PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF online or (occasionally) in Olin 103

Discussion on Lecture 14 -- Chap. 6 (F & W)
Extensions of Hamiltonian formalism

1. **Virial theorem**
2. **Canonical transformations**
3. **Hamilton-Jacobi formalism**

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment	Due
1	Wed, 8/26/2020	Chap. 1	Introduction	#1	8/31/2020
2	Fri, 8/28/2020	Chap. 1	Scattering theory	#2	9/02/2020
3	Mon, 8/31/2020	Chap. 1	Scattering theory	#3	9/04/2020
4	Wed, 9/02/2020	Chap. 1	Scattering theory		
5	Fri, 9/04/2020	Chap. 1	Scattering theory	#4	9/09/2020
6	Mon, 9/07/2020	Chap. 2	Non-inertial coordinate systems		
7	Wed, 9/09/2020	Chap. 3	Calculus of Variation	#5	9/11/2020
8	Fri, 9/11/2020	Chap. 3	Calculus of Variation	#6	9/14/2020
9	Mon, 9/14/2020	Chap. 3 & 6	Lagrangian Mechanics	#7	9/18/2020
10	Wed, 9/16/2020	Chap. 3 & 6	Lagrangian & constraints	#8	9/21/2020
11	Fri, 9/18/2020	Chap. 3 & 6	Constants of the motion		
12	Mon, 9/21/2020	Chap. 3 & 6	Hamiltonian equations of motion	#9	9/23/2020
13	Wed, 9/23/2020	Chap. 3 & 6	Liouville theorm	#10	9/25/2020
14	Fri, 9/25/2020	Chap. 3 & 6	Canonical transformations		
15	Mon, 9/28/2020	Chap. 4	Small oscillations about equilibrium		

Schedule for weekly one-on-one meetings

Nick – 11 AM Monday (ED/ST)
Tim – 9 AM Tuesday
Bamidele – 7 PM Tuesday
Zhi– 9 PM Tuesday -- possibly shift time?
Jeanette – 11 AM Friday
Derek – 12 PM Friday

Your questions –

From Nick –

1. Why is this true in the proof of the Virial theorem? Are we just saying if the average of the derivative is 0, then we get the Virial theorem?

$$\left\langle \frac{dA}{dt} \right\rangle = \frac{1}{\tau}\int_0^\tau \frac{dA(t)}{dt}dt = \frac{A(\tau)-A(0)}{\tau} \Rightarrow 0$$

From Gao –

1. About today's lecture, What's F?

$$\boxed{\sum_\sigma p_\sigma \dot{q}_\sigma - H(\{q_\sigma\},\{p_\sigma\},t) = \sum_\sigma P_\sigma \dot{Q}_\sigma - \tilde{H}(\{Q_\sigma\},\{P_\sigma\},t) + \frac{d}{dt}F(\{q_\sigma\},\{Q_\sigma\},t)}$$

?

Virial theorem (Rudolf Clausius ~ 1870)

$$2\langle T\rangle = -\left\langle \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r}_\sigma \right\rangle$$

Proof:

Define: $A \equiv \sum_\sigma \mathbf{p}_\sigma \cdot \mathbf{r}_\sigma$

$$\frac{dA}{dt} = \sum_\sigma \left(\dot{\mathbf{p}}_\sigma \cdot \mathbf{r}_\sigma + \mathbf{p}_\sigma \cdot \dot{\mathbf{r}}_\sigma \right) = \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r}_\sigma + 2T$$

Because $\dot{\mathbf{p}}_\sigma = \mathbf{F}_\sigma$

$$\left\langle \frac{dA}{dt} \right\rangle = \left\langle \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r} \right\rangle + 2\langle T\rangle$$

$$\left\langle \frac{dA}{dt} \right\rangle = \frac{1}{\tau}\int_0^\tau \frac{dA(t)}{dt}dt = \frac{A(\tau)-A(0)}{\tau} \Rightarrow 0$$

Note that this implies that the motion is periodic or bounded (not for all systems).

When it is true -- $\Rightarrow \left\langle \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r} \right\rangle + 2\langle T\rangle = 0$

Examples of the Virial Theorem

$$2\langle T\rangle = -\left\langle \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r}_\sigma \right\rangle$$

Harmonic oscillator:

$$\mathbf{F} = -kx\hat{\mathbf{x}} \qquad T = \frac{1}{2}m\dot{x}^2 \qquad \langle m\dot{x}^2\rangle = \langle kx^2\rangle$$

Check: for $x(t) = X\sin\left(\sqrt{\frac{k}{m}}t + \alpha\right)$

$$\langle 2T\rangle = \langle m\dot{x}^2\rangle = kX^2\left\langle \cos^2\left(\sqrt{\frac{k}{m}}t+\alpha\right)\right\rangle = \frac{1}{2}kX^2$$

$$-\left\langle \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r}_\sigma \right\rangle = \langle kx^2\rangle = kX^2\left\langle \sin^2\left(\sqrt{\frac{k}{m}}t+\alpha\right)\right\rangle = \frac{1}{2}kX^2$$

Premise true because of periodicity.

Examples of the Virial Theorem

$$2\langle T\rangle = -\left\langle \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r}_\sigma \right\rangle$$

Circular orbit due to gravitational field of massive object:

$$\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}} \qquad T = \frac{1}{2}mv^2 \qquad \left\langle mv^2\right\rangle = \left\langle \frac{GMm}{r}\right\rangle$$

Check: for $\dfrac{v^2}{r} = \dfrac{GM}{r^2}$ $\quad \Rightarrow \left\langle mv^2\right\rangle = \left\langle \dfrac{GMm}{r}\right\rangle$

centripetal acceleration

gravitational force

Premise true because of periodicity.

Hamiltonian formalism and the canonical equations of motion:

$$H = H\left(\{q_\sigma(t)\},\{p_\sigma(t)\},t\right)$$

Canonical equations of motion

$$\frac{dq_\sigma}{dt} = \frac{\partial H}{\partial p_\sigma}$$

$$\frac{dp_\sigma}{dt} = -\frac{\partial H}{\partial q_\sigma}$$

In the next slides we will consider finding different coordinates and momenta that can also describe the system. Why?

a. Because we can
b. Because it might be useful

Notion of "Canonical" generalized coordinate transformations

$$q_\sigma = q_\sigma\left(\{Q_1 \cdots Q_n\},\{P_1 \cdots P_n\},t\right) \qquad \text{for each } \sigma$$

$$p_\sigma = p_\sigma\left(\{Q_1 \cdots Q_n\},\{P_1 \cdots P_n\},t\right) \qquad \text{for each } \sigma$$

Note that because of the way we set up the problem we can always add such a term.

For some $\tilde{H}$ and F, using Legendre transformations

$$\sum_\sigma p_\sigma \dot{q}_\sigma - H\left(\{q_\sigma\},\{p_\sigma\},t\right) = \sum_\sigma P_\sigma \dot{Q}_\sigma - \tilde{H}\left(\{Q_\sigma\},\{P_\sigma\},t\right) + \frac{d}{dt}F\left(\{q_\sigma\},\{Q_\sigma\},t\right)$$

Apply Hamilton's principle:

$$\delta \int_{t_i}^{t_f} \left[\sum_\sigma P_\sigma \dot{Q}_\sigma - \tilde{H}\left(\{Q_\sigma\},\{P_\sigma\},t\right) + \frac{d}{dt}F\left(\{q_\sigma\},\{Q_\sigma\},t\right)\right] dt = 0$$

$$\delta \int_{t_i}^{t_f} \left[\frac{d}{dt}F\left(\{q_\sigma\},\{Q_\sigma\},t\right)\right] dt = \int_{t_i}^{t_f} \left[\frac{d}{dt}\delta F\left(\{q_\sigma\},\{Q_\sigma\},t\right)\right] dt$$

$$= \delta F\left(t_f\right) - \delta F\left(t_i\right) = 0 \qquad \text{and} \qquad \dot{Q}_\sigma = \frac{\partial \tilde{H}}{\partial P_\sigma} \qquad \dot{P}_\sigma = -\frac{\partial \tilde{H}}{\partial Q_\sigma}$$

Your question -- Why do we add the function F into the equation

Comment – We have show that adding the total derivative of F to the equation adds 0 to the result which means that the solutions are ambiguous relative to this term. We are motivated to explore the possibilities generated by this term which are guaranteed to fulfill the “canonical” equation construction. Some of the results will be beautiful and some will be ugly.

Some details --

$$q_\sigma = q_\sigma\left(\{Q_1 \cdots Q_n\},\{P_1 \cdots P_n\},t\right) \qquad \text{for each } \sigma$$

$$p_\sigma = p_\sigma\left(\{Q_1 \cdots Q_n\},\{P_1 \cdots P_n\},t\right) \qquad \text{for each } \sigma$$

For some $\tilde{H}$ and F, using Legendre transformations

$$\sum_\sigma p_\sigma \dot{q}_\sigma - H\left(\{q_\sigma\},\{p_\sigma\},t\right) = \sum_\sigma P_\sigma \dot{Q}_\sigma - \tilde{H}\left(\{Q_\sigma\},\{P_\sigma\},t\right) + \frac{d}{dt}F\left(\{q_\sigma\},\{Q_\sigma\},t\right)$$

Action integral:

$$S = \int_{t_i}^{t_f} dt \left(\sum_\sigma p_\sigma \dot{q}_\sigma - H\left(\{q_\sigma\},\{p_\sigma\},t\right) \right)$$

$$\delta S = \int_{t_i}^{t_f} dt \left(\sum_\sigma \left(\delta p_\sigma \dot{q}_\sigma + p_\sigma \delta \dot{q}_\sigma\right) - \delta H\left(\{q_\sigma\},\{p_\sigma\},t\right) \right)$$

Note that $\delta \int_{t_i}^{t_f} dt \left(\frac{dF(t)}{dt} \right) = \int_{t_i}^{t_f} dt \left(\frac{d\delta F(t)}{dt} \right) = 0$

Some relations between old and new variables:

$$\sum_\sigma p_\sigma \dot{q}_\sigma - H(\{q_\sigma\},\{p_\sigma\},t) =$$

$$\sum_\sigma P_\sigma \dot{Q}_\sigma - \widetilde{H}(\{Q_\sigma\},\{P_\sigma\},t) + \frac{d}{dt}F(\{q_\sigma\},\{Q_\sigma\},t)$$

$$\frac{d}{dt}F(\{q_\sigma\},\{Q_\sigma\},t) = \sum_\sigma \left(\left(\frac{\partial F}{\partial q_\sigma}\right)\dot{q}_\sigma + \left(\frac{\partial F}{\partial Q_\sigma}\right)\dot{Q}_\sigma \right) + \frac{\partial F}{\partial t}$$

$$\Rightarrow \sum_\sigma \left(p_\sigma - \left(\frac{\partial F}{\partial q_\sigma}\right)\right)\dot{q}_\sigma - H\left(\{q_\sigma\},\{p_\sigma\},t\right) =$$

$$\sum_\sigma \left(P_\sigma + \left(\frac{\partial F}{\partial Q_\sigma}\right)\right)\dot{Q}_\sigma - \tilde{H}\left(\{Q_\sigma\},\{P_\sigma\},t\right) + \frac{\partial F}{\partial t}$$

$$\sum_\sigma \left(p_\sigma - \left(\frac{\partial F}{\partial q_\sigma} \right) \right) \dot{q}_\sigma - H(\{q_\sigma\}, \{p_\sigma\}, t) =$$

$$\sum_\sigma \left(P_\sigma + \left(\frac{\partial F}{\partial Q_\sigma} \right) \right) \dot{Q}_\sigma - \widetilde{H}(\{Q_\sigma\}, \{P_\sigma\}, t) + \frac{\partial F}{\partial t}$$

$$\Rightarrow p_\sigma = \left(\frac{\partial F}{\partial q_\sigma} \right) \qquad P_\sigma = -\left(\frac{\partial F}{\partial Q_\sigma} \right)$$

$$\Rightarrow \widetilde{H}(\{Q_\sigma\}, \{P_\sigma\}, t) = H(\{q_\sigma\}, \{p_\sigma\}, t) + \frac{\partial F}{\partial t}$$

Note that it is conceivable that if we were extraordinarily clever, we could find all of the constants of the motion!

$$\dot{Q}_\sigma = \frac{\partial \widetilde{H}}{\partial P_\sigma} \qquad \dot{P}_\sigma = -\frac{\partial \widetilde{H}}{\partial Q_\sigma}$$

$$\text{Suppose:} \quad \dot{Q}_\sigma = \frac{\partial \widetilde{H}}{\partial P_\sigma} = 0 \quad \text{and} \quad \dot{P}_\sigma = -\frac{\partial \widetilde{H}}{\partial Q_\sigma} = 0$$

$$\Rightarrow Q_\sigma, P_\sigma \quad \text{are constants of the motion}$$

Possible solution – Hamilton-Jacobi theory:

$$\text{Suppose:} \quad F(\{q_\sigma\},\{Q_\sigma\},t) \Rightarrow -\sum_\sigma P_\sigma Q_\sigma + S(\{q_\sigma\},\{P_\sigma\},t)$$

$$\sum_\sigma p_\sigma \dot{q}_\sigma - H(\{q_\sigma\},\{p_\sigma\},t) =$$

$$\sum_\sigma P_\sigma \dot{Q}_\sigma - \widetilde{H}(\{Q_\sigma\},\{P_\sigma\},t) + \frac{d}{dt}\left(-\sum_\sigma P_\sigma Q_\sigma + S(\{q_\sigma\},\{P_\sigma\},t)\right)$$

$$= -\widetilde{H}(\{Q_\sigma\},\{P_\sigma\},t) - \sum_\sigma \dot{P}_\sigma Q_\sigma + \sum_\sigma \left(\frac{\partial S}{\partial q_\sigma}\dot{q}_\sigma + \frac{\partial S}{\partial P_\sigma}\dot{P}_\sigma\right) + \frac{\partial S}{\partial t}$$

Solution :

$$p_\sigma = \frac{\partial S}{\partial q_\sigma} \qquad\qquad Q_\sigma = \frac{\partial S}{\partial P_\sigma}$$

$$\widetilde{H}(\{Q_\sigma\},\{P_\sigma\},t) = H(\{q_\sigma\},\{p_\sigma\},t) + \frac{\partial S}{\partial t}$$

When the dust clears:

$$\text{Assume } \{Q_\sigma\}, \{P_\sigma\}, \tilde{H} \quad \text{are constants; choose } \tilde{H} = 0$$

$$\text{Need to find } S(\{q_\sigma\}, \{P_\sigma\}, t)$$

$$p_\sigma = \frac{\partial S}{\partial q_\sigma} \qquad\qquad Q_\sigma = \frac{\partial S}{\partial P_\sigma}$$

$$\Rightarrow H\left(\{q_\sigma\}, \left\{\frac{\partial S}{\partial q_\sigma}\right\}, t\right) + \frac{\partial S}{\partial t} = 0$$

Note: S is the "action":

$$\sum_\sigma p_\sigma \dot{q}_\sigma - H(\{q_\sigma\}, \{p_\sigma\}, t) =$$

$$\cancelto{0}{\sum_\sigma P_\sigma \dot{Q}_\sigma} - \cancelto{0}{\tilde{H}(\{Q_\sigma\}, \{P_\sigma\}, t)} + \frac{d}{dt}\left(-\cancelto{0}{\sum_\sigma P_\sigma Q_\sigma} + S(\{q_\sigma\}, \{P_\sigma\}, t) \right)$$

$$\sum_\sigma p_\sigma \dot{q}_\sigma - H(\{q_\sigma\},\{p_\sigma\},t) =$$

$$\cancelto{0}{\sum_\sigma P_\sigma \dot{Q}_\sigma} - \cancelto{0}{\widetilde{H}(\{Q_\sigma\},\{P_\sigma\},t)} + \frac{d}{dt}\left(-\cancelto{0}{\sum_\sigma P_\sigma Q_\sigma} + S(\{q_\sigma\},\{P_\sigma\},t) \right)$$

$$\int_{t_i}^{t_f} \left(\sum_\sigma p_\sigma \dot{q}_\sigma - H(\{q_\sigma\},\{p_\sigma\},t) \right) dt = \int_{t_i}^{t_f} \left(\frac{d}{dt}\left(S(\{q_\sigma\},\{P_\sigma\},t)\right) \right) dt$$

$$= S(\{q_\sigma\},\{P_\sigma\},t)\Big|_{t_i}^{t_f}$$

Differential equation for ***S***:

$$H\left(\{q_\sigma\},\left\{\frac{\partial S}{\partial q_\sigma}\right\},t\right)+\frac{\partial S}{\partial t}=0$$

$$\text{Example:}\qquad H(\{q\},\{p\},t)=\frac{p^2}{2m}+\frac{1}{2}m\omega^2q^2$$

$$\text{Hamilton-Jacobi Eq:}\quad H\left(\{q\},\left\{\frac{\partial S}{\partial q}\right\},t\right)+\frac{\partial S}{\partial t}=0$$

$$\frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2+\frac{1}{2}m\omega^2q^2+\frac{\partial S}{\partial t}=0$$

Does this look familiar?

$$\text{Assume:}\qquad S(q,t)\equiv W(q)-Et \qquad\qquad (E\ \text{ constant})$$

Continued:

$$\frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2+\frac{1}{2}m\omega^2q^2+\frac{\partial S}{\partial t}=0$$

$$\text{Assume:}\qquad S(q,t)\equiv W(q)-Et \qquad\qquad (E\quad\text{constant})$$

$$\frac{1}{2m}\left(\frac{dW}{dq}\right)^2+\frac{1}{2}m\omega^2q^2=E$$

$$\frac{dW}{dq}=\sqrt{2mE-(m\omega)^2q^2}$$

$$W(q)=\int\sqrt{2mE-(m\omega)^2q^2}\,dq$$

Continued:

$$W(q) = \int \sqrt{2mE - (m\omega)^2 q^2}\, dq$$

$$= \frac{1}{2} q\sqrt{2mE - (m\omega)^2 q^2} + \frac{E}{\omega}\sin^{-1}\left(\frac{m\omega q}{\sqrt{2mE}}\right) + C$$

$$S(q,E,t) = \frac{1}{2} q\sqrt{2mE - (m\omega)^2 q^2} + \frac{E}{\omega}\sin^{-1}\left(\frac{m\omega q}{\sqrt{2mE}}\right) - Et$$

$$\frac{\partial S}{\partial E} = Q = \frac{1}{\omega}\sin^{-1}\left(\frac{m\omega q}{\sqrt{2mE}}\right) - t$$

$$\Rightarrow q(t) = \frac{\sqrt{2mE}}{m\omega}\sin(\omega(t+Q))$$

Another example of Hamilton Jacobi equations

Example: $$H\left(\{y\},\{p\},t\right)=\frac{p^2}{2m}+mgy$$

Assume $y(0)=h;\qquad p(0)=0$

Hamilton-Jacobi Eq: $$H\left(\{y\},\left\{\frac{\partial S}{\partial y}\right\},t\right)+\frac{\partial S}{\partial t}=0$$

$$\frac{1}{2m}\left(\frac{\partial S}{\partial y}\right)^2+mgy+\frac{\partial S}{\partial t}=0$$

Assume: $S(y,t)\equiv W(y)-Et \qquad (E \quad \text{constant})$

Example: $$H\left(\{y\},\{p\},t\right)=\frac{p^2}{2m}+mgy$$

Assume $y(0)=h;\qquad p(0)=0$

$$\frac{1}{2m}\left(\frac{dW}{dy}\right)^2+mgy=E\equiv mgh$$

$$W(y)=m\int_y^h\sqrt{2g\left(h-y'\right)}dy'=\frac{2}{3}m\sqrt{2g}\left(h-y\right)^{3/2}$$

$$S(y,t)=W(y)-Et=\frac{2}{3}m\sqrt{2g}\left(h-y\right)^{3/2}-mght$$

Check action:

For this case: $y(t) = h - \frac{1}{2}gt^2$

$$S = \int_0^t \left(\frac{1}{2}m\dot{y}^2 - mgy \right) dt' = \frac{1}{3}mg^2t^3 - mght$$

$$S(y,t) = W(y) - Et = \frac{2}{3}m\sqrt{2g}\left(h - y\right)^{3/2} - mght$$

Agrees with Hamilton-Jacobi analysis.

Alternatively, keeping E notation:

$$W(y) = \int_y^h \sqrt{2mE - 2m^2 gy'}\,dy'$$

$$= \sqrt{\frac{2}{m}}\frac{1}{g}\left(E - mgy\right)^{3/2}$$

$$S(y,t) = W(y) - Et = \sqrt{\frac{2}{m}}\frac{2}{3g}\left(E - mgy\right)^{3/2} - Et$$

$$\frac{\partial S}{\partial E} = Q = \sqrt{\frac{2}{m}}\frac{1}{g}\left(E - mgy\right)^{1/2} - t$$

$$\Rightarrow y(t) = \frac{E}{mg} - \frac{1}{2}g\left(t + Q\right)^2$$

In our case, $Q = 0$

$E = mgh$

What do you think of Hamilton-Jacobi method

a. Historically important
b. Hysterical
c. Painful
d. Might be useful

The next 3 slides contain important equations that you will hopefully remember for this material contained in Chapters 3 & 6 of Fetter and Walecka. On Monday we will start with Chapter 4 and discuss one of the many applications of these ideas – the case of small oscillations near equilibrium.

Recap --

Lagrangian picture

For independent generalized coordinates $q_\sigma(t)$:

$$L = L\left(\{q_\sigma(t)\}, \{\dot{q}_\sigma(t)\}, t\right)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

$\Rightarrow$ Second order differential equations for $q_\sigma(t)$

Hamiltonian picture

$$H = H\left(\{q_\sigma(t)\}, \{p_\sigma(t)\}, t\right)$$

$$\frac{dq_\sigma}{dt} = \frac{\partial H}{\partial p_\sigma} \qquad \frac{dp_\sigma}{dt} = -\frac{\partial H}{\partial q_\sigma}$$

$\Rightarrow$ Coupled first order differential equations for

$q_\sigma(t)$ and $p_\sigma(t)$

General treatment of particle of mass m and charge q moving in 3 dimensions in an potential $U(\mathbf{r})$ as well as electromagnetic scalar and vector potentials $\Phi(\mathbf{r},t)$ and $\mathbf{A}(\mathbf{r},t)$:

Lagrangian: $$L(\mathbf{r},\dot{\mathbf{r}},t)=\frac{1}{2}m\dot{\mathbf{r}}^2-U(\mathbf{r})-q\Phi(\mathbf{r},t)+\frac{q}{c}\dot{\mathbf{r}}\cdot\mathbf{A}(\mathbf{r},t)$$

Hamiltonian: $$\mathbf{p}=\frac{\partial L}{\partial \dot{\mathbf{r}}}=m\dot{\mathbf{r}}+\frac{q}{c}\mathbf{A}(\mathbf{r},t)$$

$$H(\mathbf{r},\mathbf{p},t)=\mathbf{p}\cdot\dot{\mathbf{r}}-L(\mathbf{r},\dot{\mathbf{r}},t)$$

$$=\frac{1}{2m}\left(\mathbf{p}-\frac{q}{c}\mathbf{A}(\mathbf{r},t)\right)^2+U(\mathbf{r})+q\Phi(\mathbf{r},t)$$

Recipe for constructing the Hamiltonian and analyzing the equations of motion

1. Construct Lagrangian function : $L = L\left(\{q_\sigma(t)\}, \{\dot{q}_\sigma(t)\}, t\right)$

2. Compute generalized momenta : $p_\sigma \equiv \dfrac{\partial L}{\partial \dot{q}_\sigma}$

3. Construct Hamiltonian expression : $H = \sum_\sigma \dot{q}_\sigma p_\sigma - L$

4. Form Hamiltonian function : $H = H\left(\{q_\sigma(t)\}, \{p_\sigma(t)\}, t\right)$

5. Analyze canonical equations of motion :

$$\frac{dq_\sigma}{dt} = \frac{\partial H}{\partial p_\sigma} \qquad \frac{dp_\sigma}{dt} = -\frac{\partial H}{\partial q_\sigma}$$