

PHY 711 – Problem Set # 24

1. Consider the density $\rho(x, t)$ a one dimensional fluid described by an equation of the form:

$$\frac{\partial \rho}{\partial t} + u(\rho) \frac{\partial \rho}{\partial x} = 0.$$

Here, the effective velocity function is given by

$$u(\rho) \equiv \frac{d\mu(\rho)}{d\rho} = 3\rho$$

for this particular system.

- (a) Suppose that at time $t = 0$ the density function has the shape:

$$\rho(x, 0) = \begin{cases} 1 & \text{for } x < 0 \\ 2 & \text{for } x > 0. \end{cases}$$

Find the form of $\rho(x, t)$ for $t > 0$.

- (b) Suppose that at time $t = 0$ the density function has the shape:

$$\rho(x, 0) = \begin{cases} 2 & \text{for } x < 0 \\ 1 & \text{for } x > 0. \end{cases}$$

Find the form of $\rho(x, t)$ for $t > 0$. In particular, find the velocity of the shock front for this system.