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Notes for Lecture #2

Examples of solutions of the one-dimensional Poisson equation

Consider the following one dimensional charge distribution:

$$\rho(x) = \begin{cases} 0 & \text{for } x < -a \\ -\rho_0 & \text{for } -a < x < 0 \\ +\rho_0 & \text{for } 0 < x < a \\ 0 & \text{for } x > a \end{cases} \tag{1}$$

We want to find the electrostatic potential such that

$$\frac{d^2\Phi(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon_0}, \tag{2}$$

with the boundary condition $\Phi(-\infty) = 0$.

In class, we showed that the solution is given by:

$$\Phi(x) = \begin{cases} 0 & \text{for } x < -a \\ (\rho_0/(2\varepsilon_0))(x+a)^2 & \text{for } -a < x < 0 \\ -(\rho_0/(2\varepsilon_0))(x-a)^2 + (\rho_0 a^2)/\varepsilon_0 & \text{for } 0 < x < a \\ (\rho_0 a^2)/\varepsilon_0 & \text{for } x > a \end{cases} . \tag{3}$$

The electrostatic field is given by:

$$E(x) = \begin{cases} 0 & \text{for } x < -a \\ -(\rho_0/\varepsilon_0)(x+a) & \text{for } -a < x < 0 \\ (\rho_0/\varepsilon_0)(x-a) & \text{for } 0 < x < a \\ 0 & \text{for } x > a \end{cases} . \tag{4}$$

The electrostatic potential can be determined by piecewise solution within each of the four regions or by use of the Green's function $G(x, x') = x_<$, where,

$$\Phi(x) = \frac{1}{\varepsilon_0} \int_{-\infty}^{\infty} G(x, x')\rho(x')dx'. \tag{5}$$

In the expression for $G(x, x')$, $x_<$ should be taken as the smaller of x and x'. It can be shown that Eq. 5 gives the identical result for $\Phi(x)$ as given in Eq. 3.