PHY 711 – Problem Set # 5

Start reading Chapter 3 of **Fetter & Walecka**.

1. Suppose you want to extremize an integral of the form:

$$\mathcal{I} \equiv \int_{x_1}^{x_2} \phi(y, y', y''; x) \, dx,$$

   where $y = y(x)$, $y' \equiv \frac{dy}{dx}$, and $y'' \equiv \frac{d^2y}{d^2x}$, and where ϕ is a given function. Find the generalization of the Euler-Lagrange equation (Eq. 17.34 in your text) for this function ϕ. In order to solve this problem, you should assume that the end point values and slopes are all fixed (not varying). That is, $y(x_1) = A$, $y(x_2) = B$, $y'(x_1) = C$, and $y'(x_2) = D$, where A, B, and C, and D are all fixed values.

2. *Extra Credit*

   Consider an integral of the form:

$$E = \int d^3r \, F[\rho(\mathbf{r}), g(\mathbf{r})],$$

   where $\rho(\mathbf{r})$ is a function of position in three dimensions, and $g(\mathbf{r}) \equiv |\nabla \rho(\mathbf{r})|$. Show that the functional variation of E with respect to $\rho(\mathbf{r})$ is given by:

$$\delta E = \int d^3r \left\{ \frac{\partial F}{\partial \rho} - \frac{\partial F}{\partial g} \left(\frac{\nabla^2 \rho}{g} - \frac{\nabla \rho \cdot \nabla g}{g^2} \right) - \frac{\partial^2 F}{\partial \rho \partial g} g - \frac{\partial^2 F}{\partial^2 g} \frac{\nabla \rho \cdot \nabla g}{g} \right\} \delta\rho(\mathbf{r}).$$