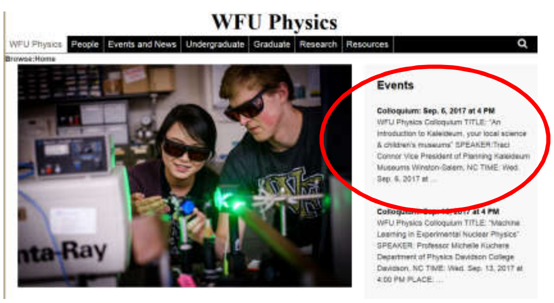
PHY 712 Quantum Mechanics
12-12:50 PM MWF Olin 103

Plan for Lecture 5:
Reading: Chapters #5 in Shankar; quantum mechanical systems in 1-dim

1. **Free particle**
2. **Particle bound in a box**



WFU Physics Colloquium

TITLE: "An Introduction to Kaleideum, your local science & children's museums"

SPEAKER: Traci Connor
Vice President of Planning
Kaleideum Museums
Winston-Salem, NC

TIME: Wed. Sep. 6, 2017 at 4:00 PM

PLACE: George P. Williams
101)

ABSTRACT

Traci Connor, VP of Planning at Kaleideum, specializes in exhibit development and evaluating the impact of exhibits. She will be providing an overview of science and children's museums including how scientists are partnering with these types of institution across the nation. She will also give us a virtual tour of our local science and children's museums, and a sneak preview into what they are planning now that they have merged into one institution, Kaleideum.

PHY 741 Quantum Mechanics

MWF 12 PM - 12:50 PM | OPL 103 | http://www.wfu.edu/~natalie/f17phy741/

Instructor: Natalie Holzwarth | Phone:758-5510 | Office:300 OPL | e-mail:natalie@wfu.edu

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment	Due
1	Mon, 8/28/2017	Chap. 1	Review of basic principles	#1	9/6/2017
2	Wed, 8/30/2017	Chap. 1	Linear vector spaces	#2	9/6/2017
3	Fri, 9/01/2017	Chap. 1	Linear vector spaces	#3	9/6/2017
4	Mon, 9/04/2017	Chap. 4	Principles of Quantum Mechanics	#4	9/8/2017
5	Wed, 9/06/2017	Chap. 5	Examples in 1 dimension		
6	Fri, 9/08/2017				
7	Mon, 9/11/2017				
8	Wed, 9/13/2017				

Formal solution of the Schrödingder equation

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H|\psi(t)\rangle$$

Solution in terms of eigenstates $|E\rangle$ of H:

$$H|E\rangle = E|E\rangle$$

$$|\psi(t)\rangle = \sum_E |E\rangle\langle E|\psi(t)\rangle \equiv \sum_E a_E(t)|E\rangle$$

coefficients to be determined

Formal solution of the Schrödingder equation -- continued

$$|\psi(t)\rangle = \sum_E a_E(t)|E\rangle$$

$$(i\hbar\, \partial/\partial t - H)|\psi(t)\rangle = \sum_E (i\hbar \dot{a}_E - E a_E)|E\rangle = 0$$

Equation for each coefficient $a_E(t)$:

$$i\hbar \frac{da_E(t)}{dt} = E a_E(t)$$

$$\rightarrow a_E(t) = a_E(0)\, e^{-iEt/\hbar}$$

Formal solution of the Schrödingder equation -- continued

$$\left|\psi(t)\right\rangle = \sum_E a_E(t)\left|E\right\rangle = \sum_E a_E(0)\left|E\right\rangle e^{-iEt/\hbar}$$

$$\left|\psi(t)\right\rangle = \sum_E \left|E\right\rangle\left\langle E\middle|\psi(0)\right\rangle e^{-iEt/\hbar}$$

$$= U(t)\left|\psi(0)\right\rangle$$

Propagator: $U(t) = \sum_E \left|E\right\rangle\left\langle E\right| e^{-iEt/\hbar}$

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Example – particle free to move in one dimension

$H = \frac{P^2}{2m}$ (Note: in this notation P is an operator while p is used to denote its eigenvalue)

Suppose we analyze our system in terms of eigenvalues of the operator P:

$$P\left|p\right\rangle = p\left|p\right\rangle$$

$$\frac{P^2}{2m}\left|p\right\rangle = H\left|p\right\rangle = \frac{p^2}{2m}\left|p\right\rangle$$

Note that in this case, $\left|p\right\rangle$ is simultaneously an eigenvector of P and H.

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Example – particle free to move in one dimension

Relationship between the eigenvalues *p* and *E*:

$$\left(\frac{p^2}{2m} - E\right)\left|p\right\rangle = 0$$

$$p = \pm(2mE)^{1/2}$$

$$\left|E\right\rangle = \beta\left|p = (2mE)^{1/2}\right\rangle + \gamma\left|p = -(2mE)^{1/2}\right\rangle$$

coefficients

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Example – particle free to move in one dimension

Determination of the propagator for this case

Propagator: $U(t) = \sum_E |E\rangle\langle E| e^{-iEt/\hbar}$

For this case:

$$U(t) = \sum_p |p\rangle\langle p| e^{-ip^2 t/(2m\hbar)}$$

In order to evaluate this expression, we need to have a functional form for the eigenstates |*p*>.

Example – particle free to move in one dimension

$P|p\rangle = p|p\rangle$

In the coordinate representation:

$-i\hbar \frac{d}{dx}|p\rangle = p|p\rangle$

$|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$

Check normalization: $\langle p|p'\rangle = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} e^{i(p'-p)x/\hbar} dx$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(p'-p)u} du = \delta(p-p')$$

Example – particle free to move in one dimension

Propagator: $U(t) = \sum_E |E\rangle\langle E| e^{-iEt/\hbar}$

For this case:

$$U(t) = \sum_p |p\rangle\langle p| e^{-ip^2 t/(2m\hbar)}$$

$|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$ formally: $\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$

$\langle x|U(t)|x'\rangle = \sum_p \langle x|p\rangle\langle p|x'\rangle e^{-ip^2 t/(2m\hbar)}$

Example – particle free to move in one dimension

$$\langle x|U(t)|x'\rangle = \sum_p \langle x|p\rangle\langle p|x'\rangle e^{-ip^2t/(2m\hbar)}$$

$$= \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} dp\ e^{ip(x-x')/\hbar} e^{-ip^2 t/(2m\hbar)}$$

$$= \left(\frac{1}{2\pi\hbar i t}\right)^{1/2} e^{im(x-x')^2/(2\hbar t)}$$

Digression – justification for integration result
(see appendix A of your text)

$$I_0(\alpha,\beta) = \int_{-\infty}^{\infty} e^{-\alpha x^2 + \beta x}\, dx$$

$$I_0(\alpha,\beta) = e^{\beta^2/4\alpha}\int_{-\infty}^{\infty} e^{-\alpha(x-\beta/2\alpha)^2}\, dx = e^{\beta^2/4\alpha}\left(\frac{\pi}{\alpha}\right)^{1/2}$$

Example – particle free to move in one dimension

$$\langle x|U(t)|x'\rangle = \left(\frac{1}{2\pi\hbar i t}\right)^{1/2} e^{im(x-x')^2/(2\hbar t)}$$

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle$$

$$\langle x|\psi(t)\rangle = \int_{-\infty}^{\infty} dx'\langle x|U(t)|x'\rangle\langle x'|\psi(0)\rangle$$

$$= \left(\frac{1}{2\pi\hbar i t}\right)^{1/2}\int_{-\infty}^{\infty} dx' e^{im(x-x')^2/(2\hbar t)}\langle x'|\psi(0)\rangle$$

Example – particle free to move in one dimension

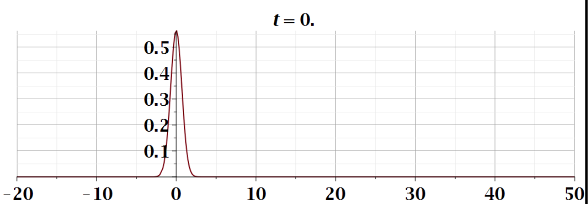
Time evolution of a Gaussian wave packet:

$$\psi(x',0)=e^{ip_0x'/\hbar}\,\frac{e^{-x'^2/2\Delta^2}}{(\pi\Delta^2)^{1/4}}$$

$$\psi(x,t)=\left[\pi^{1/2}\left(\Delta+\frac{i\hbar t}{m\Delta}\right)\right]^{-1/2}\cdot\exp\left[\frac{-(x-p_0t/m)^2}{2\Delta^2(1+i\hbar t/m\Delta^2)}\right]\exp\left[\frac{ip_0}{\hbar}\left(x-\frac{p_0t}{2m}\right)\right]$$

Probability density:

$$P(x,t)=\frac{1}{\pi^{1/2}(\Delta^2+\hbar^2t^2/m^2\Delta^2)^{1/2}}\cdot\exp\left\{\frac{-[x-(p_0/m)t]^2}{\Delta^2+\hbar^2t^2/m^2\Delta^2}\right\}$$



Summary of results for free particle

Energy eigenvalues: $E=\frac{p^2}{2m}$

Eigenvectors: $|p\rangle=\frac{1}{\sqrt{2\pi\hbar}}e^{ipx/\hbar}$

In this case, the energy spectrum is continuous
Notion of "density of states"

$$N(\varepsilon)\equiv\frac{L}{2\pi\hbar}\int_{-\infty}^{\infty}dp\,\delta\left(\varepsilon-\frac{p^2}{2m}\right)$$

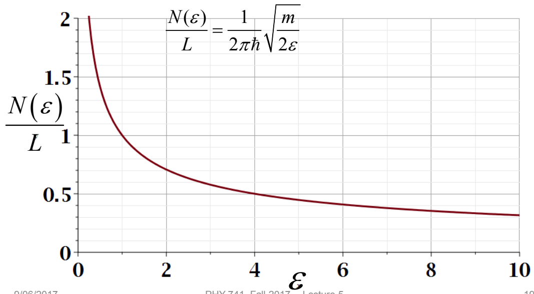
In this case, we imagine that we can account for each value of p in increments of $dp=\frac{2\pi\hbar}{L}$ in the limit of large L

Notion of density of states for continuum spectrum

Example: one-dimensional free particle

$$N(\varepsilon) \equiv \frac{L}{2\pi\hbar}\int_{-\infty}^{\infty} dp\,\delta\left(\varepsilon - \frac{p^2}{2m}\right)$$

$$\frac{N(\varepsilon)}{L} = \frac{1}{2\pi\hbar}\sqrt{\frac{m}{2\varepsilon}}$$



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