

**PHY 711 Classical Mechanics and
Mathematical Methods**
10-10:50 AM MWF Olin 103

Plan for Lecture 25:

Rotational motion (Chapter 5)

- 1. Rigid body motion**
- 2. Moment of inertia tensor**
- 3. Torque free motion**

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5	Fri, 9/07/2018	Chap. 2	Non-inertial coordinate systems	#4	9/12/2018
6	Mon, 9/10/2018	Chap. 3	Calculus of Variation	#5	9/12/2018
7	Wed, 9/12/2018	Chap. 3	Calculus of Variation	#6	9/17/2018
	Fri, 9/14/2018	No class	University closed due to weather.		
8	Mon, 9/17/2018	Chap. 3	Lagrangian Mechanics	#7	9/19/2018
9	Wed, 9/19/2018	Chap. 3 and 6	Lagrangian Mechanics and constraints	#8	9/24/2018
10	Fri, 9/21/2018	Chap. 3 and 6	Constants of the motion		
11	Mon, 9/24/2018	Chap. 3 and 6	Hamiltonian formalism	#9	9/28/2018
12	Wed, 9/26/2018	Chap. 3 and 6	Liouville theorem	#10	10/3/2018
13	Fri, 9/28/2018	Chap. 3 and 6	Canonical transformations		
14	Mon, 10/1/2018	Chap. 4	Small oscillations about equilibrium	#11	10/5/2018
15	Wed, 10/3/2018	Chap. 4	Normal modes of vibration		
16	Fri, 10/5/2018	Chap. 1-4, 6	Review		
17	Mon, 10/8/2018	Chap. 7	Strings		
18	Wed, 10/10/2018	Chap. 7	Wave equation		
	Fri, 10/12/2018	No class	Fall break		
19	Mon, 10/15/2018	Chap. 7	Wave equation		
20	Wed, 10/17/2018	Chap. 7	Fourier Transforms	#12	10/22/2018
21	Fri, 10/19/2018	Chap. 7	Laplace transforms; Contour integrals	#13	10/24/2018
22	Mon, 10/22/2018	Chap. 7	Contour integrals		
23	Wed, 10/24/2018	Chap. 5	Rigid body motion	#14	10/26/2018

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Dr. Ilias Belharouak

Distinguished Scientist and Group Leader
Oak Ridge National Laboratory

**“Science and Technology Drivers in Batteries for
Renewables Energy & Mobility Applications”**

Wednesday, October 24, 2018, at 4:00 pm

George P. Williams, Jr. Lecture Hall (Olin 101)



The worldwide growing integration of renewable energies and electric vehicles will require massive deployment of energy storage solutions. The aim of this seminar is to discuss the potential of batteries to stabilize the grid whilst storing energy, understand various types of batteries used to store energy and address battery life expectancy, and how it can be maintained for maximum efficiency. A case study centered around a 250kW/500kWh lithium-ion battery coupled with a 200kW PV-plant represents an opportunity for investigating the cycling conditions and aging behavior of a grid-connected Li-ion battery. In a second part, the application of the LNM/LTO cell chemistry will be discussed in the light of recent results which anticipate the potential deployment of this lithium-ion battery in both grid-storage and electric vehicles.

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Comparison of analysis in "inertial frame" versus "non-inertial frame"

Denote by $\hat{e}_i^0$ a fixed coordinate system

Denote by $\hat{e}_i$ a moving coordinate system

For an arbitrary vector $\mathbf{V}$: $\mathbf{V} = \sum_{i=1}^3 V_i^0 \hat{e}_i^0 = \sum_{i=1}^3 V_i \hat{e}_i$

$$\left(\frac{d\mathbf{V}}{dt} \right)_{\text{inertial}} = \sum_{i=1}^3 \frac{dV_i^0}{dt} \hat{e}_i^0 = \sum_{i=1}^3 \frac{dV_i}{dt} \hat{e}_i + \sum_{i=1}^3 V_i \frac{d\hat{e}_i}{dt}$$

$$\text{Define: } \left(\frac{d\mathbf{V}}{dt} \right)_{\text{body}} \equiv \sum_{i=1}^3 \frac{dV_i}{dt} \hat{e}_i$$

$$\Rightarrow \left(\frac{d\mathbf{V}}{dt} \right)_{\text{inertial}} = \left(\frac{d\mathbf{V}}{dt} \right)_{\text{body}} + \sum_{i=1}^3 V_i \frac{d\hat{e}_i}{dt}$$

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The physics of rigid body motion; body fixed frame vs inertial frame; results from Chapter 2:

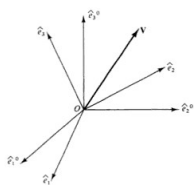


Figure 6.1 Transformation to a rotating coordinate system.

Let $\mathbf{V}$ be a general vector, e.g., the position of a particle. This vector can be characterized by its components with respect to either orthonormal triad. Thus we can write

$$\mathbf{V} = \sum_{i=1}^3 V_i^0 \hat{e}_i^0 \quad (6.1a)$$

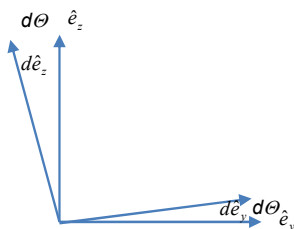
$$\mathbf{V} = \sum_{i=1}^3 V_i \hat{e}_i \quad (6.1b)$$

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Properties of the frame motion (rotation):



$$d\hat{e}_y = d\Theta \hat{e}_z$$

$$d\hat{e}_z = -d\Theta \hat{e}_y$$

$$\Rightarrow d\hat{\mathbf{e}} = d\Theta \times \hat{\mathbf{e}}$$

$$\frac{d\hat{\mathbf{e}}}{dt} = \frac{d\Theta}{dt} \times \hat{\mathbf{e}}$$

$$\frac{d\hat{\mathbf{e}}}{dt} = \boldsymbol{\omega} \times \hat{\mathbf{e}}$$

$$\begin{pmatrix} d\hat{e}_y \\ d\hat{e}_z \end{pmatrix} = \begin{pmatrix} \cos(d\Theta) & \sin(d\Theta) \\ -\sin(d\Theta) & \cos(d\Theta) \end{pmatrix} \begin{pmatrix} \hat{e}_y \\ \hat{e}_z \end{pmatrix} = \begin{pmatrix} \hat{e}_y \\ \hat{e}_z \end{pmatrix}$$

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$$\left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \sum_{i=1}^3 V_i \frac{d\hat{e}_i}{dt}$$

$$\left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \boldsymbol{\omega} \times \mathbf{V}$$

Effects on acceleration:

$$\left(\frac{d}{dt} \frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\frac{d}{dt}\right)_{body} \left\{ \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \boldsymbol{\omega} \times \mathbf{V} \right\}$$

$$\left(\frac{d^2\mathbf{V}}{dt^2}\right)_{inertial} = \left(\frac{d^2\mathbf{V}}{dt^2}\right)_{body} + 2\boldsymbol{\omega} \times \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{V} + \boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{V}$$

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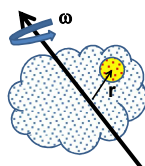
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Kinetic energy of rigid body :

$$\left(\frac{d\mathbf{r}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{r}}{dt}\right)_{body} + \boldsymbol{\omega} \times \mathbf{r}$$

$$\rightarrow \left(\frac{d\mathbf{r}}{dt}\right)_{inertial} = \boldsymbol{\omega} \times \mathbf{r}$$

=0 for rigid body



$$T = \sum_p \frac{1}{2} m_p v_p^2 = \sum_p \frac{1}{2} m_p (\boldsymbol{\omega} \times \mathbf{r}_p)^2$$

$$= \sum_p \frac{1}{2} m_p (\boldsymbol{\omega} \times \mathbf{r}_p) \cdot (\boldsymbol{\omega} \times \mathbf{r}_p)$$

$$= \sum_p \frac{1}{2} m_p [(\boldsymbol{\omega} \cdot \boldsymbol{\omega})(\mathbf{r}_p \cdot \mathbf{r}_p) - (\mathbf{r}_p \cdot \boldsymbol{\omega})^2]$$

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$$T = \sum_p \frac{1}{2} m_p [(\boldsymbol{\omega} \cdot \boldsymbol{\omega})(\mathbf{r}_p \cdot \mathbf{r}_p) - (\mathbf{r}_p \cdot \boldsymbol{\omega})^2]$$

$$= \frac{1}{2} \boldsymbol{\omega} \cdot \tilde{\mathbf{I}} \cdot \boldsymbol{\omega}$$

Moment of inertia tensor :

$$\tilde{\mathbf{I}} \equiv \sum_p m_p (\mathbf{r}_p^2 \mathbf{1} - \mathbf{r}_p \mathbf{r}_p) \quad (\text{dyad notation})$$

Matrix notation :

$$\tilde{\mathbf{I}} \equiv \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$$

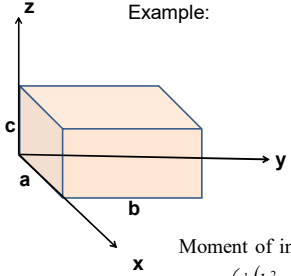
$$I_{ij} \equiv \sum_p m_p (\delta_{ij} r_p^2 - r_{pi} r_{pj})$$

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Example:



Moment of inertia tensor :

$$\vec{I} = M \begin{pmatrix} \frac{1}{3}(b^2 + c^2) & -\frac{1}{4}ab & -\frac{1}{4}ac \\ -\frac{1}{4}ab & \frac{1}{3}(a^2 + c^2) & -\frac{1}{4}bc \\ -\frac{1}{4}ac & -\frac{1}{4}bc & \frac{1}{3}(a^2 + b^2) \end{pmatrix}$$

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Properties of moment of inertia tensor:

- Symmetric matrix ➔ real eigenvalues I_1, I_2, I_3
- ➔ orthogonal eigenvectors

$\vec{I} \cdot \hat{e}_i = I_i \hat{e}_i \quad i = 1, 2, 3$

Moment of inertia tensor :

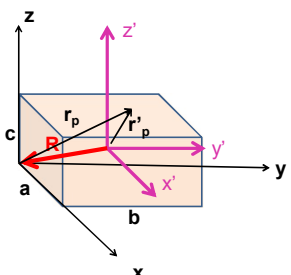
$$\vec{I} = M \begin{pmatrix} \frac{1}{3}(b^2 + c^2) & -\frac{1}{4}ab & -\frac{1}{4}ac \\ -\frac{1}{4}ab & \frac{1}{3}(a^2 + c^2) & -\frac{1}{4}bc \\ -\frac{1}{4}ac & -\frac{1}{4}bc & \frac{1}{3}(a^2 + b^2) \end{pmatrix}$$

For $a = b = c$:

$$I_1 = \frac{1}{6}Ma^2 \quad I_2 = \frac{11}{12}Ma^2 \quad I_3 = \frac{11}{12}Ma^2$$

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Changing origin of rotation



$$I_{ij} \equiv \sum_p m_p (\delta_{ij} r_p^2 - r_{pi} r_{pj})$$

$$I'_{ij} \equiv \sum_p m_p (\delta_{ij} r'^2_p - r'_{pi} r'_{pj})$$

$$\mathbf{r}'_p = \mathbf{r}_p - \mathbf{R}$$

Define the center of mass :

$$\mathbf{r}_{CM} = \frac{\sum_p m_p \mathbf{r}_p}{\sum_p m_p} \equiv \frac{\sum_p m_p \mathbf{r}_p}{M}$$

$$I'_{ij} = I_{ij} + M(R^2 \delta_{ij} - R_i R_j) + M(2\mathbf{r}_{CM} \cdot \mathbf{R} \delta_{ij} - r_{CMi} R_j - R_i r_{CMj})$$

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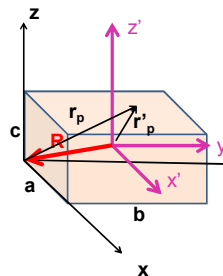
$$I'_{ij} = I_{ij} + M(R^2 \delta_{ij} - R_i R_j) + M(2\mathbf{r}_{CM} \cdot \mathbf{R} \delta_{ij} - r_{CMi} R_j - R_i r_{CMj})$$

Suppose that $\mathbf{R} = -\frac{a}{2}\hat{\mathbf{x}} - \frac{b}{2}\hat{\mathbf{y}} - \frac{c}{2}\hat{\mathbf{z}}$
 and $\mathbf{r}_{CM} = -\mathbf{R}$

$$I'_{ij} = I_{ij} - M(R^2 \delta_{ij} - R_i R_j)$$

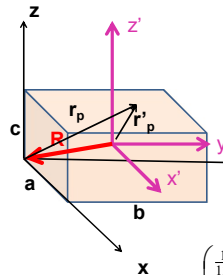
$$\tilde{\mathbf{I}}' = M \begin{pmatrix} \frac{1}{3}(b^2 + c^2) & -\frac{1}{4}ab & -\frac{1}{4}ac \\ -\frac{1}{4}ab & \frac{1}{3}(a^2 + c^2) & -\frac{1}{4}bc \\ -\frac{1}{4}ac & -\frac{1}{4}bc & \frac{1}{3}(a^2 + b^2) \end{pmatrix}$$

$$- M \begin{pmatrix} \frac{1}{4}(b^2 + c^2) & -\frac{1}{4}ab & -\frac{1}{4}ac \\ -\frac{1}{4}ab & \frac{1}{4}(a^2 + c^2) & -\frac{1}{4}bc \\ -\frac{1}{4}ac & -\frac{1}{4}bc & \frac{1}{4}(a^2 + b^2) \end{pmatrix}$$



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Note: This is a special case;
 changing the center of rotation
 does not necessarily result in a
 diagonal $\mathbf{I}'$



$$\tilde{\mathbf{I}}' = M \begin{pmatrix} \frac{1}{12}(b^2 + c^2) & 0 & 0 \\ 0 & \frac{1}{12}(a^2 + c^2) & 0 \\ 0 & 0 & \frac{1}{12}(a^2 + b^2) \end{pmatrix}$$

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Descriptions of rotation about a given origin

For general coordinate system

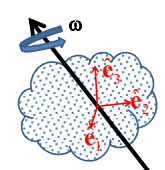
$$T = \frac{1}{2} \sum_{ij} I_{ij} \omega_i \omega_j$$

For (body fixed) coordinate system that diagonalizes
 moment of inertia tensor :

$$\tilde{\mathbf{I}} \cdot \hat{\mathbf{e}}_i = I_i \hat{\mathbf{e}}_i \quad i = 1, 2, 3$$

$$\boldsymbol{\omega} = \tilde{\omega}_1 \hat{\mathbf{e}}_1 + \tilde{\omega}_2 \hat{\mathbf{e}}_2 + \tilde{\omega}_3 \hat{\mathbf{e}}_3$$

$$\Rightarrow T = \frac{1}{2} \sum_i I_i \tilde{\omega}_i^2$$



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Descriptions of rotation about a given origin -- continued
Time rate of change of angular momentum

$$\frac{d\mathbf{L}}{dt} = \left(\frac{d\mathbf{L}}{dt} \right)_{\text{body}} + \boldsymbol{\omega} \times \mathbf{L}$$

For (body fixed) coordinate system that diagonalizes moment of inertia tensor :

$$\tilde{\mathbf{I}} \cdot \hat{\mathbf{e}}_i = I_i \hat{\mathbf{e}}_i \quad \boldsymbol{\omega} = \tilde{\omega}_1 \hat{\mathbf{e}}_1 + \tilde{\omega}_2 \hat{\mathbf{e}}_2 + \tilde{\omega}_3 \hat{\mathbf{e}}_3$$

$$\mathbf{L} = I_1 \tilde{\omega}_1 \hat{\mathbf{e}}_1 + I_2 \tilde{\omega}_2 \hat{\mathbf{e}}_2 + I_3 \tilde{\omega}_3 \hat{\mathbf{e}}_3$$

$$\begin{aligned} \frac{d\mathbf{L}}{dt} = & I_1 \dot{\tilde{\omega}}_1 \hat{\mathbf{e}}_1 + I_2 \dot{\tilde{\omega}}_2 \hat{\mathbf{e}}_2 + I_3 \dot{\tilde{\omega}}_3 \hat{\mathbf{e}}_3 + \tilde{\omega}_2 \tilde{\omega}_3 (I_3 - I_2) \hat{\mathbf{e}}_1 \\ & + \tilde{\omega}_3 \tilde{\omega}_1 (I_1 - I_3) \hat{\mathbf{e}}_2 + \tilde{\omega}_1 \tilde{\omega}_2 (I_2 - I_1) \hat{\mathbf{e}}_3 \end{aligned}$$

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Descriptions of rotation about a given origin -- continued

Note that the torque equation

$$\frac{d\mathbf{L}}{dt} = \left(\frac{d\mathbf{L}}{dt} \right)_{\text{body}} + \boldsymbol{\omega} \times \mathbf{L} = \boldsymbol{\tau}$$

is very difficult to solve directly in the body fixed frame.

For $\boldsymbol{\tau} = 0$ we can solve the Euler equations :

$$\begin{aligned} \frac{d\mathbf{L}}{dt} = & I_1 \dot{\tilde{\omega}}_1 \hat{\mathbf{e}}_1 + I_2 \dot{\tilde{\omega}}_2 \hat{\mathbf{e}}_2 + I_3 \dot{\tilde{\omega}}_3 \hat{\mathbf{e}}_3 + \tilde{\omega}_2 \tilde{\omega}_3 (I_3 - I_2) \hat{\mathbf{e}}_1 \\ & + \tilde{\omega}_3 \tilde{\omega}_1 (I_1 - I_3) \hat{\mathbf{e}}_2 + \tilde{\omega}_1 \tilde{\omega}_2 (I_2 - I_1) \hat{\mathbf{e}}_3 = 0 \end{aligned}$$

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Euler equations for rotation in body fixed frame :

$$I_1 \dot{\tilde{\omega}}_1 + \tilde{\omega}_2 \tilde{\omega}_3 (I_3 - I_2) = 0$$

$$I_2 \dot{\tilde{\omega}}_2 + \tilde{\omega}_3 \tilde{\omega}_1 (I_1 - I_3) = 0$$

$$I_3 \dot{\tilde{\omega}}_3 + \tilde{\omega}_1 \tilde{\omega}_2 (I_2 - I_1) = 0$$

→ Solution for symmetric top -- $I_2 = I_1$:

$$I_1 \dot{\tilde{\omega}}_1 + \tilde{\omega}_2 \tilde{\omega}_3 (I_3 - I_1) = 0$$

$$I_1 \dot{\tilde{\omega}}_2 + \tilde{\omega}_3 \tilde{\omega}_1 (I_1 - I_3) = 0$$

$$I_3 \dot{\tilde{\omega}}_3 = 0 \quad \Rightarrow \quad \tilde{\omega}_3 = (\text{constant})$$

$$\text{Define : } \Omega \equiv \tilde{\omega}_3 \frac{I_3 - I_1}{I_1} \quad \begin{aligned} \dot{\tilde{\omega}}_1 &= -\tilde{\omega}_2 \Omega \\ \dot{\tilde{\omega}}_2 &= \tilde{\omega}_1 \Omega \end{aligned}$$

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Solution of Euler equations for a symmetric top -- continued

$$\dot{\tilde{\omega}}_1 = -\tilde{\omega}_2 \Omega \quad \dot{\tilde{\omega}}_2 = \tilde{\omega}_1 \Omega$$

$$\text{where } \Omega \equiv \tilde{\omega}_3 \frac{I_3 - I_1}{I_1}$$

$$\text{Solution: } \tilde{\omega}_1(t) = A \cos(\Omega t + \phi)$$

$$\tilde{\omega}_2(t) = A \sin(\Omega t + \phi)$$

$$\tilde{\omega}_3(t) = \tilde{\omega}_3 \quad (\text{constant})$$

$$T = \frac{1}{2} \sum_i I_i \tilde{\omega}_i^2 = \frac{1}{2} I_1 A^2 + \frac{1}{2} I_3 \tilde{\omega}_3^2$$

$$\begin{aligned} \mathbf{L} &= I_1 \tilde{\omega}_1 \hat{\mathbf{e}}_1 + I_2 \tilde{\omega}_2 \hat{\mathbf{e}}_2 + I_3 \tilde{\omega}_3 \hat{\mathbf{e}}_3 \\ &= I_1 A (\cos(\Omega t + \phi) \hat{\mathbf{e}}_1 + \sin(\Omega t + \phi) \hat{\mathbf{e}}_2) + I_3 \tilde{\omega}_3 \hat{\mathbf{e}}_3 \end{aligned}$$

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Euler equations for rotation in body fixed frame :

$$I_1 \dot{\tilde{\omega}}_1 + \tilde{\omega}_2 \tilde{\omega}_3 (I_3 - I_2) = 0$$

$$I_2 \dot{\tilde{\omega}}_2 + \tilde{\omega}_3 \tilde{\omega}_1 (I_1 - I_3) = 0$$

$$I_3 \dot{\tilde{\omega}}_3 + \tilde{\omega}_1 \tilde{\omega}_2 (I_2 - I_1) = 0$$

→ Solution for asymmetric top -- $I_3 \neq I_2 \neq I_1$:

$$I_1 \dot{\tilde{\omega}}_1 + \tilde{\omega}_2 \tilde{\omega}_3 (I_3 - I_2) = 0$$

$$I_2 \dot{\tilde{\omega}}_2 + \tilde{\omega}_3 \tilde{\omega}_1 (I_1 - I_3) = 0$$

$$I_3 \dot{\tilde{\omega}}_3 + \tilde{\omega}_1 \tilde{\omega}_2 (I_2 - I_1) = 0$$

$$\text{Suppose : } \dot{\tilde{\omega}}_3 \approx 0 \quad \text{Define : } \Omega_1 \equiv \tilde{\omega}_3 \frac{I_3 - I_2}{I_1}$$

$$\text{Define : } \Omega_2 \equiv \tilde{\omega}_3 \frac{I_3 - I_1}{I_2}$$

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Euler equations for rotation in body fixed frame :

$$I_1 \dot{\tilde{\omega}}_1 + \tilde{\omega}_2 \tilde{\omega}_3 (I_3 - I_2) = 0$$

$$I_2 \dot{\tilde{\omega}}_2 + \tilde{\omega}_3 \tilde{\omega}_1 (I_1 - I_3) = 0$$

$$I_3 \dot{\tilde{\omega}}_3 + \tilde{\omega}_1 \tilde{\omega}_2 (I_2 - I_1) = 0$$

Solution for asymmetric top -- $I_3 \neq I_2 \neq I_1$:

Approximate solution --

$$\text{Suppose: } \dot{\tilde{\omega}}_3 \approx 0 \quad \text{Define: } \Omega_1 \equiv \tilde{\omega}_3 \frac{I_3 - I_2}{I_1}$$

$$\text{Define: } \Omega_2 \equiv \tilde{\omega}_3 \frac{I_3 - I_1}{I_2}$$

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Euler equations for asymmetric top -- continued

$$I_1 \dot{\tilde{\omega}}_1 + \tilde{\omega}_2 \tilde{\omega}_3 (I_3 - I_2) = 0$$

$$I_2 \dot{\tilde{\omega}}_2 + \tilde{\omega}_3 \tilde{\omega}_1 (I_1 - I_3) = 0$$

$$I_3 \dot{\tilde{\omega}}_3 + \tilde{\omega}_1 \tilde{\omega}_2 (I_2 - I_1) = 0$$

If $\dot{\tilde{\omega}}_3 \approx 0$, Define: $\Omega_1 \equiv \tilde{\omega}_3 \frac{I_3 - I_2}{I_1}$ $\Omega_2 \equiv \tilde{\omega}_3 \frac{I_3 - I_1}{I_2}$

$$\dot{\tilde{\omega}}_1 = -\Omega_1 \tilde{\omega}_2 \quad \dot{\tilde{\omega}}_2 = \Omega_2 \tilde{\omega}_1$$

If Ω_1 and Ω_2 are both positive or both negative :

$$\tilde{\omega}_1(t) \approx A \cos(\sqrt{\Omega_1 \Omega_2} t + \varphi)$$

$$\tilde{\omega}_2(t) \approx A \sqrt{\frac{\Omega_2}{\Omega_1}} \sin(\sqrt{\Omega_1 \Omega_2} t + \varphi)$$

$\Rightarrow$ If Ω_1 and Ω_2 have opposite signs, solution is unstable.

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