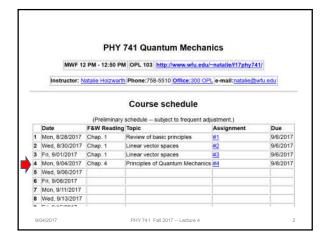
PHY 712 Quantum Mechanics 12-12:50 PM MWF Olin 103

Plan for Lecture 4:

Reading: Skim chapters #2 & #3 and start reading #4 in Shankar

- 1. Mathematical formalisms that can represent quantum phenomena
- 2. State vector
- 3. Physical variables and their representations

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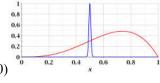


Dirac delta funct	tions $\delta(k)$	- k') ≡	$\equiv \frac{1}{2\pi} \int_{-\infty}^{\infty}$	$e^{i(k-k')x}$	dx
		=	$=\frac{1}{2\pi} \lim_{L\to\infty}$	$ \underset{\to^{\infty}}{m} \left(\int_{-I}^{L} e^{-\frac{t}{2}} \right) $	$dx^{i(k-k')x}dx$
		4	L=50		
-4	-2	₩₩₩ 0 -2	L=2	2 .	4
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Properties of the Dirac delta function

$$\delta(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ \infty & \text{for } x = 0 \end{cases}$$

$$\int_{0}^{\epsilon} \delta(x) dx = 1$$



Properties of the Dirac delta function -- continued

$$\int_{-\infty}^{\infty} G(x)\delta(f(x))dx = \int_{-\infty}^{\infty} G(x)\delta(f(x))\frac{df}{df}dx$$

$$\int_{-\infty}^{\infty} G(x)\delta(f(x))dx = \int_{-\infty}^{\infty} G(x)\delta(f(x))\frac{df}{df}dx$$
$$\int_{-\infty}^{\infty} G(x)\frac{\delta(f(x))}{df/dx}df = \sum_{i} \frac{G(x_{i})}{\left|\frac{df}{dx}\right|_{f(x_{i})=0}}$$

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Properties of the Dirac delta function -- continued

$$\int_{-\infty}^{\infty} G(x)\delta'(x)dx = -\frac{dG(x)}{dx}\bigg|_{x=0}$$



Details:

$$\int_{-\infty}^{\infty} G(x) \frac{d\delta(x)}{dx} dx = \int_{-\infty}^{\infty} \frac{d\left(G(x)\delta(x)\right)}{dx} dx - \int_{-\infty}^{\infty} \frac{dG(x)}{dx} \delta(x) dx$$



=0 provided that $G(\infty)$ is well-behaved

Note: Shankar has minus sign issue....

All Is Not Well with Classical Mechanics It was mentioned in the Prelude that as we keep expanding our domain of observations we must constantly check to see if the existing laws of physics continue to explain the new phonennean, and that, if they do not, we must try to find new laws that do. In this chapter you will get acquainted with experiments that betray the inadequacy of the classical scheme. The experiments to be described were never performed exactly as described here, but they contain the essential features of the actual experiments that were performed (in the first quarter of this century) with none of their inessential complications.	Material to skim:	2
In this chapter we will develop the Lagrangian and Hamiltonian formulations of mechanics starting from Newton's laws. These subsequent reformulations of mechanics being with them a gest uited of degane and computational ease but our principal to be computed to the computation of mechanics are sensitially the same theory, in that their domains of validity and predictions are identical. Nonetheless, in a given context, one or the other may be more inviting for conceptual, computational, or simply setheric reasons. 21. The Principle of Least Action and Lagrangian Mechanics All Is Not Well with Classical Mechanics It was mentioned in the Prelude that as we keep expanding our domain of observations we must constantly check to see if the existing laws of physics continue to explain the new photocomes, and that, if they does, we must perform the one when the content of the computation of the compu	Review of C	Classical Mechanics
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Classical Mechanics	Quantum Mechanics
II. Every dynamical variable ω is a function of x and $p: \omega = \omega(x, p)$.	II. The independent variables x and p of classical mechanics are represented
	by Hermitian operators X and P with the following matrix elements
	in the eigenbasis of X^{\ddagger}
	$\langle x X x'\rangle = x\delta(x-x')$
	$\langle x P x'\rangle = -i\hbar\delta'(x-x')$
	The operators corresponding to dependent variables $\omega(x, p)$ are
	given Hermitian operators
	$\Omega(X, P) = \omega(x \to X, p \to P) $
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Classical Mechanics	Quantum Mechanics
III. If the particle is in a state given by x and p , the measurement $ $ of the	III. If the particle is in a state $ \psi\rangle$, meas urement of the variable (corre
variable ω will yield a value $\omega(x, p)$. The state will remain unaffected.	sponding to) Ω will yield one of the
The state will remain unanected.	eigenvalues ω with probability $P(\omega) \propto \langle \omega \psi \rangle ^2$. The state of the
	system will change from $ \psi\rangle$ to $ \omega\rangle$ as a result of the measurement.
9/04/2017 PHY 741 Fall 20	17 Lecture 4 11
Classical Mechanics	Quantum Mechanics
IV. The state variables change with time	
according to Hamilton's equations:	Schrödinger equation
$\dot{\mathbf{r}} = \frac{\partial \mathscr{H}}{\partial \mathbf{r}}$	$i\hbar \frac{d}{dt} \psi(t)\rangle = H \psi(t)\rangle$
$\dot{x} = \frac{\partial \mathscr{H}}{\partial p}$	$\frac{d}{dt} \psi(t)\rangle - H \psi(t)\rangle$
$\dot{p} = -\frac{\partial \mathcal{H}}{\partial x}$	where $H(X, P) = \mathcal{H}(x \rightarrow X, p \rightarrow P)$ is
∂x	the quantum Hamiltonian operator and \mathcal{H} is the Hamiltonian for the
	corresponding classical problem.
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Expectation values

For an operator **O** and a system in a state $|\psi\rangle$:

$$\langle \mathbf{O} \rangle = \langle \psi | \mathbf{O} | \psi \rangle$$

Uncertainty values

For an operator **O** and a system in a state $|\psi\rangle$:

$$\langle \Delta \mathbf{O} \rangle = (\langle \psi | (\mathbf{O} - \langle \mathbf{O} \rangle)^2 | \psi \rangle)^{1/2}$$

Consider the following example of the four 2-dimensional operators:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

Eigenvectors of σ_z and σ^2 :

$$|\psi_{\downarrow}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
 $|\psi_{\uparrow}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Expectation values:

$$\begin{split} & \left\langle \stackrel{\bot}{\psi_{\downarrow}} \middle| \sigma_x \middle| \psi_{\downarrow} \right\rangle = 0 \qquad \left\langle \psi_{\downarrow} \middle| \sigma_y \middle| \psi_{\downarrow} \right\rangle = 0 \qquad \left\langle \psi_{\downarrow} \middle| \sigma_z \middle| \psi_{\downarrow} \right\rangle = -1 \qquad \left\langle \psi_{\uparrow} \middle| \sigma^2 \middle| \psi_{\uparrow} \right\rangle = 3 \\ & \left\langle \psi_{\uparrow} \middle| \sigma_x \middle| \psi_{\uparrow} \right\rangle = 0 \qquad \left\langle \psi_{\uparrow} \middle| \sigma_y \middle| \psi_{\uparrow} \right\rangle = 0 \qquad \left\langle \psi_{\uparrow} \middle| \sigma_z \middle| \psi_{\uparrow} \right\rangle = +1 \qquad \left\langle \psi_{\uparrow} \middle| \sigma^2 \middle| \psi_{\uparrow} \right\rangle = 3 \end{split}$$

Uncertainty values:

Uncertainty values:
$$\langle \psi_{\downarrow} \big| \Delta \sigma_x \big| \psi_{\downarrow} \rangle = 1 \quad \langle \psi_{\downarrow} \big| \Delta \sigma_y \big| \psi_{\downarrow} \rangle = 1 \quad \langle \psi_{\downarrow} \big| \Delta \sigma_z \big| \psi_{\downarrow} \rangle = 0 \quad \langle \psi_{\downarrow} \big| \Delta \sigma^2 \big| \psi_{\downarrow} \rangle = 0$$

$$\langle \psi_{\uparrow} \big| \Delta \sigma_x \big| \psi_{\uparrow} \rangle = 1 \quad \langle \psi_{\uparrow} \big| \Delta \sigma_y \big| \psi_{\uparrow} \rangle = 1 \quad \langle \psi_{\uparrow} \big| \Delta \sigma_z \big| \psi_{\uparrow} \rangle = 0 \quad \langle \psi_{\uparrow} \big| \Delta \sigma^2 \big| \psi_{\uparrow} \rangle = 0$$

$$\langle \psi_{\uparrow} \big| \Delta \sigma^2 \big| \psi_{\uparrow} \rangle = 0 \quad \langle \psi_{\uparrow} \big| \Delta \sigma^2 \big| \psi_{\uparrow} \rangle = 0$$

Further considerations --

Consider the following example of the four 2-dimensional operators:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

Eigenvectors of σ_x :

$$|\psi_{x1}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1\\-1 \end{pmatrix}$$
 $|\psi_{x2}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1\\1 \end{pmatrix}$ Expectation values:

$$\begin{split} & \left\langle \psi_{x1} \middle| \sigma_{x} \middle| \psi_{x1} \right\rangle = -1 & \left\langle \psi_{x1} \middle| \sigma_{y} \middle| \psi_{x1} \right\rangle = 0 & \left\langle \psi_{x1} \middle| \sigma_{z} \middle| \psi_{x1} \right\rangle = 0 & \left\langle \psi_{x1} \middle| \sigma^{2} \middle| \psi_{x1} \right\rangle = 3 \\ & \left\langle \psi_{x2} \middle| \sigma_{x} \middle| \psi_{x2} \right\rangle = +1 & \left\langle \psi_{x2} \middle| \sigma_{y} \middle| \psi_{x2} \right\rangle = 0 & \left\langle \psi_{x2} \middle| \sigma_{z} \middle| \psi_{x2} \right\rangle = 0 & \left\langle \psi_{x2} \middle| \sigma^{2} \middle| \psi_{x2} \right\rangle = 3 \end{split}$$

Uncertainty values:

$$\begin{split} & \left\langle \psi_{x1} \left| \Delta \sigma_{x} \left| \psi_{x1} \right\rangle = 0 \quad \left\langle \psi_{x1} \left| \Delta \sigma_{y} \right| \psi_{x1} \right\rangle = 1 \quad \left\langle \psi_{x1} \left| \Delta \sigma_{z} \right| \psi_{x1} \right\rangle = 1 \quad \left\langle \psi_{x1} \left| \Delta \sigma^{z} \right| \psi_{x1} \right\rangle = 0 \\ & \left\langle \psi_{x2} \left| \Delta \sigma_{x} \left| \psi_{x2} \right\rangle = 0 \quad \left\langle \psi_{x2} \left| \Delta \sigma_{y} \right| \psi_{x2} \right\rangle = 1 \quad \left\langle \psi_{x2} \left| \Delta \sigma_{z} \right| \psi_{x2} \right\rangle = 1 \quad \left\langle \psi_{x2} \left| \Delta \sigma^{z} \right| \psi_{x2} \right\rangle = 0 \end{split}$$

Time evolution

Suppose that the Hamiltonian for this system has the form:

$$H = \sigma_z B = \begin{pmatrix} -B & 0 \\ 0 & B \end{pmatrix}$$
 where B is a constant

Schroedinger equation:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle$$

Note that if
$$|\psi(t=0)\rangle = |\psi_{\downarrow}\rangle = \begin{pmatrix} 1\\0 \end{pmatrix}$$
, then $|\psi(t)\rangle = e^{iBt/\hbar} \begin{pmatrix} 1\\0 \end{pmatrix}$

Note that if
$$|\psi(t=0)\rangle = |\psi_{\uparrow}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
, then $|\psi(t)\rangle = e^{-iBt/\hbar} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Note that if
$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1\\1 \end{pmatrix}$$
, then $|\psi(t)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{iBt/\hbar}\\e^{-iBt/\hbar} \end{pmatrix}$

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Time evolution -- continued

Note that if
$$|\psi(t=0)\rangle = |\psi_{\downarrow}\rangle = \begin{pmatrix} 1\\0 \end{pmatrix}$$
, then $|\psi(t)\rangle = e^{iBt/\hbar} \begin{pmatrix} 1\\0 \end{pmatrix}$
 $\Rightarrow \langle H \rangle = \langle \psi(t)|B|\psi(t)\rangle = -B$

Note that if
$$|\psi(t=0)\rangle = |\psi_{\uparrow}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
, then $|\psi(t)\rangle = e^{-iBt/\hbar} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\Rightarrow \langle H \rangle = \langle \psi(t)|H|\psi(t)\rangle = B$$

Note that if
$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1\\1 \end{pmatrix}$$
, then $|\psi(t)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{iBt/h}\\e^{-iBt/h} \end{pmatrix}$
 $\Rightarrow \langle H \rangle = \langle \psi(t)|H|\psi(t)\rangle = 0$

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Time evolution -- continued

Note that if
$$|\psi(t=0)\rangle = |\psi_{\downarrow}\rangle = \begin{pmatrix} 1\\0 \end{pmatrix}$$
, then $|\psi(t)\rangle = e^{i\hbar t/\hbar} \begin{pmatrix} 1\\0 \end{pmatrix}$

$$\Rightarrow \langle \sigma_x \rangle = \langle \psi(t)|\sigma_x|\psi(t)\rangle = 0$$

Note that if
$$|\psi(t=0)\rangle = |\psi_{\uparrow}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
, then $|\psi(t)\rangle = e^{-iBt/\hbar} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\Rightarrow \langle \sigma_x \rangle = \langle \psi(t)|\sigma_x|\psi(t)\rangle = 0$$

Note that if
$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
, then $|\psi(t)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{iBt/\hbar} \\ e^{-iBt/\hbar} \end{pmatrix}$

$$\Rightarrow \langle \sigma_x \rangle = \langle \psi(t) | \sigma_x | \psi(t) \rangle = \cos(2Bt/\hbar)$$

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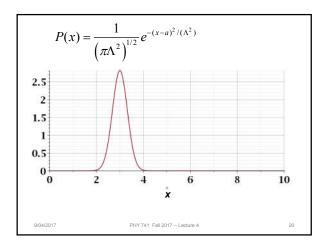
Other examples

Suppose we have a state vector which is a continuous function of x:

$$\begin{aligned} |\psi(x)\rangle &= \frac{1}{\left(\pi\Lambda^{2}\right)^{1/4}} e^{-(x-a)^{2}/(2\Lambda^{2})} \\ \langle x\rangle &= \langle \psi(x)|x|\psi(x)\rangle = \frac{1}{\left(\pi\Lambda^{2}\right)^{1/2}} \int_{-\infty}^{\infty} x e^{-(x-a)^{2}/(\Lambda^{2})} dx = a \\ \langle x^{2}\rangle &= \langle \psi(x)|x^{2}|\psi(x)\rangle = \frac{1}{\left(\pi\Lambda^{2}\right)^{1/2}} \int_{-\infty}^{\infty} x^{2} e^{-(x-a)^{2}/(\Lambda^{2})} dx = a^{2} + \frac{\Lambda^{2}}{2} \\ \langle \Delta x\rangle &= \sqrt{\langle x^{2}\rangle - \langle x\rangle^{2}} = \frac{\Lambda}{\sqrt{2}} \end{aligned}$$

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Example continued

$$|\psi(x)\rangle = \frac{1}{(\pi\Lambda^2)^{1/4}} e^{-(x-a)^2/(2\Lambda^2)}$$

Expectation value and uncertainty of momentum

$$\langle p \rangle = \langle \psi(x) | p | \psi(x) \rangle = \langle \psi(x) | -i\hbar \frac{d}{dx} | \psi(x) \rangle = 0$$

$$\langle p^2 \rangle = \langle \psi(x) | p^2 | \psi(x) \rangle = \langle \psi(x) | -\hbar^2 \frac{d^2}{dx^2} | \psi(x) \rangle = \frac{\hbar^2}{2\Lambda^2}$$

$$\langle \Delta p \rangle = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \frac{\hbar}{\sqrt{2}\Lambda}$$

Note that:

$$\langle \Delta x \rangle \langle \Delta p \rangle = \frac{\Lambda}{\sqrt{2}} \frac{\hbar}{\sqrt{2}\Lambda} = \frac{\hbar}{2}$$

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