March 21, 2001

Notes for Lecture #24

Reflectivity – Section 7.3 in Jackson's text

In Section 7.3 of your text, the electric field amplitudes of the reflected and transmitted electromagnetic waves are derived, for the cases of s-polarization ($\mathbf{E}$ perpendicular to the plane of incidence) and p-polarization ($\mathbf{E}$ parallel to the plane of incidence). The reflectivity can by determined from the ratio of the surface normal components of Poynting vectors. Denoting the surface normal as $\hat{\mathbf{z}}$ and using the text's notation the reflectivity is given by,

$$\mathcal{R} = Re\left\{\frac{\mathbf{S_r}\cdot\hat{\mathbf{z}}}{\mathbf{S_i}\cdot\hat{\mathbf{z}}}\right\} = \left|\frac{E_0''}{E_0}\right|^2. \tag{1}$$

Similarly, the transmittance is given by

$$\mathcal{T} = Re\left\{\frac{\mathbf{S_t}\cdot\hat{\mathbf{z}}}{\mathbf{S_i}\cdot\hat{\mathbf{z}}}\right\} = \left|\frac{E_0'}{E_0}\right|^2 \; Re\left\{\frac{(n'/\mu')^*\cos t}{(n/\mu)^*\;\cos i}\right\}. \tag{2}$$

It is apparent that these equations are consistent with energy conservation at the interface: $\mathcal{R}+\mathcal{T}=1$.

For the case that $\mu'=\mu$ and that the refractive indices are real, using the equations 7.39 and 7.41 and some algebra, we can show that the ratio of the reflectivity at p-polarization to that at s-polarization is:

$$\frac{\mathcal{R}_p}{\mathcal{R}_s} = \left|\frac{\tan^2 i - \sqrt{\left(\frac{n'}{n}\right)^2 + \tan^2 i\left[\left(\frac{n'}{n}\right)^2 - 1\right]}}{\tan^2 i + \sqrt{\left(\frac{n'}{n}\right)^2 + \tan^2 i\left[\left(\frac{n'}{n}\right)^2 - 1\right]}}\right|^2. \tag{3}$$

This ratio vanishes at the Brewster angle – $\tan i = \frac{n'}{n}$, and goes to 1 at normal incidence – $\tan i = 0$.