PHY 712 – Problem Set # 15

Complete reading of Chapter 5 of Jackson.

You may choose one of the following problems. (Extra credit for working both problems.)

1. Work Problem #5.13 in Jackson.

2. Consider a uniform cylindrical current expressed in cylindrical equations in the form:

   \[ \mathbf{J}(\rho) \equiv j_0 \Theta(a - \rho) \mathbf{\hat{z}}, \]  

   where \( j_0 \) is a constant current density, \( a \) is the radius of the cylinder, and \( \Theta(a - \rho) \) denotes the Heaviside function.

   (a) Find the vector potential \( \mathbf{A} \) in the Coulomb gauge (\( \nabla \cdot \mathbf{A} = 0 \)). Assuming the appropriate boundary conditions at \( \rho = a \), find the form of \( \mathbf{A} \) for both \( \rho < a \) and for \( \rho > a \) up to an arbitrary constant.

   (b) Find the magnetic field \( \mathbf{B} \) for both \( \rho < a \) and for \( \rho > a \).

   (c) Sketch \( \mathbf{A}(\rho) \) and \( \mathbf{B}(\rho) \) as a function of \( \rho \).