PHY 741 Quantum Mechanics
12-12:50 PM MWF Olin 103

Plan for Lecture 12:
Read Chapter #10 in Shankar
Quantum mechanics of multiparticle systems

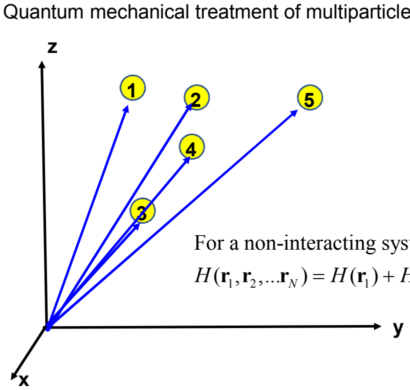
1. **Non-interacting particles**
   a. **Distinguishable, Fermi, Bose**
   b. **Second quantized formalism**
2. **Interacting particles**

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment	Due
1	Mon, 8/28/2017	Chap. 1	Review of basic principles	#1	9/6/2017
2	Wed, 8/30/2017	Chap. 1	Linear vector spaces	#2	9/6/2017
3	Fri, 9/01/2017	Chap. 1	Linear vector spaces	#3	9/6/2017
4	Mon, 9/04/2017	Chap. 4	Principles of Quantum Mechanics	#4	9/8/2017
5	Wed, 9/06/2017	Chap. 5	Examples in 1 dimension		
6	Fri, 9/08/2017	Chap. 5	Schrödinger equation in one-dimension	#5	9/13/2017
7	Mon, 9/11/2017	Chap. 5	Schrödinger equation in one-dimension		
8	Wed, 9/13/2017	Chap. 7	Schrödinger equation in one-dimension	#6	9/15/2017
9	Fri, 9/15/2017	Chap. 7	Schrödinger equation in one-dimension	#7	9/20/2017
10	Mon, 9/18/2017	Chap. 5 and 7	Schrödinger equation in one-dimension		
11	Wed, 9/20/2017	Chap. 9	Commutator formalism	#8	9/22/2017
12	Fri, 9/22/2017	Chap. 10	Quantum mechanics of multiparticle systems	#9	9/25/2017
13	Mon, 9/25/2017				
14	Wed, 9/27/2017				
15	Fri, 9/29/2017				
	Mon, 10/02/2017		*Take-home exam -- No class*		
	Wed, 10/04/2017		*Take-home exam -- No class*		

Quantum mechanical treatment of multiparticle systems



For a non-interacting system:

$$H(\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_N) = H(\mathbf{r}_1) + H(\mathbf{r}_2) + \ldots H(\mathbf{r}_N)$$

Quantum mechanical treatment of multiparticle systems

For a non-interacting system:

$$H(\mathbf{r}_1,\mathbf{r}_2,...\mathbf{r}_N) = H(\mathbf{r}_1) + H(\mathbf{r}_2) + ...H(\mathbf{r}_N)$$

Energy eigenstates:

$$H(\mathbf{r}_1,\mathbf{r}_2,...\mathbf{r}_N)\psi(\mathbf{r}_1,\mathbf{r}_2,...\mathbf{r}_N) = E\psi(\mathbf{r}_1,\mathbf{r}_2,...\mathbf{r}_N)$$

Simplification for separable Hamiltonian

For: $H(\mathbf{r}_1)\varphi_a(\mathbf{r}_1) = \varepsilon_a\varphi_a(\mathbf{r}_1)$

$H(\mathbf{r}_2)\varphi_b(\mathbf{r}_2) = \varepsilon_b\varphi_b(\mathbf{r}_2)$

Solution to the many particle problem

$$\psi(\mathbf{r}_1,\mathbf{r}_2,...\mathbf{r}_N) = \varphi_a(\mathbf{r}_1)\varphi_b(\mathbf{r}_2)...\varphi_z(\mathbf{r}_N)$$

$$E = \varepsilon_a + \varepsilon_b + ...\varepsilon_z$$

Quantum mechanical treatment of multiparticle systems – non-interacting particles

The treatment given on previous slides, assumes that the particles are distinguishable.
A more sophisticated treatment is needed for indistinguishable particles.

Two types of indistinguishable particles:

Fermi particles: $\psi(\mathbf{r}_1,\mathbf{r}_2,..\mathbf{r}_i...\mathbf{r}_j...\mathbf{r}_N) = -\psi(\mathbf{r}_1,\mathbf{r}_2,..\mathbf{r}_j...\mathbf{r}_i...\mathbf{r}_N)$

Bose particles: $\psi(\mathbf{r}_1,\mathbf{r}_2,..\mathbf{r}_i...\mathbf{r}_j...\mathbf{r}_N) = +\psi(\mathbf{r}_1,\mathbf{r}_2,..\mathbf{r}_j...\mathbf{r}_i...\mathbf{r}_N)$

Quantum mechanical treatment of multiparticle systems – non-interacting **Fermi** particles

Fermi particles: $\psi(\mathbf{r}_1,\mathbf{r}_2,..\mathbf{r}_i...\mathbf{r}_j...\mathbf{r}_N) = -\psi(\mathbf{r}_1,\mathbf{r}_2,..\mathbf{r}_j...\mathbf{r}_i...\mathbf{r}_N)$

Example for two particles:

$$H(\mathbf{r}_1,\mathbf{r}_2)\psi(\mathbf{r}_1,\mathbf{r}_2) = E\psi(\mathbf{r}_1,\mathbf{r}_2)$$

For: $H(\mathbf{r}_1)\varphi_a(\mathbf{r}_1) = \varepsilon_a\varphi_a(\mathbf{r}_1)$

$H(\mathbf{r}_2)\varphi_b(\mathbf{r}_2) = \varepsilon_b\varphi_b(\mathbf{r}_2)$

$$\psi(\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{\sqrt{2}}\left(\varphi_a(\mathbf{r}_1)\varphi_b(\mathbf{r}_2) - \varphi_a(\mathbf{r}_2)\varphi_b(\mathbf{r}_1)\right)$$

$$E = \varepsilon_a + \varepsilon_b$$

Quantum mechanical treatment of multiparticle systems – non-interacting **Fermi** particles

Fermi particles: $\psi(\mathbf{r}_1,\mathbf{r}_2,..\mathbf{r}_i...\mathbf{r}_j...\mathbf{r}_N) = -\psi(\mathbf{r}_1,\mathbf{r}_2,..\mathbf{r}_j...\mathbf{r}_i...\mathbf{r}_N)$

Example for N particles using Slater determinant:

$$\psi(\mathbf{r}_1,\mathbf{r}_2....\mathbf{r}_N) = \frac{1}{\sqrt{N!}}\begin{vmatrix} \varphi_a(\mathbf{r}_1) & \varphi_a(\mathbf{r}_2) & \varphi_a(\mathbf{r}_3) & \cdots & \varphi_a(\mathbf{r}_N) \\ \varphi_b(\mathbf{r}_1) & \varphi_b(\mathbf{r}_2) & \varphi_b(\mathbf{r}_3) & \cdots & \varphi_b(\mathbf{r}_N) \\ \varphi_c(\mathbf{r}_1) & \varphi_c(\mathbf{r}_2) & \varphi_c(\mathbf{r}_3) & \cdots & \varphi_c(\mathbf{r}_N) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \varphi_z(\mathbf{r}_1) & \varphi_z(\mathbf{r}_2) & \varphi_z(\mathbf{r}_3) & \cdots & \varphi_z(\mathbf{r}_N) \end{vmatrix}$$

$E = \varepsilon_a + \varepsilon_b + ...\varepsilon_z$

Quantum mechanical treatment of multiparticle systems – non-interacting **Bose** particles

Bose particles: $\psi(\mathbf{r}_1,\mathbf{r}_2,..\mathbf{r}_i...\mathbf{r}_j...\mathbf{r}_N) = \psi(\mathbf{r}_1,\mathbf{r}_2,..\mathbf{r}_j...\mathbf{r}_i...\mathbf{r}_N)$

Example for two particles:

$H(\mathbf{r}_1,\mathbf{r}_2)\psi(\mathbf{r}_1,\mathbf{r}_2) = E\psi(\mathbf{r}_1,\mathbf{r}_2)$

For: $H(\mathbf{r}_1)\varphi_a(\mathbf{r}_1) = \varepsilon_a\varphi_a(\mathbf{r}_1)$

$H(\mathbf{r}_2)\varphi_b(\mathbf{r}_2) = \varepsilon_b\varphi_b(\mathbf{r}_2)$

$$\psi(\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{\sqrt{2}}\left(\varphi_a(\mathbf{r}_1)\varphi_b(\mathbf{r}_2) + \varphi_a(\mathbf{r}_2)\varphi_b(\mathbf{r}_1)\right)$$

$E = \varepsilon_a + \varepsilon_b$

Quantum mechanical treatment of multiparticle systems – non-interacting particles; multiplicity of eigenstates

Consider a system with two independent particle states and two particles:

ε_b $\qquad H(\mathbf{r}_1)\varphi_a(\mathbf{r}_1) = \varepsilon_a\varphi_a(\mathbf{r}_1)$

ε_a $\qquad H(\mathbf{r}_2)\varphi_b(\mathbf{r}_2) = \varepsilon_b\varphi_b(\mathbf{r}_2)$

Possible states for distinguishable particles:

$\psi_I(\mathbf{r}_1,\mathbf{r}_2) = \varphi_a(\mathbf{r}_1)\varphi_b(\mathbf{r}_2) \qquad E_I = \varepsilon_a + \varepsilon_b$

$\psi_{II}(\mathbf{r}_1,\mathbf{r}_2) = \varphi_a(\mathbf{r}_2)\varphi_b(\mathbf{r}_1) \qquad E_{II} = \varepsilon_a + \varepsilon_b$

$\psi_{III}(\mathbf{r}_1,\mathbf{r}_2) = \varphi_a(\mathbf{r}_1)\varphi_a(\mathbf{r}_2) \qquad E_{III} = 2\varepsilon_a$

$\psi_{IV}(\mathbf{r}_1,\mathbf{r}_2) = \varphi_b(\mathbf{r}_1)\varphi_b(\mathbf{r}_2) \qquad E_{IV} = 2\varepsilon_b$

Quantum mechanical treatment of multiparticle systems – non-interacting particles; multiplicity of eigenstates

Consider a system with two independent particle states and two particles:

ε_b $\qquad H(\mathbf{r}_1)\varphi_a(\mathbf{r}_1)=\varepsilon_a\varphi_a(\mathbf{r}_1)$

ε_a $\qquad H(\mathbf{r}_2)\varphi_b(\mathbf{r}_2)=\varepsilon_b\varphi_b(\mathbf{r}_2)$

Possible states for Fermi particles:

$$\psi_I(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{\sqrt{2}}\left(\varphi_a(\mathbf{r}_1)\varphi_b(\mathbf{r}_2)-\varphi_a(\mathbf{r}_2)\varphi_b(\mathbf{r}_1)\right) \qquad E_I=\varepsilon_a+\varepsilon_b$$

Quantum mechanical treatment of multiparticle systems – non-interacting particles; multiplicity of eigenstates

Consider a system with two independent particle states and two particles:

ε_b $\qquad H(\mathbf{r}_1)\varphi_a(\mathbf{r}_1)=\varepsilon_a\varphi_a(\mathbf{r}_1)$

ε_a $\qquad H(\mathbf{r}_2)\varphi_b(\mathbf{r}_2)=\varepsilon_b\varphi_b(\mathbf{r}_2)$

Possible states for Bose particles:

$$\psi_I(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{\sqrt{2}}\left(\varphi_a(\mathbf{r}_1)\varphi_b(\mathbf{r}_2)+\varphi_a(\mathbf{r}_2)\varphi_b(\mathbf{r}_1)\right) \qquad E_I=\varepsilon_a+\varepsilon_b$$

$$\psi_{II}(\mathbf{r}_1,\mathbf{r}_2)=\varphi_a(\mathbf{r}_1)\varphi_a(\mathbf{r}_2) \qquad E_{II}=2\varepsilon_a$$

$$\psi_{III}(\mathbf{r}_1,\mathbf{r}_2)=\varphi_b(\mathbf{r}_1)\varphi_b(\mathbf{r}_2) \qquad E_{III}=2\varepsilon_b$$

Treating multiparticle systems using "second" quantization formalism

Consider a non-interacting system:

$$H(\mathbf{r}_1,\mathbf{r}_2,\ldots\mathbf{r}_N)=H(\mathbf{r}_1)+H(\mathbf{r}_2)+\ldots H(\mathbf{r}_N)$$

For a system of non-interacting identical particles, the single particle Hamiltonians $H(\mathbf{r}_1)$ are also identical.

Suppose we have a complete basis set that describes each single-particle state;

$$\psi(\mathbf{r},t)=\sum_\alpha C_\alpha\varphi_\alpha(\mathbf{r})e^{-i\varepsilon_\alpha t/\hbar}$$

These basis functions can be used to represent the many particle wavefunctions.

Example for Fermi particles

Slater determinant for N particles:

$$\left|\psi(\mathbf{r}_1,\mathbf{r}_2....\mathbf{r}_N)\right\rangle = \frac{1}{\sqrt{N!}}\begin{vmatrix} \varphi_a(\mathbf{r}_1) & \varphi_a(\mathbf{r}_2) & \varphi_a(\mathbf{r}_3) & \cdots & \varphi_a(\mathbf{r}_N) \\ \varphi_b(\mathbf{r}_1) & \varphi_b(\mathbf{r}_2) & \varphi_b(\mathbf{r}_3) & \cdots & \varphi_b(\mathbf{r}_N) \\ \varphi_c(\mathbf{r}_1) & \varphi_c(\mathbf{r}_2) & \varphi_c(\mathbf{r}_3) & \cdots & \varphi_c(\mathbf{r}_N) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \varphi_z(\mathbf{r}_1) & \varphi_z(\mathbf{r}_2) & \varphi_z(\mathbf{r}_3) & \cdots & \varphi_z(\mathbf{r}_N) \end{vmatrix}$$

Second quantization representation:

$$\left|\psi(\mathbf{r}_1,\mathbf{r}_2....\mathbf{r}_N)\right\rangle \Rightarrow \left|n_a n_b n_c....n_z\right\rangle$$

For Fermi particles, the occupation eigenvalues can be n_α=*0* or *1*

Operators for Fermi system

Creation operator:

$$a_\alpha^\dagger\left|0\right\rangle = \left|1_\alpha\right\rangle$$

Destruction operator:

$$a_\alpha\left|1_\alpha\right\rangle = \left|0_\alpha\right\rangle$$

$$a_\alpha^\dagger a_\beta^\dagger = -a_\beta^\dagger a_\alpha^\dagger$$

$$a_\alpha a_\beta = -a_\beta a_\alpha$$

Second quantization creation and annhilation operators

$$a_\alpha\left|n_\alpha\right\rangle = \sqrt{n_\alpha}\left|n_\alpha - 1\right\rangle$$

$$a^\dagger{}_\alpha\left|n_\alpha\right\rangle = \sqrt{n_\alpha - 1}\left|n_\alpha + 1\right\rangle$$

$$a_\alpha^\dagger a_\alpha\left|n_\alpha\right\rangle = n_\alpha\left|n_\alpha\right\rangle$$

$$a_\alpha a_\alpha^\dagger\left|n_\alpha\right\rangle = \left(n_\alpha - 1\right)\left|n_\alpha\right\rangle$$