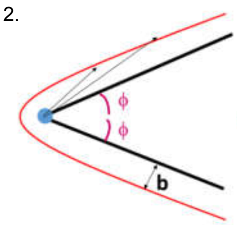
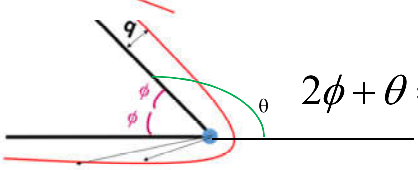
2.



$$\cos\phi = \left(1 + (2CEb)^2\right)^{-1/2}.$$



$$2\phi + \theta = \pi$$

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Solving for b:

Valid for $m \ll M$

$$b = \left|\frac{1}{2CE}\frac{\sin\phi}{\cos\phi}\right| = \left|\frac{1}{2CE}\frac{\cos(\theta/2)}{\sin(\theta/2)}\right|$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{1}{(4CE)^2}\frac{1}{\sin^4(\theta/2)}$$

For $m \sim M$, need to evaluate CM cross section

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{1}{(4CE_{rel})^2}\frac{1}{\sin^4(\theta/2)}$$

$$E_{rel} = \frac{1}{2}\mu v_{rel}^2 = \frac{1}{2}\frac{mM}{m+M}v_m^2 = \frac{M}{m+M}E$$

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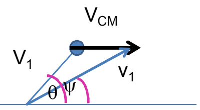
Relationship between center of mass and laboratory frames of reference – for elastic scattering



$$\mathbf{v}_1 = \mathbf{V}_1 + \mathbf{V}_{CM}$$

$$v_1 \sin\psi = V_1 \sin\theta$$

$$v_1 \cos\psi = V_1 \cos\theta + V_{CM}$$

$$\tan\psi = \frac{\sin\theta}{\cos\theta + V_{CM}/V_1} = \frac{\sin\theta}{\cos\theta + m/M}$$

$$\text{Also:} \quad \cos\psi = \frac{\cos\theta + m/M}{\sqrt{1 + 2m/M\cos\theta + (m/M)^2}}$$

Differential cross sections in different reference frames – continued:

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}}\right) = \left(\frac{d\sigma_{CM}(\theta)}{d\Omega_{CM}}\right)\left|\frac{d\cos\theta}{d\cos\psi}\right|$$

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}}\right) = \left(\frac{d\sigma_{CM}(\theta)}{d\Omega_{CM}}\right)\frac{\left(1 + 2m/M\cos\theta + (m/M)^2\right)^{3/2}}{(m/M)\cos\theta + 1}$$

$$\text{where:} \quad \tan\psi = \frac{\sin\theta}{\cos\theta + m/M}$$

For $m = M$:

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}}\right) = \left(\frac{d\sigma_{CM}(\theta)}{d\Omega_{CM}}\right)\frac{\left(1 + 2m/M\cos\theta + (m/M)^2\right)^{3/2}}{(m/M)\cos\theta + 1}$$

$$= \frac{1}{(CE)^2}\frac{\cos\psi}{\sin^4\psi}$$

3. A particle of mass m and charge q moves in the field of a vector potential $\mathbf{A}(\mathbf{r},t) = -(E_0ct + B_0z)\hat{\mathbf{y}}$. (Note that we are using the cgs Gaussian units of your text book.) Here E_0 denotes a constant electric field amplitude and B_0 denotes a constant magnetic field amplitude. The initial particle position is $\mathbf{r}(0) = 0$ and the initial particle velocity is $\dot{\mathbf{r}}(0) = 0$.

   (a) Determine the Lagrangian $L(x, y, z, \dot{x}, \dot{y}, \dot{z}, t)$ which describes the particle's motion.

   (b) Write the Euler-Lagrange equations for this system.

   (c) Find the particle trajectories $x(t)$, $y(t)$, $z(t)$ by solving the equations and imposing the initial conditions.

   (d) Determine the Hamiltonian $H(x, y, z, p_x, p_y, p_z, t)$ for this system.

   (e) Convince yourself that the canonical equations of motion are consistent with results of the Lagrangian analysis. (It is not necessary to repeat your solution.)

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + \frac{q}{c}\dot{\mathbf{r}}\cdot\mathbf{A}(x,y,z,t)$$

$$= \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \frac{q}{c}\dot{y}\left(E_0ct + B_0z\right)$$

$$\frac{d}{dt}m\dot{x} = 0 \qquad x(t) = 0; \quad \dot{x}(t) = 0$$

$$\frac{d}{dt}\left(m\dot{y} - \frac{q}{c}\left(E_0ct + B_0z\right)\right) = 0$$

$$\dot{y} = \frac{q}{mc}\left(E_0ct + B_0z\right)$$

$$\frac{d}{dt}m\dot{z} = -\frac{q}{c}\mathrm{B}_0\dot{y}$$

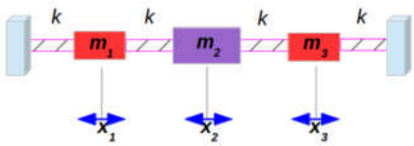
When the dust clears:

$$y(t) = \frac{mc}{qB_0}\frac{E_0c}{B_0}\left(1 - \cos\left(\frac{qB_0}{mc}t\right)\right)$$

$$z(t) = \frac{E_0c}{B_0}\left(\frac{mc}{qB_0}\sin\left(\frac{qB_0}{mc}t\right) - t\right)$$

$$H = \frac{p_x^2}{2m} + \frac{p_z^2}{2m} + \frac{\left(p_y + \frac{q}{c}\left(E_0ct + B_0z\right)\right)^2}{2m}$$

4.



$$L = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_3^2 + \frac{1}{2}3m\dot{x}_2^2$$
$$-\frac{1}{2}kx_1^2 - \frac{1}{2}kx_3^2 - \frac{1}{2}k\left(x_3 - x_2\right)^2 - \frac{1}{2}k\left(x_2 - x_1\right)^2$$

$$m\ddot{x}_1 = -2kx_1 + kx_2$$
$$3m\ddot{x}_2 = -2kx_2 + kx_1 + kx_3 \qquad x_i(t) = X_i e^{-i\omega t}$$
$$m\ddot{x}_3 = -2kx_3 + kx_2$$

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$$m\ddot{x}_3 = -2kx_3 + kx_2$$

$$\frac{k}{m}\begin{pmatrix} 2 & -1 & 0 \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ & -1 & 2 \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \omega^2 \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$$

Maple says:

$$\omega^2 = 2\frac{k}{m}, \left(\frac{4}{3} + \frac{\sqrt{10}}{3}\right)\frac{k}{m}, \left(\frac{4}{3} - \frac{\sqrt{10}}{3}\right)\frac{k}{m}$$

Picking topic for "project"

- Prepare ~ 10 minute presentation to teach us (and yourself) something related to classical mechanics
- Similar effort to take-home exam
- May combine projects for PHY 711 and 741