Start reading Chapter 7 of Marion.

Consider the Lagrangian representing a mass $m$ moving vertically in a uniform gravitational field:

$$L(y, \dot{y}; t) \equiv \frac{1}{2}m\dot{y}^2 - mgy$$

such that $y(0) = h$ and $y(T) = 0$, where the fixed time $T$ is defined to be $T \equiv \sqrt{\frac{2h}{g}}$.

1. Solve the Euler-Lagrange equations to find the particle trajectory $y(t)$ and evaluate the action integral for that trajectory.

2. Consider the following alternative trajectories and evaluate the action integrals for them:

   (a) $y_1(t) = h(1 - t/T)$
   (b) $y_2(t) = h\left(1 - \left(t/T\right)^3\right)$

   Are your results consistent with Hamilton’s principle?