PHY 711 Classical Mechanics and Mathematical Methods
9-9:50 AM MWF Olin 107

Plan for Lecture 2:

1. Brief comment on quiz
2. Particle interactions
3. Notion of center of mass reference fame
4. Introduction to scattering theory

PHY 711 Classical Mechanics and Mathematical Methods

MWF 9 AM-9:50 AM	OPL 107	http://www.wfu.edu/~natalie/f17phy711/

Instructor: Natalie Holzwarth	Phone:758-5510	Office:300 OPL	e-mail:natalie@wfu.edu

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment	Due
1	Mon, 8/28/2017	Chap. 1	Introduction	#1	9/6/2017
2	Wed, 8/30/2017	Chap. 1	Scattering theory	#2	9/6/2017
3	Fri, 9/01/2017				
4	Mon, 9/04/2017				
5	Wed, 9/06/2017				
6	Fri, 9/08/2017				
7	Mon, 9/11/2017				

PHY 711 -- Assignment #2

Aug. 30, 2017

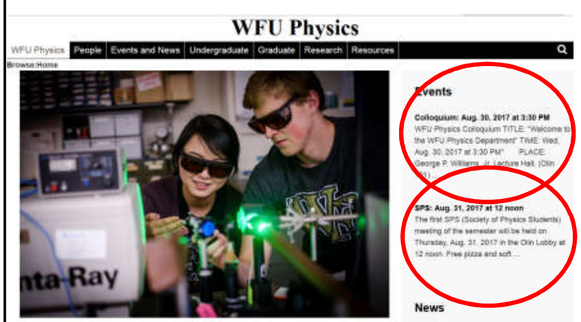
Read Chapter 1 in **Fetter & Walecka**.

1. In class, we "derived" the differential cross section for the scattering of two hard spheres of mutual radius *D* in the center of mass frame. Find the differential cross section for this system in the lab frame in which m_{target} is initially at rest and evaluate the expression for the following cases.
   a. $m_1/m_{target}=0.1$
   b. $m_1/m_{target}=1$
   c. $m_1/m_{target}=1000$

Fall 2017 Schedule
for N. A. W. Holzwarth

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:00	Classical Mechanics PHY711		Classical Mechanics PHY711		Classical Mechanics PHY711
10:00-12:00	Lecture Preparation/ Office Hours		Lecture Preparation/ Office Hours		Lecture Preparation/ Office Hours
12:00-1:00	Quantum Mechanics PHY741	Physics Research	Quantum Mechanics PHY741	Physics Research	Quantum Mechanics PHY741
1:00-2:15	Condensed Matter Theory Journal Club		Physics Research		Physics Research
2:15-3:30					
3:30-5:00	Physics Research		Physics Colloquium		

Schedule additional office hours by email: natalie@wfu.edu



Comment on quiz questions

1. $g(t)=\int_0^t \left(x^2+t\right)dx \qquad \frac{dg}{dt}=\int_0^t \frac{d\left(x^2+t\right)}{dt}dt+\left(x^2+t\right)\Big|_{x=t}$

$$=\int_0^t dt+\left(t^2+t\right)=t^2+2t$$

2. Evaluate the integral $\oint \frac{dz}{z}$ for a closed contour about the origin.

Suppose that $z=e^{i\theta} \qquad dz=e^{i\theta}id\theta \qquad \oint\frac{dz}{z}=\int_0^{2\pi}\frac{e^{i\theta}id\theta}{e^{i\theta}}=2\pi i$

3. $\frac{df}{dx}=f \qquad \Rightarrow f(x)=Ae^x \qquad f(x)=1 \quad \Rightarrow A=1$

4. $\sum_{n=1}^{N}a^n=\frac{a-a^{N+1}}{1-a} \qquad$ Let $S\equiv\sum_{n=1}^{N}a^n$ Note that $aS-S=a^{N+1}-a$

Scattering theory:

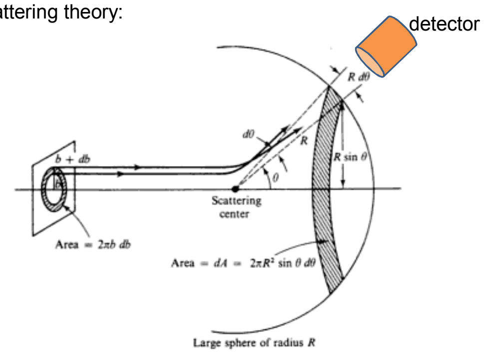
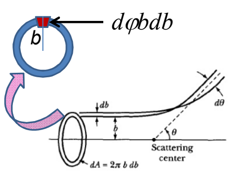


Figure 5.5 The scattering problem and relation of cross section to impact parameter.

Differential cross section

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\text{Number of detected particles at } \theta \text{ per target particle}}{\text{Number of incident particles per unit area}}$$

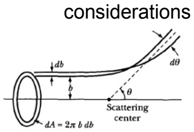
$$= \text{Area of incident beam that is scattered into detector at angle } \theta$$



$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{d\varphi\, b\, db}{d\varphi \sin\theta\, d\theta} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Figure from Marion & Thorton, Classical Dynamics

Note: Notion of cross section is common to many areas of physics including classical mechanics, quantum mechanics, optics, etc. Only in the classical mechanics can we calculate it using geometric considerations



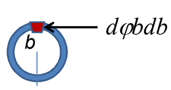
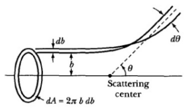


Figure from Marion & Thorton, Classical Dynamics

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{d\varphi\, b\, db}{d\varphi \sin\theta\, d\theta} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Note: We are assuming that the process is isotropic in ϕ

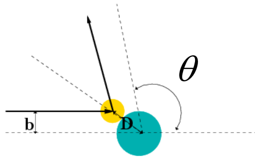
Simple example – collision of hard spheres



$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Microscopic view:

$$b(\theta) = ?$$

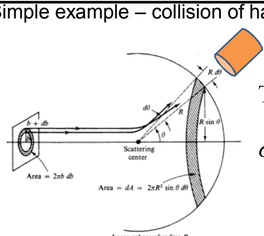


$$b(\theta) = D\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{D^2}{4}$$

8/30/2017 PHY 711 Fall 2017 -- Lecture 2 10

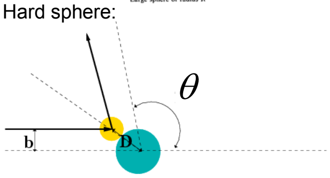
Simple example – collision of hard spheres -- continued



Total scattering cross section:

$$\sigma = \int \left(\frac{d\sigma}{d\Omega}\right) d\Omega$$

Hard sphere:



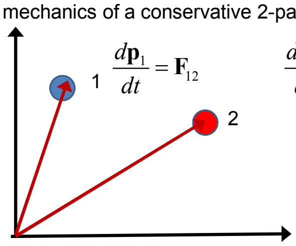
$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{D^2}{4}$$

$$\sigma = \pi D^2$$

8/30/2017 PHY 711 Fall 2017 -- Lecture 2 11

Relationship of scattering cross-section to particle interactions --
Classical mechanics of a conservative 2-particle system.

$$\frac{d\mathbf{p}_1}{dt} = \mathbf{F}_{12} \qquad \frac{d\mathbf{p}_2}{dt} = \mathbf{F}_{21}$$



$$\mathbf{F}_{12} = -\nabla_1 V(\mathbf{r}_1 - \mathbf{r}_2) \quad \Rightarrow E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + V(\mathbf{r}_1 - \mathbf{r}_2)$$

8/30/2017 PHY 711 Fall 2017 -- Lecture 2 12

Typical two-particle interactions –

Central potential: $V(\mathbf{r}_1 - \mathbf{r}_2) = V(|\mathbf{r}_1 - \mathbf{r}_2|) \equiv V(r)$

Hard sphere: $V(r) = \begin{cases} \infty & r \le a \\ 0 & r > a \end{cases}$

Coulomb or gravitational: $V(r) = \frac{K}{r}$

Lennard-Jones: $V(r) = \frac{A}{r^{12}} - \frac{B}{r^6}$

Relationship between center of mass and laboratory frames of reference

Definition of center of mass $\mathbf{R}_{CM}$

$$m_1\mathbf{r}_1 + m_2\mathbf{r}_2 = (m_1 + m_2)\mathbf{R}_{CM}$$

$$m_1\dot{\mathbf{r}}_1 + m_2\dot{\mathbf{r}}_2 = (m_1 + m_2)\dot{\mathbf{R}}_{CM} = (m_1 + m_2)\mathbf{V}_{CM}$$

$$E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + V(\mathbf{r}_1 - \mathbf{r}_2)$$

$$= \frac{1}{2}(m_1 + m_2)V_{CM}^2 + \frac{1}{2}\mu|\mathbf{v}_1 - \mathbf{v}_2|^2 + V(\mathbf{r}_1 - \mathbf{r}_2)$$

where: $\mu \equiv \frac{m_1m_2}{m_1 + m_2}$

Classical mechanics of a conservative 2-particle system -- continued

$$E = \frac{1}{2}(m_1 + m_2)V_{CM}^2 + \frac{1}{2}\mu|\mathbf{v}_1 - \mathbf{v}_2|^2 + V(\mathbf{r}_1 - \mathbf{r}_2)$$

For central potentials: $V(\mathbf{r}_1 - \mathbf{r}_2) = V(|\mathbf{r}_1 - \mathbf{r}_2|) \equiv V(r_{12})$

Relative angular momentum is also conserved:

$$\mathbf{L}_{12} \equiv \mathbf{r}_{12} \times \mu\mathbf{v}_{12}$$

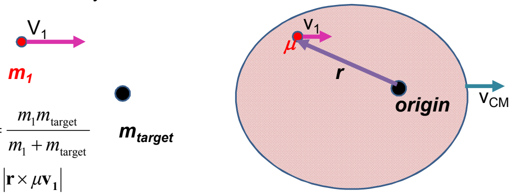
$$E = \frac{1}{2}(m_1 + m_2)V_{CM}^2 + \frac{1}{2}\mu v_{12}^2 + \frac{L_{12}^2}{2\mu r_{12}^2} + V(r_{12})$$

Simpler notation:

$$E = \frac{1}{2}(m_1 + m_2)V_{CM}^2 + \frac{1}{2}\mu\dot{r}^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

Note: The following analysis will be carried out in the center of mass frame of reference.

In laboratory frame: In center-of-mass frame:



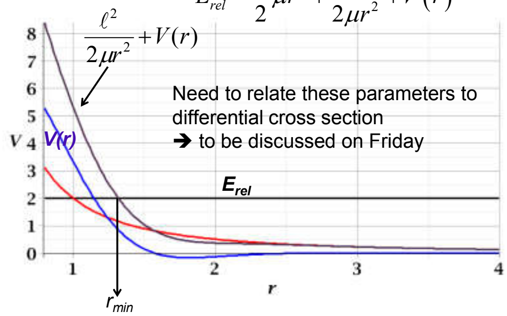
$$\mu = \frac{m_1 m_{\text{target}}}{m_1 + m_{\text{target}}}$$

$$\ell = |\mathbf{r} \times \mu \mathbf{v}_1|$$

Also note: We are assuming that the interaction between particle and target $V(r)$ conserves energy and angular momentum.

For a continuous potential interaction in center of mass reference frame:

$$E_{rel} = \frac{1}{2}\mu\dot{r}^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

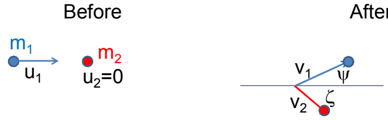


It is often convenient to analyze the scattering cross section in the center of mass reference frame.

Relationship between normal laboratory reference and center of mass:

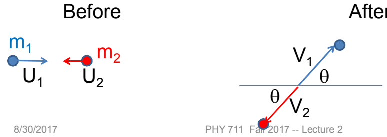
Laboratory reference frame:

Before After



Center of mass reference frame:

Before After



Relationship between center of mass and laboratory frames of reference -- continued

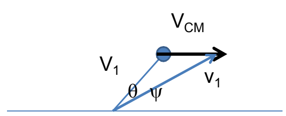
Since m_2 is initially at rest :

$$\mathbf{V}_{CM} = \frac{m_1}{m_1+m_2}\mathbf{u}_1 \qquad \mathbf{u}_1 = \mathbf{U}_1 + \mathbf{V}_{CM} \quad \Rightarrow \mathbf{U}_1 = \frac{m_2}{m_1+m_2}\mathbf{u}_1 = \frac{m_2}{m_1}\mathbf{V}_{CM}$$

$$\mathbf{u}_2 = \mathbf{U}_2 + \mathbf{V}_{CM} \quad \Rightarrow \mathbf{U}_2 = -\frac{m_1}{m_1+m_2}\mathbf{u}_1 = -\mathbf{V}_{CM}$$

$$\mathbf{v}_1 = \mathbf{V}_1 + \mathbf{V}_{CM}$$
$$\mathbf{v}_2 = \mathbf{V}_2 + \mathbf{V}_{CM}$$

Relationship between center of mass and laboratory frames of reference



$$\mathbf{v}_1 = \mathbf{V}_1 + \mathbf{V}_{CM}$$
$$v_1 \sin\psi = V_1 \sin\theta$$
$$v_1 \cos\psi = V_1 \cos\theta + V_{CM}$$
$$\tan\psi = \frac{\sin\theta}{\cos\theta + V_{CM}/V_1} = \frac{\sin\theta}{\cos\theta + m_1/m_2}$$

For elastic scattering

Digression – elastic scattering

$$\tfrac{1}{2}m_1U_1^2 + \tfrac{1}{2}m_2U_2^2 + \tfrac{1}{2}(m_1+m_2)V_{CM}^2$$
$$= \tfrac{1}{2}m_1V_1^2 + \tfrac{1}{2}m_2V_2^2 + \tfrac{1}{2}(m_1+m_2)V_{CM}^2$$

Also note:

$$m_1\mathbf{U}_1 + m_2\mathbf{U}_2 = 0 \qquad m_1\mathbf{V}_1 + m_2\mathbf{V}_2 = 0$$

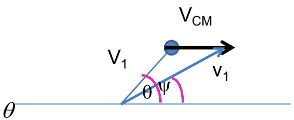
$$\mathbf{U}_1 = \frac{m_2}{m_1}\mathbf{V}_{CM} \qquad \mathbf{U}_2 = -\mathbf{V}_{CM}$$

$$\Rightarrow |\mathbf{U}_1| = |\mathbf{V}_1| \quad \text{and} \quad |\mathbf{U}_2| = |\mathbf{V}_2| = |\mathbf{V}_{CM}|$$

Also note that : $m_1|\mathbf{U}_1| = m_2|\mathbf{U}_2|$

So that : $V_{CM}/V_1 = V_{CM}/U_1 = m_1/m_2$

Relationship between center of mass and laboratory frames of reference – continued (elastic scattering)



$$\mathbf{v}_1 = \mathbf{V}_1 + \mathbf{V}_{CM}$$

$$v_1 \sin\psi = V_1 \sin\theta$$

$$v_1 \cos\psi = V_1 \cos\theta + V_{CM}$$

$$\tan\psi = \frac{\sin\theta}{\cos\theta + V_{CM}/V_1} = \frac{\sin\theta}{\cos\theta + m_1/m_2}$$

$$\text{Also:} \quad \cos\psi = \frac{\cos\theta + m_1/m_2}{\sqrt{1 + 2m_1/m_2\cos\theta + (m_1/m_2)^2}}$$

Differential cross sections in different reference frames

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}}\right) = \left(\frac{d\sigma_{CM}(\theta)}{d\Omega_{CM}}\right)\frac{d\Omega_{CM}}{d\Omega_{LAB}}$$

$$\frac{d\Omega_{CM}}{d\Omega_{LAB}} = \left|\frac{\sin\theta}{\sin\psi}\frac{d\theta}{d\psi}\right| = \left|\frac{d\cos\theta}{d\cos\psi}\right|$$

Using:

$$\cos\psi = \frac{\cos\theta + m_1/m_2}{\sqrt{1 + 2(m_1/m_2)\cos\theta + (m_1/m_2)^2}}$$

$$\left|\frac{d\cos\psi}{d\cos\theta}\right| = \frac{(m_1/m_2)\cos\theta + 1}{\left(1 + 2(m_1/m_2)\cos\theta + (m_1/m_2)^2\right)^{3/2}}$$

Differential cross sections in different reference frames – continued:

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}}\right) = \left(\frac{d\sigma_{CM}(\theta)}{d\Omega_{CM}}\right)\left|\frac{d\cos\theta}{d\cos\psi}\right|$$

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}}\right) = \left(\frac{d\sigma_{CM}(\theta)}{d\Omega_{CM}}\right)\frac{\left(1 + 2m_1/m_2\cos\theta + (m_1/m_2)^2\right)^{3/2}}{(m_1/m_2)\cos\theta + 1}$$

$$\text{where:} \quad \tan\psi = \frac{\sin\theta}{\cos\theta + m_1/m_2}$$

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}}\right) = \left(\frac{d\sigma_{CM}(\theta)}{d\Omega_{CM}}\right)\frac{\left(1+2m_1/m_2\cos\theta+(m_1/m_2)^2\right)^{3/2}}{(m_1/m_2)\cos\theta+1}$$

where : $\tan\psi = \dfrac{\sin\theta}{\cos\theta + m_1/m_2}$

Example: suppose $m_1 = m_2$

In this case : $\tan\psi = \dfrac{\sin\theta}{\cos\theta+1} \quad \Rightarrow \psi = \dfrac{\theta}{2}$

note that $0 \le \psi \le \dfrac{\pi}{2}$

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}}\right) = \left(\frac{d\sigma_{CM}(2\psi)}{d\Omega_{CM}}\right)\cdot 4\cos\psi$$

8/30/2017 PHY 711 Fall 2017 -- Lecture 2 25