

```

In[9]:= M = {{9, -2, 0, 0, 0}, {-2, 9, -2, 0, 0}, {0, -2, 9, -2, 0}, {0, 0, -2, 9, -2}, {0, 0, 0, -2, 9}}
Out[9]= {{9, -2, 0, 0, 0}, {-2, 9, -2, 0, 0}, {0, -2, 9, -2, 0}, {0, 0, -2, 9, -2}, {0, 0, 0, -2, 9} }

In[10]:= Eigenvalues[M]
Out[10]= {9 + 2 Sqrt[3], 11, 9, 7, 9 - 2 Sqrt[3]}

In[11]:= MM = {{9, -2, 0, 0, 0, 0, 0}, {-2, 9, -2, 0, 0, 0, 0}, {0, -2, 9, -2, 0, 0, 0}, {0, 0, -2, 9, -2, 0, 0}, {0, 0, 0, -2, 9, -2, 0}, {0, 0, 0, 0, -2, 9, -2}, {0, 0, 0, 0, 0, -2, 9}}
Out[11]= {{9, -2, 0, 0, 0, 0, 0}, {-2, 9, -2, 0, 0, 0, 0}, {0, -2, 9, -2, 0, 0, 0}, {0, 0, -2, 9, -2, 0, 0}, {0, 0, 0, -2, 9, -2, 0}, {0, 0, 0, 0, -2, 9, -2}, {0, 0, 0, 0, 0, -2, 9} }

In[12]:= Eigenvalues[MM]
Out[12]= {9 + 2 Sqrt[2 + Sqrt[2]], 9 + 2 Sqrt[2], 9 + 2 Sqrt[2 - Sqrt[2]], 9, 9 - 2 Sqrt[2 - Sqrt[2]], 9 - 2 Sqrt[2], 9 - 2 Sqrt[2 + Sqrt[2]]}

In[13]:= N[%]
Out[13]= {12.6955, 11.8284, 10.5307, 9., 7.46927, 6.17157, 5.30448}

In[14]:= Sort[%]
Out[14]= {5.30448, 6.17157, 7.46927, 9., 10.5307, 11.8284, 12.6955}

In[15]:= MMM = {{9, -2, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {-2, 9, -2, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -2, 9, -2, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -2, 9, -2, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -2, 9, -2, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -2, 9, -2, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -2, 9, -2, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -2, 9, -2, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -2, 9, -2, 0}, {0, 0, 0, 0, 0, 0, 0, 0, -2, 9}}
Out[15]= {{9, -2, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {-2, 9, -2, 0, 0, 0, 0, 0, 0, 0, 0}, {0, -2, 9, -2, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -2, 9, -2, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -2, 9, -2, 0, 0, 0, 0, 0}, {0, 0, 0, 0, -2, 9, -2, 0, 0, 0, 0}, {0, 0, 0, 0, 0, -2, 9, -2, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -2, 9, -2, 0, 0}, {0, 0, 0, 0, 0, 0, 0, -2, 9, -2, 0}, {0, 0, 0, 0, 0, 0, 0, 0, -2, 9}}

```

In[16]:= **Eigenvalues**[**MM**]

$$\text{Out}[16]= \left\{ 9 + \sqrt{81 + \text{Root}[2160801389 + 147165045 \#1 + 4003858 \#1^2 + 54394 \#1^3 + 369 \#1^4 + \#1^5 \&, 5]}, \right. \\ 9 + \sqrt{81 + \text{Root}[2160801389 + 147165045 \#1 + 4003858 \#1^2 + 54394 \#1^3 + 369 \#1^4 + \#1^5 \&, 4]}, \\ 9 + \sqrt{81 + \text{Root}[2160801389 + 147165045 \#1 + 4003858 \#1^2 + 54394 \#1^3 + 369 \#1^4 + \#1^5 \&, 3]}, \\ 9 + \sqrt{81 + \text{Root}[2160801389 + 147165045 \#1 + 4003858 \#1^2 + 54394 \#1^3 + 369 \#1^4 + \#1^5 \&, 2]}, \\ 9 + \sqrt{81 + \text{Root}[2160801389 + 147165045 \#1 + 4003858 \#1^2 + 54394 \#1^3 + 369 \#1^4 + \#1^5 \&, 1]}, \\ 9 - \sqrt{81 + \text{Root}[2160801389 + 147165045 \#1 + 4003858 \#1^2 + 54394 \#1^3 + 369 \#1^4 + \#1^5 \&, 1]}, \\ 9 - \sqrt{81 + \text{Root}[2160801389 + 147165045 \#1 + 4003858 \#1^2 + 54394 \#1^3 + 369 \#1^4 + \#1^5 \&, 2]}, \\ 9 - \sqrt{81 + \text{Root}[2160801389 + 147165045 \#1 + 4003858 \#1^2 + 54394 \#1^3 + 369 \#1^4 + \#1^5 \&, 3]}, \\ 9 - \sqrt{81 + \text{Root}[2160801389 + 147165045 \#1 + 4003858 \#1^2 + 54394 \#1^3 + 369 \#1^4 + \#1^5 \&, 4]}, \\ \left. 9 - \sqrt{81 + \text{Root}[2160801389 + 147165045 \#1 + 4003858 \#1^2 + 54394 \#1^3 + 369 \#1^4 + \#1^5 \&, 5]} \right\}$$

In[17]:= **N**[%]

$$\text{Out}[17]= \{12.838, 12.365, 11.6194, 10.6617, 9.56926, 8.43074, 7.33834, 6.38056, 5.63499, 5.16203\}$$