January 31, 2002

PHY 712 – Problem Set #3

1. Consider a three-dimensional charge distribution of the form:

$$\rho(\mathbf{r}) = \frac{q}{\pi^{3/2}a^3}\mathrm{e}^{-(r/a)^2}$$

where q and a are constants. In the following, you may which to use the result that for this particular charge density,

$$\frac{1}{4\pi\epsilon_0}\int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} = \frac{q}{4\pi\epsilon_0}\frac{\mathrm{erf}(r/a)}{r}.$$

   (a) Find the electrostatic potential $\Phi(\mathbf{r})$, as a function of the distance $\mathbf{r}$ from the center of the charge distribution $\rho(\mathbf{r})$.

   (b) Now suppose that a grounded metal plate is placed at a distance $-d\hat{\mathbf{z}}$ from the center of the charge distribution. Find the electrostatic potential due to $\rho(\mathbf{r})$ and the boundary condition $\Phi(x, y, z = -d) = 0$.

   (c) How would your result change if you maintained a potential V_0 on the grounded sheet? (That is that the charge distribution is still $\rho(\mathbf{r})$ and the boundary condition $\Phi(x, y, z = -d) = V_0$).