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PHY 712 – Problem Set # 16

Read Chapter 6 of **Jackson**.

Consider the electric field produced by a point charge q moving on a trajectory described by $\mathbf{r_0(t)}$ with $\rho(r,t) \equiv q\delta^3(\mathbf{r} - \mathbf{r_0}(t))$. Assume that $\mathbf{v_0}(t) \equiv \partial\mathbf{r_0}(t)/\partial t$ and $\partial^2\mathbf{r_0}(t)/\partial t^2 = 0$. Show that the electric field can be written in the form:

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0}\frac{(1 - v_0^2/c^2)(\mathbf{R} - \mathbf{v_0}R/c)}{(R - \mathbf{v_0}\cdot\mathbf{R}/c)^3}, \tag{1}$$

where $R \equiv |\mathbf{R}(t_r)|$, $\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{r_0}(t_r)$, and where all quantities which depend on time on the right hand side of the equation are evaluated at the retarded time $t_r \equiv t - R(t_r)/c$. This is the result which we will obtain from the Lienard-Wiechert potentials in Chapter 14. (See equation 14.14.) You may wish to consult Problem #6.2 and section 6.5 of your text.

If you can prove them, you may wish to use some of the following results:

$$\int d^3r'dt'\delta^3(\mathbf{r'} - \mathbf{r_0}(t')\delta\left(t' - t + \frac{|\mathbf{r} - \mathbf{r\prime}|}{c}\right) = \int dt'\delta(t' - t + R(t')/c) = \frac{1}{1 - \mathbf{v_0}\cdot\mathbf{R}/cR}. \tag{2}$$

$$\frac{\partial t_r}{\partial t} = \frac{R}{R - \mathbf{v_0}\cdot\mathbf{R}/c} \tag{3}$$

$$\frac{\partial R(t_r)}{\partial t} = -\frac{\mathbf{v_0}\cdot\mathbf{R}}{R - \mathbf{v_0}\cdot\mathbf{R}/c}. \tag{4}$$