PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Online or (occasionally) in Olin 103

Plan for Lecture 2: -- Chap. 1 of F&W

1. Brief comment on assessment exercise
2. Review of the physics of two particle interactions
3. Two particles in the scattering geometry
4. Introduction to scattering theory
5. Example of scattering analysis
6. Laboratory and center of mass reference fame

In this lecture, we will review the basic physics of two interacting particles. When the particles are not in a bound orbit, their motion may be assessed in terms of scattering. Scattering is a very powerful analysis tool that often allows for the comparison of theory and experiment.

PHY 711 Classical Mechanics and Mathematical Methods

MWF 10 AM-10:50 AM	**OPL 103**	**http://www.wfu.edu/~natalie/f20phy711/**

Instructor: Natalie Holzwarth	**Office:**300 OPL	**e-mail:**natalie@wfu.edu

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment	Due
1	Wed, 8/26/2020	Chap. 1	Introduction	#1	8/31/2020
2	Fri, 8/28/2020	Chap. 1	Scattering theory	#2	9/02/2020
3	Mon, 8/31/2020	Chap. 1	Scattering theory		
4	Wed, 9/02/2020	Chap. 1	Scattering theory		
5	Fri, 9/04/2020	Chap. 2	Non-inertial coordinate systems		
6	Mon, 9/07/2020	Chap. 3	Calculus of Variation		

This is an image of the webpage with the schedule and the homework assignment.

PHY 711 – Assignment #2

08/28/2020

1. Consider a particle of mass m moving in the vicinity of another particle of mass M where $m << M$. The particles interact with a conservative central potential of the form

$$V(r) = V_0 \left(\left(\frac{r_0}{r} \right)^2 - \left(\frac{r_0}{r} \right) \right),$$

   where r denotes the magnitude of the particle separation and V_0 and r_0 denote energy and length constants, respectively. The total energy of the system is V_0.

   (a) First consider the case where the impact parameter $b = 0$. Find the distance of closest approach of the particles.

   (b) Now consider the case where the impact parameter $b = r_0$. Find the distance of closest approach of the particles.

This assignment reviews some of the basic concepts of conservative potentials, conservation of energy, and conservation of momentum. The potential form is not physically motivated, but the solutions can be determined by simple analysis, and the qualitative results are illustrative of the particle interactions.

Comment on quiz questions

1. $$g(t) = \int_0^t \left(x^2 + t\right) dx \qquad \frac{dg}{dt} = \int_0^t \frac{d\left(x^2+t\right)}{dt} dt + \left(x^2+t\right)\Big|_{x=t}$$

$$= \int_0^t dt + \left(t^2 + t\right) = t^2 + 2t$$

2. **Evaluate the integral $\oint \frac{dz}{z}$ for a closed contour about the origin.**

Suppose that $z = e^{i\theta}$ $\qquad dz = e^{i\theta} i d\theta \qquad \oint \frac{dz}{z} = \int_0^{2\pi} \frac{e^{i\theta} i d\theta}{e^{i\theta}} = 2\pi i$

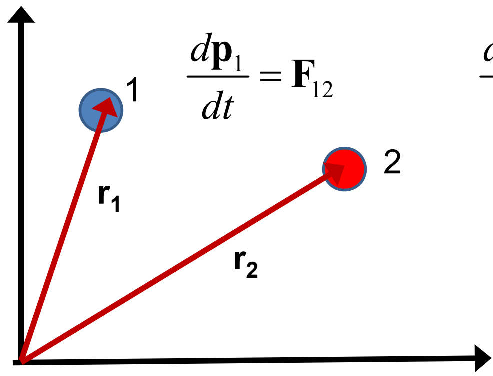
3. $$\frac{df}{dx} = f \qquad \Rightarrow f(x) = Ae^x \qquad f(x=0) = A = 1 \quad \Rightarrow A = 1$$

4. $$\sum_{n=1}^{N} a^n = \frac{a - a^{N+1}}{1-a}$$ Let $S \equiv \sum_{n=1}^{N} a^n$ Note that $aS - S = a^{N+1} - a$

These are solutions to the “assessment” exercise. It will help us figure out which mathematical methods should be usefully reviewed in this class. Most of you had particular trouble with the geometric series. It may be one of those things that you will want to remember to look up when you need it, rather than an important concept.

First consider fundamental picture of particle interactions

Classical mechanics of a conservative 2-particle system.



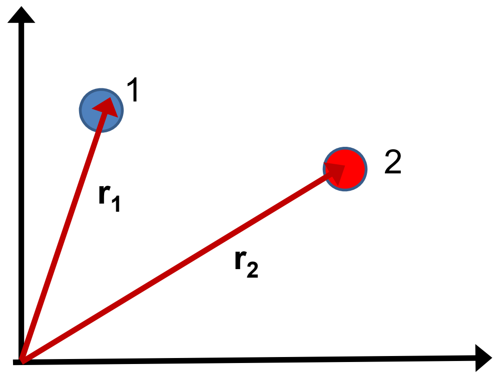
$$\frac{d\mathbf{p}_1}{dt} = \mathbf{F}_{12} \qquad \frac{d\mathbf{p}_2}{dt} = \mathbf{F}_{21}$$

$$\mathbf{F}_{12} = -\nabla_1 V\left(\mathbf{r}_1 - \mathbf{r}_2\right) \quad \Rightarrow E = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + V\left(\mathbf{r}_1 - \mathbf{r}_2\right)$$

In general, we will assume that $V(\mathbf{r})=V(r)$ (a central potential).

First consider the basic particle interactions which govern their motion.

Energy is conserved: $E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + V(\mathbf{r}_1 - \mathbf{r}_2)$



For a central potential $V(\mathbf{r})=V(r)$, angular momentum is conserved. For the moment we also make the simplifying assumption that $m_2 >> m_1$ so that particle 1 dominates the motion.

The last assumption is being made for convenience. We will consider the center of mass and relative motion much more carefully later.

Typical two-particle interactions –

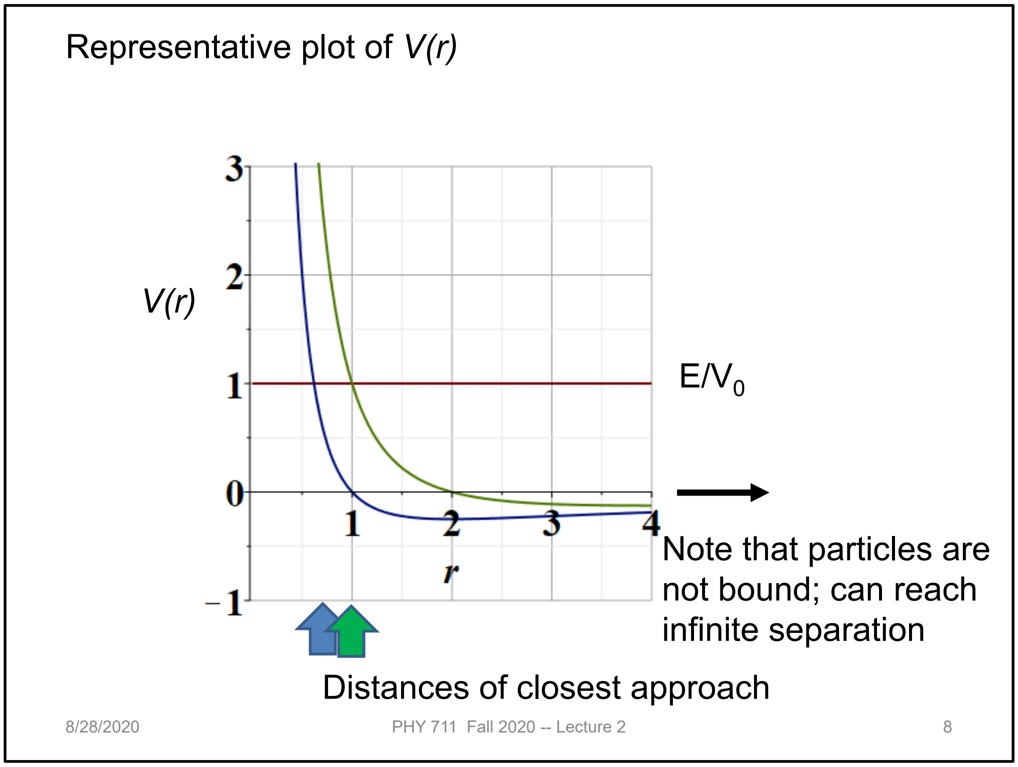
Central potential: $V(\mathbf{r}_1 - \mathbf{r}_2) = V(|\mathbf{r}_1 - \mathbf{r}_2|) \equiv V(r)$

Hard sphere: $V(r) = \begin{cases} \infty & r \leq a \\ 0 & r > a \end{cases}$

Coulomb or gravitational: $V(r) = \frac{K}{r}$

Lennard-Jones: $V(r) = \frac{A}{r^{12}} - \frac{B}{r^6}$

These represent typical particle interactions. Only the Coulomb or gravitational forms are precisely found in nature. The others serve as convenient models.



Potential energy diagram showing important aspects of the particle trajectory.

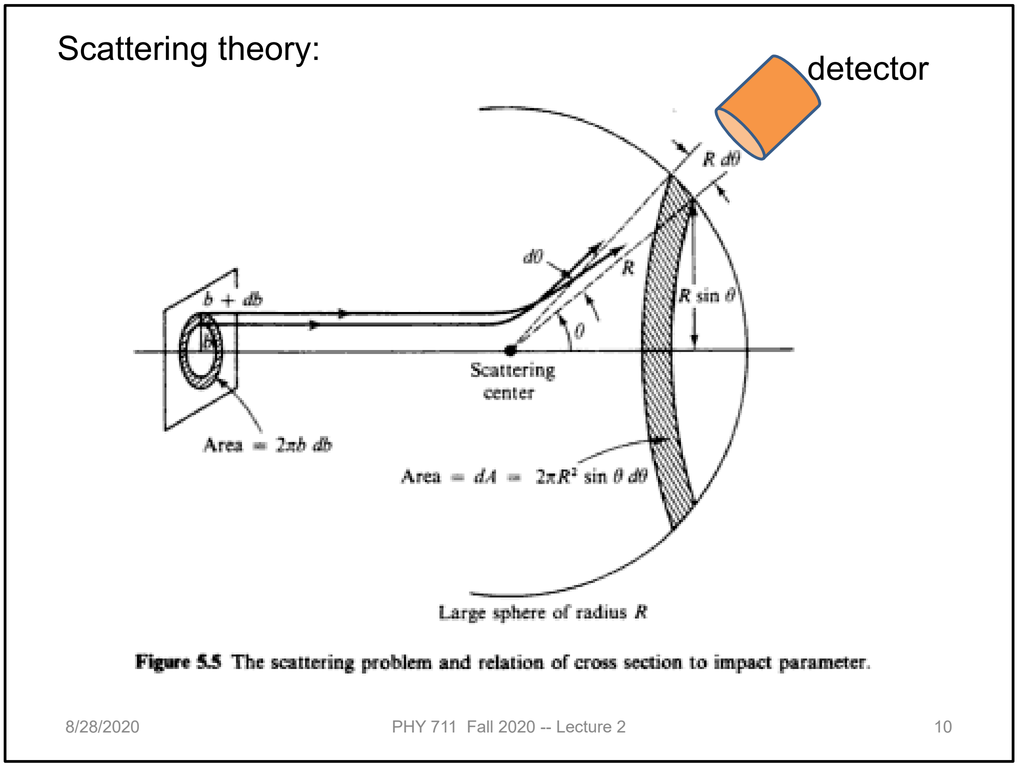
Which of the following are true:

a. The particle moves in a plane.
b. For any interparticle potential the trajectory can be determined/calculated.
c. Only for a few special interparticle potential forms can the trajectory be determined.

Why should we care about this?

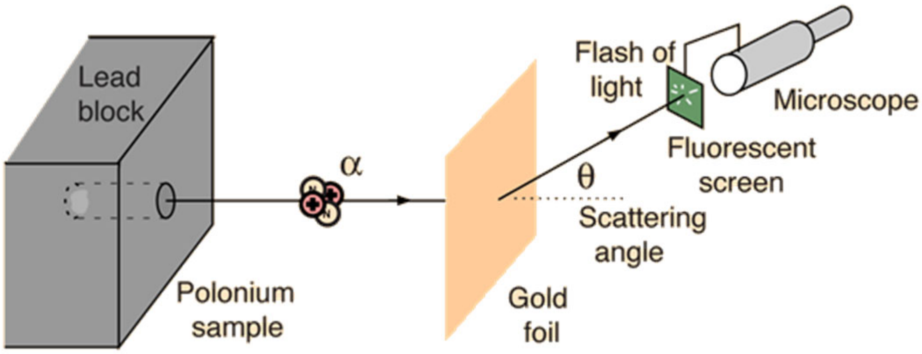
a. We shouldn’t really care.
b. It is only of academic interest
c. It is of academic interest but can be measured.
d. Many experiments can be analyzed in terms of the particle trajectory.

What do you think about these?



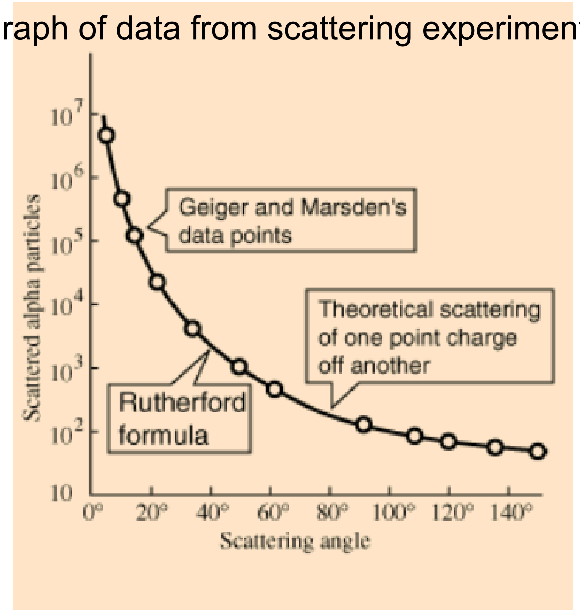
This is the ideal configuration of a scattering experiment.

Example: Diagram of Rutherford scattering experiment
http://hyperphysics.phy-astr.gsu.edu/hbase/rutsca.html



This illustrates the setup of the famous Rutherford scattering experiment.

Graph of data from scattering experiment



From website: http://hyperphysics.phy-astr.gsu.edu/hbase/Nuclear/rutsca2.html

8/28/2020 PHY 711 Fall 2020 -- Lecture 2 12

Reconstruction experimental data from the Rutherford experiment.

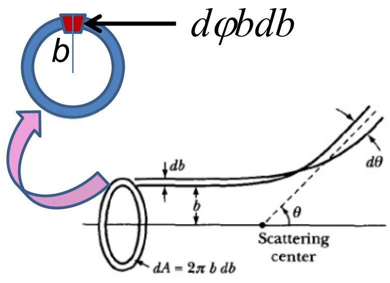
Standardization of scattering experiments --

Differential cross section

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\text{Number of detected particles at } \theta \text{ per target particle}}{\text{Number of incident particles per unit area}}$$

$$= \text{Area of incident beam that is scattered into detector at angle } \theta$$

Impact parameter: *b*

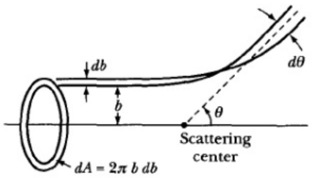


$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{d\varphi\, b\, db}{d\varphi \sin\theta\, d\theta} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Figure from Marion & Thorton, Classical Dynamics

Some details of scattering geometry and analysis.

Note: The notion of cross section is common to many areas of physics including classical mechanics, quantum mechanics, optics, etc. Only in the **classical mechanics** can we calculate it from a knowledge of the particle trajectory as it relates to the scattering geometry.



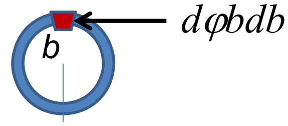


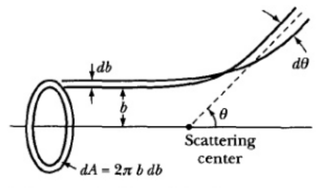
Figure from Marion & Thorton, Classical Dynamics

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{d\varphi\, b\, db}{d\varphi \sin\theta\, d\theta} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Note: We are assuming that the process is isotropic in φ

More details.

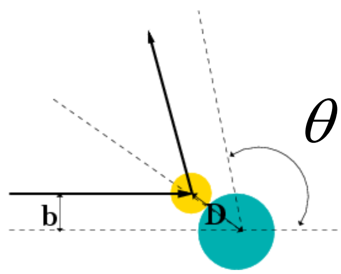
Simple example – collision of hard spheres
having mutual radius D; very large target mass



$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Microscopic view:

$$b(\theta) = ?$$

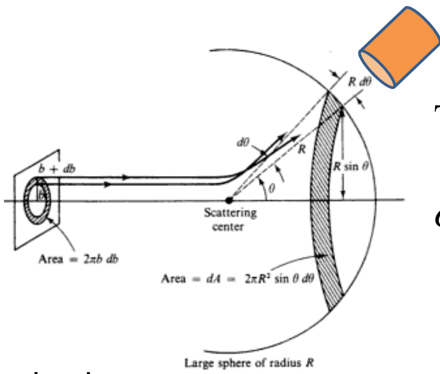


$$b(\theta) = D\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{D^2}{4}$$

Illustration of the analysis for the scattering of a hard sphere on a massive hard sphere.

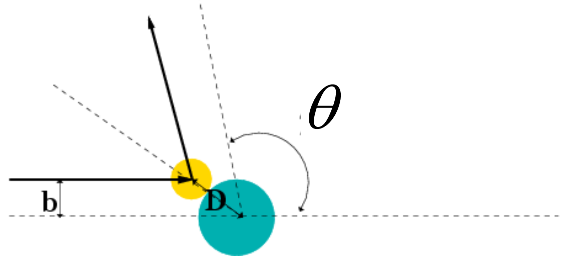
Simple example – collision of hard spheres -- continued



Total scattering cross section:

$$\sigma = \int \left(\frac{d\sigma}{d\Omega} \right) d\Omega$$

Hard sphere:



$$\left(\frac{d\sigma}{d\Omega} \right) = \frac{D^2}{4}$$

$$\sigma = \pi D^2$$

Total cross section versus partical cross section.

More details of hard sphere scattering –

Hidden in the analysis are assumptions about the scattering process such as:

- No external forces ➔ linear momentum is conserved
- No dissipative phenomena ➔ energy is conserved
- No torque on the system ➔ angular momentum is conserved
- Target particle is much more massive than scattering particle
- Other assumptions??

Note that for quantum mechanical hard spheres at low energy the total cross section is 4 times as large.

Some questions and comments. The discussion of scattering theory will be continued next time.