PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF in Olin 103

Notes on Lecture 14 -- Chap. 6 (F & W)
Extensions of Hamiltonian formalism

1. Virial theorem
2. Canonical transformations
3. Hamilton-Jacobi formalism

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment	Due
1	Mon, 8/23/2021	Chap. 1	Introduction	#1	8/27/2021
2	Wed, 8/25/2021	Chap. 1	Scattering theory	#2	8/30/2021
3	Fri, 8/27/2021	Chap. 1	Scattering theory		
4	Mon, 8/30/2021	Chap. 1	Scattering theory	#3	9/01/2021
5	Wed, 9/01/2021	Chap. 1	Summary of scattering theory	#4	9/03/2021
6	Fri, 9/03/2021	Chap. 2	Non-inertial coordinate systems	#5	9/06/2021
7	Mon, 9/06/2021	Chap. 3	Calculus of Variation	#6	9/10/2021
8	Wed, 9/08/2021	Chap. 3	Calculus of Variation		
9	Fri, 9/10/2021	Chap. 3 & 6	Lagrangian Mechanics	#7	9/13/2021
10	Mon, 9/13/2021	Chap. 3 & 6	Lagrangian Mechanics	#8	9/17/2021
11	Wed, 9/15/2021	Chap. 3 & 6	Constants of the motion		
12	Fri, 9/17/2021	Chap. 3 & 6	Hamiltonian equations of motion	#9	9/20/2021
13	Mon, 9/20/2021	Chap. 3 & 6	Liouville theorm	#10	9/22/2021
14	Wed, 9/22/2021	Chap. 3 & 6	Canonical transformations		
15	Fri, 9/24/2021	Chap. 4	Small oscillations about equilibrium		

PHYSICS COLLOQUIUM

THURSDAY

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SEPTEMBER 23, 2021

Part II
Theoretical and Computational Projects

Wake Forest University
Physics Department
Research Opportunities

This colloquium is designed to give a snapshot of the physics theoretical and computational research projects currently in progress at Wake Forest University.

The hope is that these presentations will foster collaborations between groups, inspire beginning students to think about physics research and possibly become engaged in it themselves, and inform more senior students about how their mentors and other mentors approach physics research.

4 PM in Olin 101 and via zoom

Comment on HW --

PHY 711 – Assignment #10

September 20, 2021

1. Consider a Lagrangian describing one dimensional motion of a particle of mass m in a mechanical potential $V(x)$ with an addition time dependent function $s(t)$ and extra constants Q and K having the form

$$L(x,\dot{x},s,\dot{s}) = \frac{1}{2}m\dot{x}^2 - V(x) + Q\dot{s}^2 - K\ln(s).$$

   (a) Find the constants of motion for this system.

   (b) Find the corresponding Hamiltonian in canonical form $H(x,p_x,s,p_s)$.

Inspired by Nose''s form, but only approximately since there is no coupling between *x* and *s*. The equations of motion for *x(t)* and *s(t)* are not trivial.

Your questions –

From Can:
What is the definition of constant of motion? Why is energy some time part of the constant of motion?

Comment – Generally we mean anything that is constant wrt time.

Example: $L(y,\dot{y},t)=\frac{1}{2}m\dot{y}^2-mgy$

Euler-Lagrange equation: $\frac{d^2y}{dt^2}=-g$ Here, no obvious constant of motion

General solution: $y(t)=y_0+v_0t-\frac{1}{2}gt^2$

Integration constants

Virial theorem (Rudolf Clausius ~ 1870)

$$2\langle T\rangle = -\left\langle \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r}_\sigma \right\rangle$$

Proof:

Define: $A \equiv \sum_\sigma \mathbf{p}_\sigma \cdot \mathbf{r}_\sigma$

$$\frac{dA}{dt} = \sum_\sigma \left(\dot{\mathbf{p}}_\sigma \cdot \mathbf{r}_\sigma + \mathbf{p}_\sigma \cdot \dot{\mathbf{r}}_\sigma \right) = \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r}_\sigma + 2T$$

Because $\dot{\mathbf{p}}_\sigma = \mathbf{F}_\sigma$

$$\left\langle \frac{dA}{dt} \right\rangle = \left\langle \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r} \right\rangle + 2\langle T\rangle$$

$$\left\langle \frac{dA}{dt} \right\rangle = \frac{1}{\tau}\int_0^\tau \frac{dA(t)}{dt}dt = \frac{A(\tau) - A(0)}{\tau} \Rightarrow 0$$

Note that this implies that the motion is periodic or bounded (not for all systems).

When it is true -- $\Rightarrow \left\langle \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r} \right\rangle + 2\langle T\rangle = 0$

Examples of the Virial Theorem

$$2\langle T\rangle = -\left\langle \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r}_\sigma \right\rangle$$

Harmonic oscillator:

$$\mathbf{F} = -kx\hat{\mathbf{x}} \qquad T = \frac{1}{2}m\dot{x}^2 \qquad \langle m\dot{x}^2\rangle = \langle kx^2\rangle$$

Check: for $x(t) = X\sin\left(\sqrt{\frac{k}{m}}t + \alpha\right)$

$$\langle 2T\rangle = \langle m\dot{x}^2\rangle = kX^2\left\langle \cos^2\left(\sqrt{\frac{k}{m}}t+\alpha\right)\right\rangle = \frac{1}{2}kX^2$$

$$-\left\langle \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r}_\sigma \right\rangle = \langle kx^2\rangle = kX^2\left\langle \sin^2\left(\sqrt{\frac{k}{m}}t+\alpha\right)\right\rangle = \frac{1}{2}kX^2$$

Premise true because of periodicity.

Examples of the Virial Theorem

$$2\langle T\rangle = -\left\langle \sum_\sigma \mathbf{F}_\sigma \cdot \mathbf{r}_\sigma \right\rangle$$

Circular orbit due to gravitational field of massive object:

$$\mathbf{F} = -\frac{GMm}{r^2}\hat{\mathbf{r}} \qquad T = \frac{1}{2}mv^2$$

$$\langle mv^2\rangle = \left\langle \frac{GMm}{r}\right\rangle$$

Check: for $\dfrac{v^2}{r} = \dfrac{GM}{r^2}$

$$\Rightarrow \langle mv^2\rangle = \left\langle \frac{GMm}{r}\right\rangle$$

centripetal acceleration

gravitational force

Premise true because of periodicity.

Hamiltonian formalism and the canonical equations of motion:

$$H = H\left(\{q_\sigma(t)\},\{p_\sigma(t)\},t\right)$$

Canonical equations of motion

$$\frac{dq_\sigma}{dt} = \frac{\partial H}{\partial p_\sigma}$$

$$\frac{dp_\sigma}{dt} = -\frac{\partial H}{\partial q_\sigma}$$

In the next slides we will consider finding different coordinates and momenta that can also describe the system. Why?

a. Because we can
b. Because it might be useful

Notion of "Canonical" generalized coordinate transformations

$$q_\sigma = q_\sigma\left(\{Q_1 \cdots Q_n\},\{P_1 \cdots P_n\},t\right) \qquad \text{for each } \sigma$$

$$p_\sigma = p_\sigma\left(\{Q_1 \cdots Q_n\},\{P_1 \cdots P_n\},t\right) \qquad \text{for each } \sigma$$

Note that because of the way we set up the problem we can always add such a term.

For some $\tilde{H}$ and F, using Legendre transformations

$$\sum_\sigma p_\sigma \dot{q}_\sigma - H\left(\{q_\sigma\},\{p_\sigma\},t\right) = \sum_\sigma P_\sigma \dot{Q}_\sigma - \tilde{H}\left(\{Q_\sigma\},\{P_\sigma\},t\right) + \frac{d}{dt}F\left(\{q_\sigma\},\{Q_\sigma\},t\right)$$

Apply Hamilton's principle:

$$\delta \int_{t_i}^{t_f} \left[\sum_\sigma P_\sigma \dot{Q}_\sigma - \tilde{H}\left(\{Q_\sigma\},\{P_\sigma\},t\right) + \frac{d}{dt}F\left(\{q_\sigma\},\{Q_\sigma\},t\right)\right] dt = 0$$

$$\delta \int_{t_i}^{t_f} \left[\frac{d}{dt}F\left(\{q_\sigma\},\{Q_\sigma\},t\right)\right] dt = \int_{t_i}^{t_f} \left[\frac{d}{dt}\delta F\left(\{q_\sigma\},\{Q_\sigma\},t\right)\right] dt$$

$$= \delta F\left(t_f\right) - \delta F\left(t_i\right) = 0 \qquad \text{and} \qquad \dot{Q}_\sigma = \frac{\partial \tilde{H}}{\partial P_\sigma} \qquad \dot{P}_\sigma = -\frac{\partial \tilde{H}}{\partial Q_\sigma}$$

Some details --

$$q_\sigma = q_\sigma\left(\{Q_1 \cdots Q_n\},\{P_1 \cdots P_n\},t\right) \qquad \text{for each } \sigma$$

$$p_\sigma = p_\sigma\left(\{Q_1 \cdots Q_n\},\{P_1 \cdots P_n\},t\right) \qquad \text{for each } \sigma$$

For some $\tilde{H}$ and F, using Legendre transformations

$$\sum_\sigma p_\sigma \dot{q}_\sigma - H\left(\{q_\sigma\},\{p_\sigma\},t\right) = \sum_\sigma P_\sigma \dot{Q}_\sigma - \tilde{H}\left(\{Q_\sigma\},\{P_\sigma\},t\right) + \frac{d}{dt}F\left(\{q_\sigma\},\{Q_\sigma\},t\right)$$

Action integral:

$$S = \int_{t_i}^{t_f} dt \left(\sum_\sigma p_\sigma \dot{q}_\sigma - H\left(\{q_\sigma\},\{p_\sigma\},t\right) \right)$$

$$\delta S = \int_{t_i}^{t_f} dt \left(\sum_\sigma \left(\delta p_\sigma \dot{q}_\sigma + p_\sigma \delta \dot{q}_\sigma\right) - \delta H\left(\{q_\sigma\},\{p_\sigma\},t\right) \right)$$

Note that $\delta \int_{t_i}^{t_f} dt \left(\frac{dF(t)}{dt} \right) = \int_{t_i}^{t_f} dt \left(\frac{d\delta F(t)}{dt} \right) = 0$

Some relations between old and new variables:

$$\sum_\sigma p_\sigma \dot{q}_\sigma - H(\{q_\sigma\},\{p_\sigma\},t) =$$

$$\sum_\sigma P_\sigma \dot{Q}_\sigma - \tilde{H}(\{Q_\sigma\},\{P_\sigma\},t) + \frac{d}{dt}F(\{q_\sigma\},\{Q_\sigma\},t)$$

$$\frac{d}{dt}F(\{q_\sigma\},\{Q_\sigma\},t) = \sum_\sigma \left(\left(\frac{\partial F}{\partial q_\sigma}\right)\dot{q}_\sigma + \left(\frac{\partial F}{\partial Q_\sigma}\right)\dot{Q}_\sigma \right) + \frac{\partial F}{\partial t}$$

$$\Rightarrow \sum_\sigma \left(p_\sigma - \left(\frac{\partial F}{\partial q_\sigma}\right)\right)\dot{q}_\sigma - H(\{q_\sigma\},\{p_\sigma\},t) =$$

$$\sum_\sigma \left(P_\sigma + \left(\frac{\partial F}{\partial Q_\sigma}\right)\right)\dot{Q}_\sigma - \tilde{H}(\{Q_\sigma\},\{P_\sigma\},t) + \frac{\partial F}{\partial t}$$

$$\sum_\sigma\left(p_\sigma-\left(\frac{\partial F}{\partial q_\sigma}\right)\right)\dot{q}_\sigma-H(\{q_\sigma\},\{p_\sigma\},t)=$$

$$\sum_\sigma\left(P_\sigma+\left(\frac{\partial F}{\partial Q_\sigma}\right)\right)\dot{Q}_\sigma-\widetilde{H}(\{Q_\sigma\},\{P_\sigma\},t)+\frac{\partial F}{\partial t}$$

$$\Rightarrow p_\sigma=\left(\frac{\partial F}{\partial q_\sigma}\right) \qquad P_\sigma=-\left(\frac{\partial F}{\partial Q_\sigma}\right)$$

$$\Rightarrow \widetilde{H}(\{Q_\sigma\},\{P_\sigma\},t)=H(\{q_\sigma\},\{p_\sigma\},t)+\frac{\partial F}{\partial t}$$

Note that it is conceivable that if we were extraordinarily clever, we could find all of the constants of the motion!

$$\dot{Q}_\sigma = \frac{\partial \widetilde{H}}{\partial P_\sigma} \qquad \dot{P}_\sigma = -\frac{\partial \widetilde{H}}{\partial Q_\sigma}$$

$$\text{Suppose: } \dot{Q}_\sigma = \frac{\partial \widetilde{H}}{\partial P_\sigma} = 0 \text{ and } \dot{P}_\sigma = -\frac{\partial \widetilde{H}}{\partial Q_\sigma} = 0$$

$$\Rightarrow Q_\sigma, P_\sigma \quad \text{are constants of the motion}$$

Possible solution – Hamilton-Jacobi theory:

$$\text{Suppose: } F(\{q_\sigma\},\{Q_\sigma\},t) \Rightarrow -\sum_\sigma P_\sigma Q_\sigma + S(\{q_\sigma\},\{P_\sigma\},t)$$

$$\sum_\sigma p_\sigma \dot{q}_\sigma - H(\{q_\sigma\},\{p_\sigma\},t) =$$

$$\sum_\sigma P_\sigma \dot{Q}_\sigma - \widetilde{H}(\{Q_\sigma\},\{P_\sigma\},t) + \frac{d}{dt}\left(-\sum_\sigma P_\sigma Q_\sigma + S(\{q_\sigma\},\{P_\sigma\},t)\right)$$

$$= -\widetilde{H}(\{Q_\sigma\},\{P_\sigma\},t) - \sum_\sigma \dot{P}_\sigma Q_\sigma + \sum_\sigma \left(\frac{\partial S}{\partial q_\sigma}\dot{q}_\sigma + \frac{\partial S}{\partial P_\sigma}\dot{P}_\sigma\right) + \frac{\partial S}{\partial t}$$

Solution :

$$p_\sigma = \frac{\partial S}{\partial q_\sigma} \qquad\qquad Q_\sigma = \frac{\partial S}{\partial P_\sigma}$$

$$\widetilde{H}(\{Q_\sigma\},\{P_\sigma\},t) = H(\{q_\sigma\},\{p_\sigma\},t) + \frac{\partial S}{\partial t}$$

When the dust clears:

Assume $\{Q_\sigma\}, \{P_\sigma\}, \widetilde{H}$ are constants; choose $\widetilde{H} = 0$

Need to find $S(\{q_\sigma\}, \{P_\sigma\}, t)$

$$p_\sigma = \frac{\partial S}{\partial q_\sigma} \qquad Q_\sigma = \frac{\partial S}{\partial P_\sigma}$$

$$\Rightarrow H\left(\{q_\sigma\}, \left\{\frac{\partial S}{\partial q_\sigma}\right\}, t\right) + \frac{\partial S}{\partial t} = 0$$

Note: S is the "action":

$$\sum_\sigma p_\sigma \dot{q}_\sigma - H(\{q_\sigma\}, \{p_\sigma\}, t) =$$

$$\underbrace{\sum_\sigma P_\sigma \dot{Q}_\sigma}_{0} - \underbrace{\widetilde{H}(\{Q_\sigma\}, \{P_\sigma\}, t)}_{0} + \frac{d}{dt}\left(\underbrace{-\sum_\sigma P_\sigma Q_\sigma}_{0} + S(\{q_\sigma\}, \{P_\sigma\}, t)\right)$$

$$\sum_\sigma p_\sigma \dot{q}_\sigma - H(\{q_\sigma\},\{p_\sigma\},t) =$$

$$\sum_\sigma P_\sigma \dot{Q}_\sigma - \widetilde{H}(\{Q_\sigma\},\{P_\sigma\},t) + \frac{d}{dt}\left(-\sum_\sigma P_\sigma Q_\sigma + S(\{q_\sigma\},\{P_\sigma\},t)\right)$$

0 0 0

$$\int_{t_i}^{t_f}\left(\sum_\sigma p_\sigma \dot{q}_\sigma - H(\{q_\sigma\},\{p_\sigma\},t)\right)dt = \int_{t_i}^{t_f}\left(\frac{d}{dt}\left(S(\{q_\sigma\},\{P_\sigma\},t)\right)\right)dt$$

$$= S(\{q_\sigma\},\{P_\sigma\},t)\Big|_{t_i}^{t_f}$$

Differential equation for ***S***:

$$H\left(\{q_\sigma\},\left\{\frac{\partial S}{\partial q_\sigma}\right\},t\right)+\frac{\partial S}{\partial t}=0$$

$$\text{Example:}\qquad H(\{q\},\{p\},t)=\frac{p^2}{2m}+\frac{1}{2}m\omega^2 q^2$$

$$\text{Hamilton - Jacobi Eq:}\quad H\left(\{q\},\left\{\frac{\partial S}{\partial q}\right\},t\right)+\frac{\partial S}{\partial t}=0$$

$$\frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2+\frac{1}{2}m\omega^2 q^2+\frac{\partial S}{\partial t}=0$$

Does this look familiar?

$$\text{Assume:}\qquad S(q,t)\equiv W(q)-Et \qquad (E\ \ \text{constant})$$

Continued:

$$\frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2+\frac{1}{2}m\omega^2q^2+\frac{\partial S}{\partial t}=0$$

Assume: $S(q,t)\equiv W(q)-Et$ $\quad$ (E constant)

$$\frac{1}{2m}\left(\frac{dW}{dq}\right)^2+\frac{1}{2}m\omega^2q^2=E$$

$$\frac{dW}{dq}=\sqrt{2mE-(m\omega)^2q^2}$$

$$W(q)=\int\sqrt{2mE-(m\omega)^2q^2}\,dq$$

Continued:

$$W(q)=\int\sqrt{2mE-(m\omega)^2q^2}\,dq$$

$$=\frac{1}{2}q\sqrt{2mE-(m\omega)^2q^2}+\frac{E}{\omega}\sin^{-1}\left(\frac{m\omega q}{\sqrt{2mE}}\right)+C$$

$$S(q,E,t)=\frac{1}{2}q\sqrt{2mE-(m\omega)^2q^2}+\frac{E}{\omega}\sin^{-1}\left(\frac{m\omega q}{\sqrt{2mE}}\right)-Et$$

$$\frac{\partial S}{\partial E}=Q=\frac{1}{\omega}\sin^{-1}\left(\frac{m\omega q}{\sqrt{2mE}}\right)-t$$

$$\Rightarrow q(t)=\frac{\sqrt{2mE}}{m\omega}\sin(\omega(t+Q))$$

Another example of Hamilton Jacobi equations

Example: $$H\left(\{y\},\{p\},t\right)=\frac{p^2}{2m}+mgy$$

Assume $y(0)=h; \qquad p(0)=0$

Hamilton-Jacobi Eq: $$H\left(\{y\},\left\{\frac{\partial S}{\partial y}\right\},t\right)+\frac{\partial S}{\partial t}=0$$

$$\frac{1}{2m}\left(\frac{\partial S}{\partial y}\right)^2+mgy+\frac{\partial S}{\partial t}=0$$

Assume: $S(y,t)\equiv W(y)-Et$ $\qquad$ (E constant)

Example: $$H\left(\{y\},\{p\},t\right)=\frac{p^2}{2m}+mgy$$

Assume $y(0)=h; \qquad p(0)=0$

$$\frac{1}{2m}\left(\frac{dW}{dy}\right)^2+mgy=E\equiv mgh$$

$$W(y)=m\int_y^h\sqrt{2g\left(h-y'\right)}dy'=\frac{2}{3}m\sqrt{2g}\left(h-y\right)^{3/2}$$

$$S(y,t)=W(y)-Et=\frac{2}{3}m\sqrt{2g}\left(h-y\right)^{3/2}-mght$$

Check action:

For this case: $y(t) = h - \frac{1}{2}gt^2$

$$S = \int_0^t \left(\frac{1}{2}m\dot{y}^2 - mgy \right) dt' = \frac{1}{3}mg^2t^3 - mght$$

$$S(y,t) = W(y) - Et = \frac{2}{3}m\sqrt{2g}\,(h-y)^{3/2} - mght$$

Agrees with Hamilton-Jacobi analysis.

Alternatively, keeping E notation:

$$W(y) = \int_y^h \sqrt{2mE - 2m^2 gy'}\,dy'$$

$$= \sqrt{\frac{2}{m}}\frac{1}{g}\left(E - mgy\right)^{3/2}$$

$$S(y,t) = W(y) - Et = \sqrt{\frac{2}{m}}\frac{2}{3g}\left(E - mgy\right)^{3/2} - Et$$

$$\frac{\partial S}{\partial E} = Q = \sqrt{\frac{2}{m}}\frac{1}{g}\left(E - mgy\right)^{1/2} - t$$

$$\Rightarrow y(t) = \frac{E}{mg} - \frac{1}{2}g\left(t + Q\right)^2$$

In our case, $Q = 0$

$E = mgh$

What do you think of Hamilton-Jacobi method

a. Historically important
b. Hysterical
c. Painful
d. Might be useful

The next 3 slides contain important equations that you will hopefully remember for this material contained in Chapters 3 & 6 of Fetter and Walecka. On Friday we will start with Chapter 4 and discuss one of the many applications of these ideas – the case of small oscillations near equilibrium.

Recap --

Lagrangian picture

For independent generalized coordinates $q_\sigma(t)$:

$$L = L(\{q_\sigma(t)\},\{\dot{q}_\sigma(t)\},t)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

$\Rightarrow$ Second order differential equations for $q_\sigma(t)$

Hamiltonian picture

$$H = H(\{q_\sigma(t)\},\{p_\sigma(t)\},t)$$

$$\frac{dq_\sigma}{dt} = \frac{\partial H}{\partial p_\sigma} \qquad \frac{dp_\sigma}{dt} = -\frac{\partial H}{\partial q_\sigma}$$

$\Rightarrow$ Coupled first order differential equations for

$q_\sigma(t)$ and $p_\sigma(t)$

General treatment of particle of mass m and charge q moving in 3 dimensions in an potential $U(\mathbf{r})$ as well as electromagnetic scalar and vector potentials $\Phi(\mathbf{r},t)$ and $\mathbf{A}(\mathbf{r},t)$:

Lagrangian: $$L(\mathbf{r},\dot{\mathbf{r}},t)=\frac{1}{2}m\dot{\mathbf{r}}^2-U(\mathbf{r})-q\Phi(\mathbf{r},t)+\frac{q}{c}\dot{\mathbf{r}}\cdot\mathbf{A}(\mathbf{r},t)$$

Hamiltonian: $$\mathbf{p}=\frac{\partial L}{\partial \dot{\mathbf{r}}}=m\dot{\mathbf{r}}+\frac{q}{c}\mathbf{A}(\mathbf{r},t)$$

$$H(\mathbf{r},\mathbf{p},t)=\mathbf{p}\cdot\dot{\mathbf{r}}-L(\mathbf{r},\dot{\mathbf{r}},t)$$

$$=\frac{1}{2m}\left(\mathbf{p}-\frac{q}{c}\mathbf{A}(\mathbf{r},t)\right)^2+U(\mathbf{r})+q\Phi(\mathbf{r},t)$$

Recipe for constructing the Hamiltonian and analyzing the equations of motion

1. Construct Lagrangian function : $L = L\left(\{q_\sigma(t)\}, \{\dot{q}_\sigma(t)\}, t\right)$

2. Compute generalized momenta : $p_\sigma \equiv \dfrac{\partial L}{\partial \dot{q}_\sigma}$

3. Construct Hamiltonian expression : $H = \sum_\sigma \dot{q}_\sigma p_\sigma - L$

4. Form Hamiltonian function : $H = H\left(\{q_\sigma(t)\}, \{p_\sigma(t)\}, t\right)$

5. Analyze canonical equations of motion :

$$\frac{dq_\sigma}{dt} = \frac{\partial H}{\partial p_\sigma} \qquad \frac{dp_\sigma}{dt} = -\frac{\partial H}{\partial q_\sigma}$$