PHY 712 – Problem Set # 9

Read Chapter 3 of **Jackson**.

1. Convince yourself that Eqs. 3.62 and 3.70 are correct by expanding the expressions to second order. That is, verify the following:

$$P_l(\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}') = \frac{4\pi}{2l+1} \sum_{m=-l}^{l} Y_{lm}^*(\hat{\mathbf{r}}) Y_{lm}(\hat{\mathbf{r}}')$$
(1)

and

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{4\pi}{2l+1} \frac{r_{<}^{l}}{r_{>}^{l+1}} Y_{lm}^{*}(\hat{\mathbf{r}}) Y_{lm}(\hat{\mathbf{r}}').$$
(2)

2. Using these results consider the equation we had from Problem Set #5:

$$\frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} = \frac{q}{4\pi\epsilon_0} \frac{\operatorname{erf}(r/a)}{r}.$$
 (3)

Show how you can obtain this result.