PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF in Olin 103

Notes on Lecture 32: Chap. 9 of F&W

Non-linear effects

1. Introduction to non-linear effects
2. Analysis of instability – shock phenomena

11/10/2021 PHY 711 Fall 2021 -- Lecture 32 1

In this lecture, we will consider traveling wave solutions to the sound wave equations.

28	Mon, 11/01/2021	Chap. 9	Mechanics of 3 dimensional fluids	#19	11/03/2021
29	Wed, 11/03/2021	Chap. 9	Linearized hydrodynamics equations	#20	11/05/2021
30	Fri, 11/05/2021	Chap. 9	Linear sound waves	#21	11/08/2021
31	Mon, 11/08/2021	Chap. 9	Sound sources and scattering	#22	11/10/2021
32	Wed, 11/10/2021	Chap. 9	Non linear effects in sound waves and shocks	Topic due	11/12/2021
33	Fri, 11/12/2021	Chap. 10	Surface waves in fluids		
34	Mon, 11/15/2021	Chap. 10	Surface waves in fluids; soliton solutions		
35	Wed, 11/17/2021	Chap. 11	Heat conduction		
	Fri, 11/19/2021		Presentations I		
	Mon, 11/22/2021		Presentations II		
	Wed, 11/24/2021		Thanksgiving		
	Fri, 11/26/2021		Thanksgiving		
36	Mon, 11/29/2021	Chap. 12	Viscous effects on hydrodynamics		
37	Wed, 12/01/2021	Chap. 1-12	Review		
38	Fri, 12/03/2021	Chap. 1-12	Review		

Schedule.

PHYSICS COLLOQUIUM

4 PM Olin 101

THURSDAY

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NOVEMBER 11, 2021

"Spin Response of Hybrid Organic-Inorganic Halide Perovskites"

The field of thin-film photovoltaics has been recently enriched by the introduction of the 2D and 3D hybrid organic inorganic lead halide perovskites (HOIPs) as absorber materials, which allow low-cost synthesis of solar cells with efficiencies exceeding 22%. In addition, it has been shown that these compounds may be effective active layers in spintronics applications [1] due to their large spin orbit coupling [2], Rashba effect in the continuum bands [3], and efficient luminescence emission. More recently chirality has been introduced in 2D-HOIP via chiral organic moieties [4], which has brought novel physics and myriad of spin-optoelectronic applications [5]. The impact of the crystal structure especially regarding the inversion symmetry, and the dimensionality on the optoelectronic and spin response properties of these compounds has been the focus of intense research at the present time. I will briefly outline notable achievements to date, describe the unique attributes of these perovskites that has led to their rapid emergence as serious candidates for spintronics applications, and specifically discuss spin-optoelectronic devices based on chiral 2D-HOIP [6].

Z. Valy Vardeny, Ph.D.
Distinguished Professor
Physics And Astronomy
The University of Utah
Salt Lake City, UT

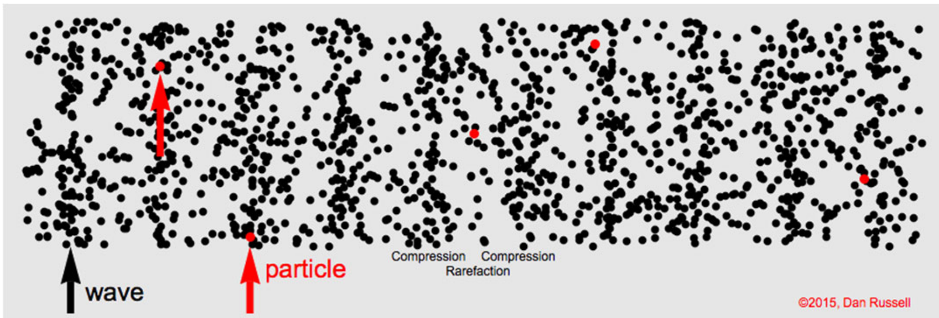
4:00 pm - Olin 101*

***Link provide for those unable to attend in person.**

Note: For additional information on the seminar or to obtain the video conference link, contact wfuphys@wfu.edu

Visualization of longitudinal wave motion

From the website:
https://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html



Now consider some non-linear effects

Effects of nonlinearities in fluid equations
-- one dimensional case

Newton - Euler equation of motion :

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{f}_{applied} - \frac{\nabla p}{\rho}$$

Continuity equation : $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$

Assume spatial variation confined to x direction ; assume that $\mathbf{v} = v\hat{\mathbf{x}}$ and $\mathbf{f}_{applied} = 0.$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial x} = 0$$

Review of basic equations, specializing in one spatial dimension.

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial x} = 0$$

Expressing p in terms of ρ: $\quad p = p(\rho)$

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial \rho}\frac{\partial \rho}{\partial x} \equiv c^2(\rho)\frac{\partial \rho}{\partial x} \qquad \text{where} \quad \frac{\partial p}{\partial \rho} \equiv c^2(\rho)$$

For adiabatic ideal gas: $\quad \frac{dp}{p} = \gamma\frac{d\rho}{\rho} \qquad p = p_0\left(\frac{\rho}{\rho_0}\right)^{\gamma}$

$$c^2(\rho) = \frac{\gamma p}{\rho} = c_0^2\left(\frac{\rho}{\rho_0}\right)^{\gamma-1} \qquad \text{where} \quad c_0^2 \equiv \frac{\gamma p_0}{\rho_0}$$

Decoupling the variables.

Digression – What is gamma?

Internal energy for ideal gas: $pV = Nk_BT$

$E_{\text{int}} = \frac{f}{2} Nk_BT$ $f \equiv$ degrees of freedom; 3 for atom, 5 for diatomic molecule

In terms of specific heat ratio: $\gamma \equiv \frac{C_p}{C_V}$

$$dE_{\text{int}} = dQ - dW$$

$$C_V = \left(\frac{dQ}{dT}\right)_V = \left(\frac{\partial E}{\partial T}\right)_V = \frac{f}{2} Nk_B$$

$$C_p = \left(\frac{dQ}{dT}\right)_p = \left(\frac{\partial E}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p = \frac{f}{2} Nk_B + Nk_B$$

$$\gamma = \frac{C_p}{C_V} = \frac{\frac{f}{2}+1}{\frac{f}{2}} = 1 + \frac{2}{f} \qquad \Rightarrow \frac{f}{2} = \frac{1}{\gamma - 1} \qquad E_{\text{int}} = \frac{1}{\gamma - 1} Nk_B$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{c^2(\rho)}{\rho}\frac{\partial \rho}{\partial x} = 0$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial x} = 0$$

Expressing variation of v in terms of $v(\rho)$:

$$\frac{\partial v}{\partial \rho}\frac{\partial \rho}{\partial t} + v\frac{\partial v}{\partial \rho}\frac{\partial \rho}{\partial x} + \frac{c^2(\rho)}{\rho}\frac{\partial \rho}{\partial x} = 0$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial \rho}\frac{\partial \rho}{\partial x} = 0$$

More analysis.

Some more algebra :

From Euler equation : $\frac{\partial v}{\partial \rho}\left(\frac{\partial \rho}{\partial t}+v\frac{\partial \rho}{\partial x}\right)+\frac{c^2(\rho)}{\rho}\frac{\partial \rho}{\partial x}=0$

From continuity equation : $\frac{\partial \rho}{\partial t}+v\frac{\partial \rho}{\partial x}=-\rho\frac{\partial v}{\partial \rho}\frac{\partial \rho}{\partial x}$

Combined equation : $\frac{\partial v}{\partial \rho}\left(-\rho\frac{\partial v}{\partial \rho}\frac{\partial \rho}{\partial x}\right)+\frac{c^2(\rho)}{\rho}\frac{\partial \rho}{\partial x}=0$

$$\Rightarrow \left(\frac{\partial v}{\partial \rho}\right)^2=\frac{c^2(\rho)}{\rho^2} \qquad \frac{\partial v}{\partial \rho}=\pm\frac{c}{\rho}$$

$$\Rightarrow \frac{\partial \rho}{\partial t}+(v\pm c)\frac{\partial \rho}{\partial x}=0$$

Further derivations.

Assuming adiabatic process: $c^2 = c_0^2\left(\frac{\rho}{\rho_0}\right)^{\gamma-1}$ $\quad c_0^2 = \frac{\gamma p_0}{\rho_0}$

$$\frac{\partial v}{\partial \rho} = \frac{dv}{d\rho} = \pm\frac{c}{\rho} \qquad \Rightarrow v = \pm c_0 \int_{\rho_0}^{\rho}\left(\frac{\rho'}{\rho_0}\right)^{(\gamma-1)/2}\frac{d\rho'}{\rho'}$$

$$\Rightarrow v = \pm\frac{2c_0}{\gamma-1}\left(\left(\frac{\rho}{\rho_0}\right)^{(\gamma-1)/2} - 1\right)$$

$$\Rightarrow c = c_0\left(\frac{\rho}{\rho_0}\right)^{(\gamma-1)/2}$$

Using adiabatic relationships.

Summary :

$$\frac{dv}{d\rho} = \pm \frac{c}{\rho}$$

$$\frac{\partial \rho}{\partial t} + (v \pm c)\frac{\partial \rho}{\partial x} = 0$$

Assuming adiabatic process : $c^2 = c_0^2 \left(\frac{\rho}{\rho_0}\right)^{\gamma-1}$ $\quad c_0^2 = \frac{\gamma p_0}{\rho_0}$

$$c = c_0 \left(\frac{\rho}{\rho_0}\right)^{(\gamma-1)/2} \qquad v = \pm \frac{2c_0}{\gamma-1}\left(\left(\frac{\rho}{\rho_0}\right)^{(\gamma-1)/2} - 1\right)$$

Analysis of fluid velocity from a knowledge of the wave velocity.

Traveling wave solution:

Assume: $\rho = \rho_0 + f(x - u(\rho)t)$

Need to find self-consistent equations for propagation velocity $u(\rho)$ using equations

From previous derivations: $\frac{\partial \rho}{\partial t} + (v \pm c)\frac{\partial \rho}{\partial x} = 0$

Apparently: $u(\rho) \Leftrightarrow v \pm c$

For adiabatic ideal gas and + signs:

$$u = v + c = c_0\left(\frac{\gamma+1}{\gamma-1}\left(\frac{\rho}{\rho_0}\right)^{(\gamma-1)/2} - \frac{2}{\gamma-1}\right)$$

Analysis for a traveling wave.

Traveling wave solution -- continued:

$$\frac{\partial \rho}{\partial t} + (v \pm c)\frac{\partial \rho}{\partial x} = 0$$

Assume: $\rho = \rho_0 + f(x - u(\rho)t) = \rho_0 + f(x - (v \pm c)t)$

For adiabatic ideal gas and + signs:

$$u = v + c = c_0\left(\frac{\gamma+1}{\gamma-1}\left(\frac{\rho}{\rho_0}\right)^{(\gamma-1)/2} - \frac{2}{\gamma-1}\right)$$

Solution in linear approxiation:

$$u = v + c \approx v_0 + c_0 = c_0\left(\frac{\gamma+1}{\gamma-1} - \frac{2}{\gamma-1}\right) = c_0$$

$$\Rightarrow \rho = \rho_0 + f\left(x - c_0 t\right)$$

Checking the linear result

Some details

Assume: $\rho = \rho_0 + f(x - u(\rho)t)$

Need to find self-consistent equations for propagation velocity $u(\rho)$ using equations

From previous derivations: $\frac{\partial \rho}{\partial t} + (v \pm c)\frac{\partial \rho}{\partial x} = 0$

Apparently: $u(\rho) \Leftrightarrow v \pm c$

Note that for $u = v + c$ (choice of + solution)

$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} = 0$ is satisfied by a function of the form

$\rho(x,t) = \rho_0 + f(x - u(\rho(x,t))t)$

Let $w \equiv x - u(\rho(x,t))t$

$$\frac{df}{dw}\frac{\partial w}{\partial t} + u\frac{df}{dw}\frac{\partial w}{\partial x} = \frac{df}{dw}(-u + u) = 0$$

Traveling wave solution -- full non-linear case:

Visualization for particular waveform: $\rho = \rho_0 + f(\underbrace{x - u(\rho)t}_{w})$

Assume: $f(w) \equiv \rho_0 s(w)$

$$\frac{\rho}{\rho_0} = 1 + s(x - ut)$$

For adiabatic ideal gas:

$$u = c_0\left(\frac{\gamma+1}{\gamma-1}\left(\frac{\rho}{\rho_0}\right)^{(\gamma-1)/2} - \frac{2}{\gamma-1}\right)$$

$$u = c_0\left(\frac{\gamma+1}{\gamma-1}\big(1 + s(x-ut)\big)^{(\gamma-1)/2} - \frac{2}{\gamma-1}\right)$$

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Analysis of how to visualize the traveling wave solution.

Visualization continued:

$$u = c_0\left(\frac{\gamma+1}{\gamma-1}\left(1+s(x-ut)\right)^{(\gamma-1)/2} - \frac{2}{\gamma-1}\right)$$

Plot $s(x-ut)$ for fixed t, as a function of x:

Let $w = x - ut$

$$x = w + ut = w + u(w)t \equiv x(w,t)$$

$$u(w) = c_0\left(\frac{\gamma+1}{\gamma-1}\left(1+s(w)\right)^{(\gamma-1)/2} - \frac{2}{\gamma-1}\right)$$

Parametric equations:

plot $s(w)$ vs $x(w,t)$ for range of w at each t

More details.

Summary

$$\frac{\partial \rho}{\partial t} + u(\rho)\frac{\partial \rho}{\partial x} = 0$$

Solution: $\rho = \rho_0 + f(x - u(\rho)t) = \rho_0\left(1 + s(x - u(\rho)t)\right)$

For linear case: $u(\rho) = c_0$

For non-linear case: $u(\rho) = c_0\left(\frac{\gamma+1}{\gamma-1}\left(1 + s(x-ut)\right)^{(\gamma-1)/2} - \frac{2}{\gamma-1}\right)$

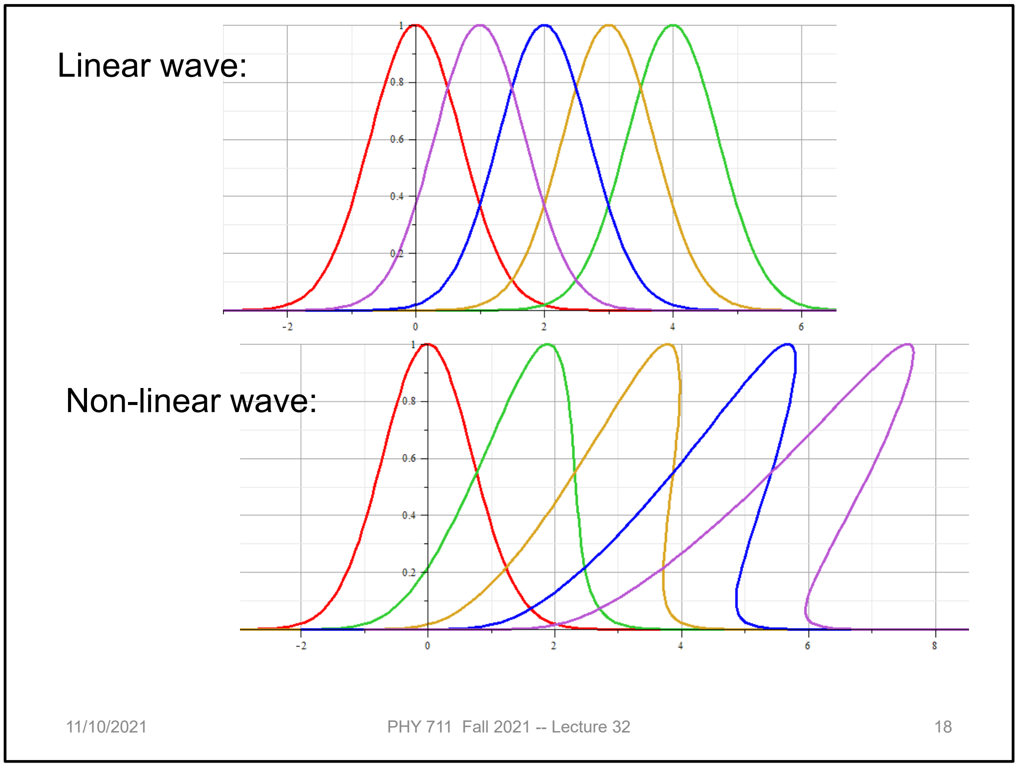
Plot $s(x-ut)$ for fixed t, as a function of x:

Let $w = x - ut \quad \Rightarrow \quad x = w + ut = w + u(w)t \equiv x(w,t)$

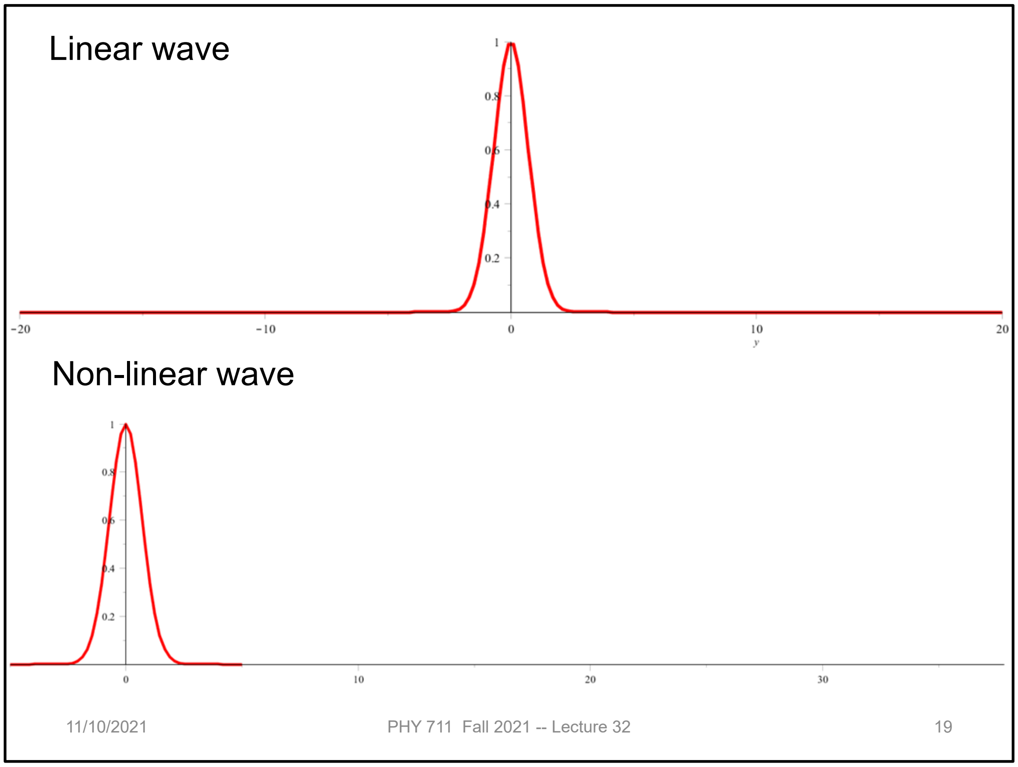
$$u(w) = c_0\left(\frac{\gamma+1}{\gamma-1}\left(1 + s(w)\right)^{(\gamma-1)/2} - \frac{2}{\gamma-1}\right)$$

Parametric equations: plot $s(w)$ vs $x(w,t)$ for range of w

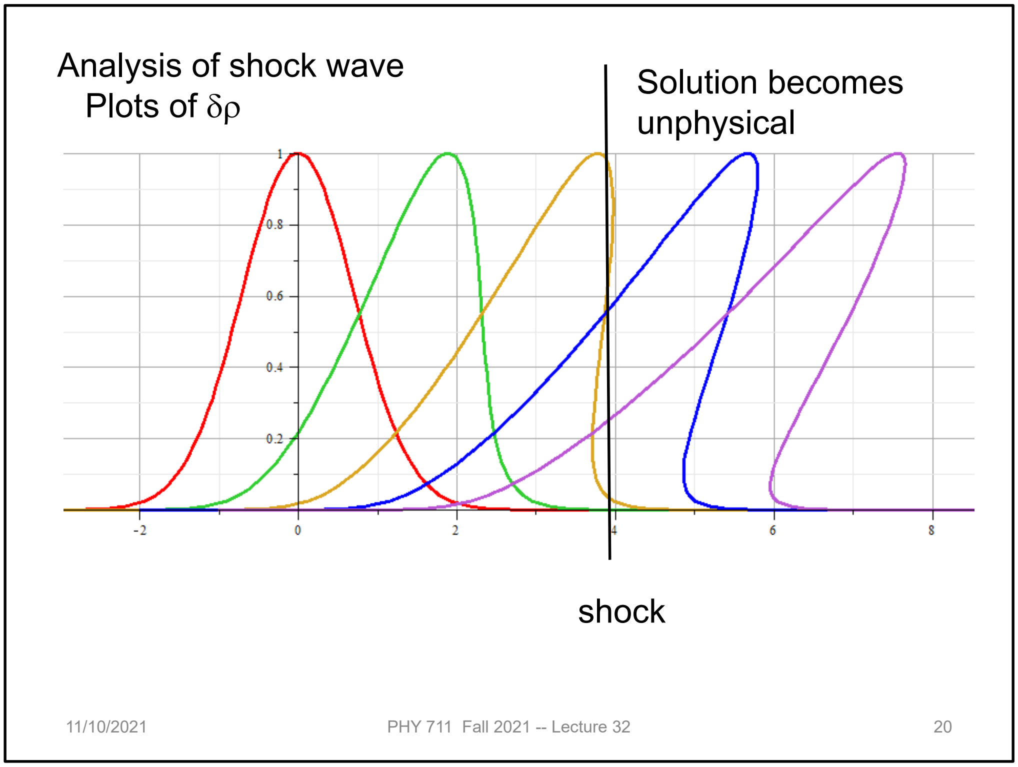
Summary.



Example visualization.

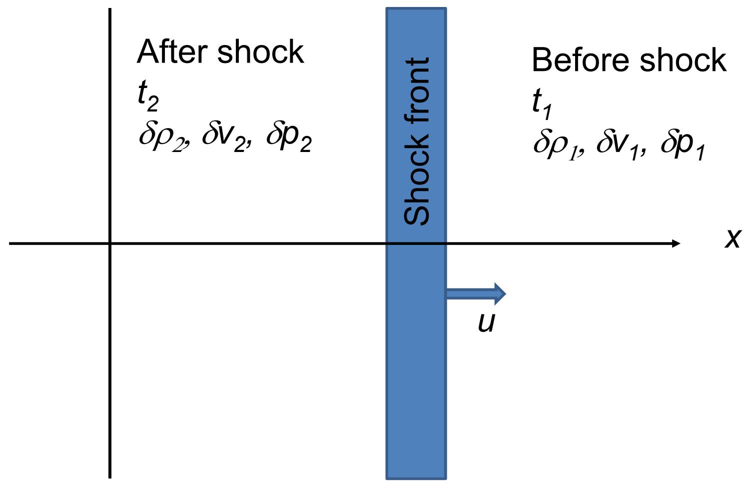


Animations from Maple.



Note that the vertical axis represents the longitudinal wave displacement. When this displacement becomes multivalued for a given coordinate x as shown, the solution becomes unphysical. At this point we need to consider the analysis in a different way.

Analysis of shock wave – assumed to moving at velocity u

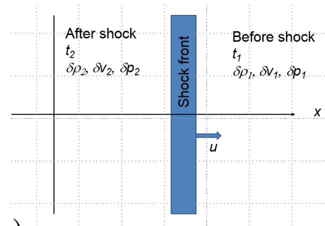


Note that in this case u is assumed to be a given parameter of the system.

Your textbook discusses the shock wave analysis. Here we assume that there is a region (blue) where the analysis fails, but assumes that we can properly analyze the physics before and after the shock. The notation given here is similar to that given in your text.

Analysis of shock wave – continued

While analysis in the shock region is complicated, we can use conservation laws to analyze regions 1 and 2



Assume $\rho(x,t) = \rho\left(x - ut\right)$

$$p(x,t) = p\left(x - ut\right)$$

$$v(x,t) = v\left(x - ut\right)$$

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial\left(\rho v\right)}{\partial x} = 0 = \frac{\partial\left(\rho v - \rho u\right)}{\partial x} \qquad \Rightarrow \left(v_2 - u\right)\rho_2 = \left(v_1 - u\right)\rho_1$$

Conservation of energy and momentum:

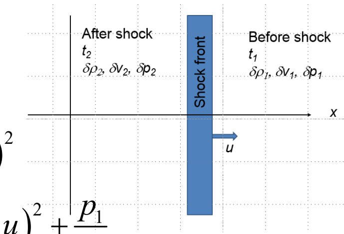
$$\Rightarrow p_2 + \rho_2\left(v_2 - u\right)^2 = p_1 + \rho_1\left(v_1 - u\right)^2$$

$$\Rightarrow \epsilon_2 + \frac{1}{2}\left(v_2 - u\right)^2 + \frac{p_2}{\rho_2} = \epsilon_1 + \frac{1}{2}\left(v_1 - u\right)^2 + \frac{p_1}{\rho_1}$$

Some of the details of the analysis before and after the shock event.

Analysis of shock wave – continued

While analysis in the shock region is complicated, we can use conservation laws to analyze regions 1 and 2



Summary of equations

$$\Rightarrow (v_2 - u)\rho_2 = (v_1 - u)\rho_1$$

$$\Rightarrow p_2 + \rho_2 (v_2 - u)^2 = p_1 + \rho_1 (v_1 - u)^2$$

$$\Rightarrow \epsilon_2 + \frac{1}{2}(v_2 - u)^2 + \frac{p_2}{\rho_2} = \epsilon_1 + \frac{1}{2}(v_1 - u)^2 + \frac{p_1}{\rho_1}$$

Assume that within each regions (1 & 2), the ideal gas equations apply

$$\epsilon_1 + \frac{p_1}{\rho_1} = \frac{\gamma}{\gamma - 1}\frac{p_1}{\rho_1} \qquad \epsilon_2 + \frac{p_2}{\rho_2} = \frac{\gamma}{\gamma - 1}\frac{p_2}{\rho_2}$$

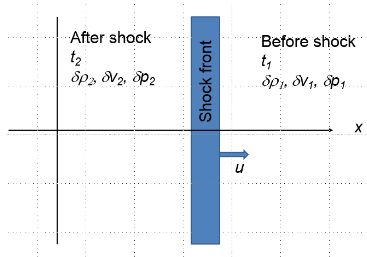
It follows that $$\frac{\gamma}{\gamma - 1}\frac{p_2}{\rho_2} + \frac{1}{2}(v_2 - u)^2 = \frac{\gamma}{\gamma - 1}\frac{p_1}{\rho_1} + \frac{1}{2}(v_1 - u)^2$$

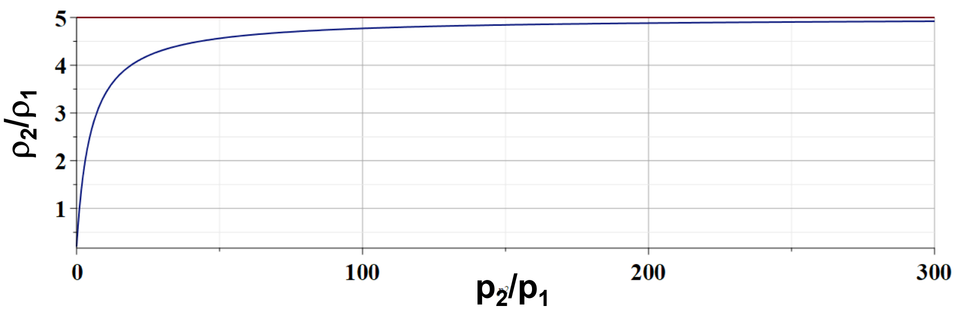
Analyzing the equations.

Analysis of shock wave – continued

For adiabatic ideal gas, also considering energy and momentum conservation:

$$\frac{\rho_2}{\rho_1} = \frac{\frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1}+1}{\frac{\gamma+1}{\gamma-1}+\frac{p_2}{p_1}} \le \frac{\gamma+1}{\gamma-1}$$





Analyzing ratio of the density after and before the shock wave.

Analysis of shock wave – continued

For adiabatic ideal gas, entropy considerations::

Internal energy density: $\varepsilon = \frac{p}{(\gamma - 1)\rho} = C_V T$

First law of thermo: $d\varepsilon = Tds - pd\left(\frac{1}{\rho}\right)$

$$ds = \frac{1}{T}\left(d\left(\frac{p}{(\gamma-1)\rho}\right) - pd\left(\frac{1}{\rho}\right)\right) = C_V d\ln\left(\frac{p}{\rho^\gamma}\right)$$

$$s = C_V \ln\left(\frac{p}{\rho^\gamma}\right) + (\text{constant})$$

$$s_2 - s_1 = C_V \ln\left(\frac{p_2}{p_1}\left(\frac{\rho_1}{\rho_2}\right)^\gamma\right) \qquad 0 < s_2 - s_1 < C_V\left(\ln\left(\frac{p_2}{p_1}\right) - \gamma\ln\left(\frac{\gamma+1}{\gamma-1}\right)\right)$$

Analyzing the entropy before and after the shock wave. In general, many more relationships can be analyzed. Consult your textbook for more details.