

PHY 712 – Problem Set # 9

Read Chapter 3 of **Jackson**.

1. Convince yourself that Eqs. 3.62 and 3.70 are correct by expanding the expressions to second order. That is, verify the following:

$$P_l(\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}') = \frac{4\pi}{2l+1} \sum_{m=-l}^l Y_{lm}^*(\hat{\mathbf{r}}) Y_{lm}(\hat{\mathbf{r}}') \quad (1)$$

and

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \sum_{l=0}^{\infty} \sum_{m=-l}^l \frac{4\pi}{2l+1} \frac{r^l}{r^{l+1}} Y_{lm}^*(\hat{\mathbf{r}}) Y_{lm}(\hat{\mathbf{r}}'). \quad (2)$$

2. Consider a charge distribution of the form:

$$\rho(r, \theta, \phi) = \rho_0 r^2 e^{-\alpha r} \cos^2(\theta), \quad (3)$$

where ρ_0 and α are constants.

- (a) Express $\rho(r, \theta, \phi)$ as a sum of radial functions time spherical harmonic functions in the form:

$$\rho(r, \theta, \phi) = \sum_{lm} \rho_{lm}(r) Y_{lm}(\theta, \phi). \quad (4)$$

- (b) Using the results of the previous problem, find the corresponding electrostatic potential which vanishes at ∞ .