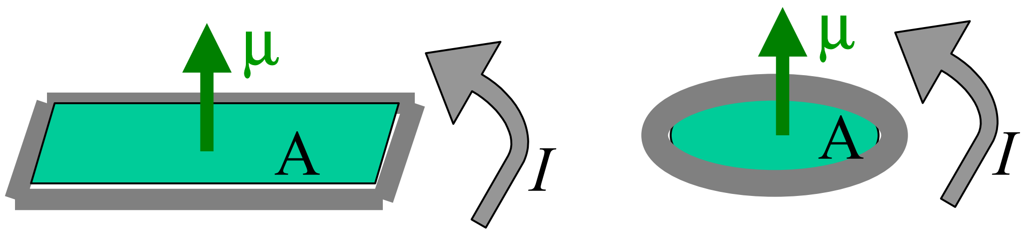
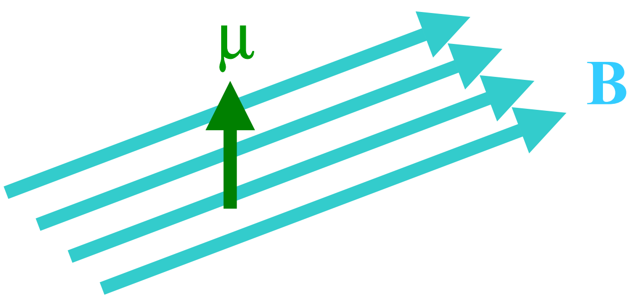
Announcements

1. Remember – Reworked exams are due now (11 AM on Wednesday (2/19/03)).
2. Presentation opportunities – after spring break if there is sufficient interest.   Collect additional problems for presentations...
3. Because of Monday’s snow day, due date for HW’s 12 & 13 are shifted to Friday and Monday, respectively. (HW 14 also due Monday.)
4. Today’s topics
   a. Magnetic dipole interactions
   b. Sources of magnetic fields
   c. Faraday’s law

Magnetic moment associated with current loop:



$\mu = NI\,\mathbf{A}$



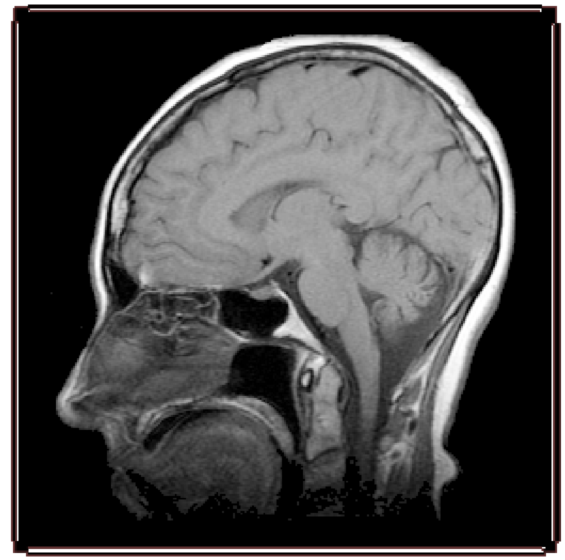
Torque:

$\tau = \mu \times \mathbf{B}$

Potential energy:

$U = -\mu \cdot \mathbf{B}$

MRI – Magnetic resonance imaging

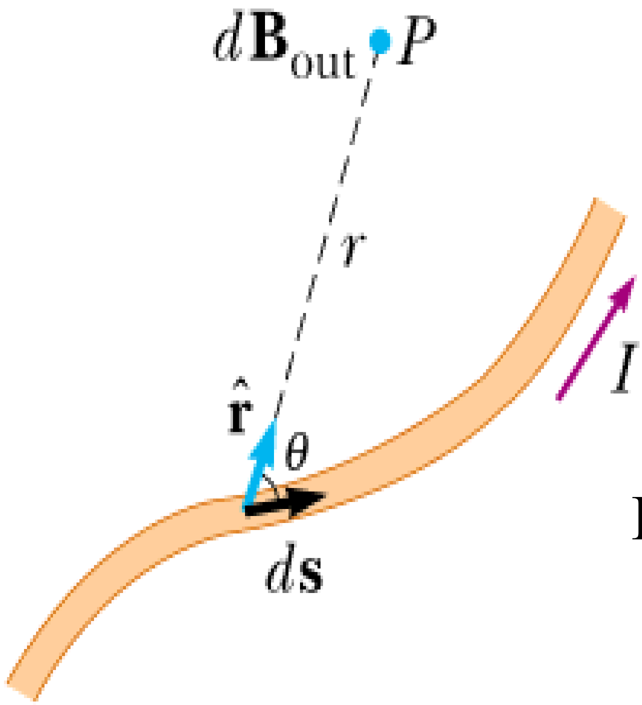


$U = -\mu \cdot \mathbf{B}$ plus time varying B_1 ➔ signal from μ of protons

Ref: http://www.cis.rit.edu/htbooks/mri/inside.htm

Sources of magnetic field – currents

Biot-Savart law

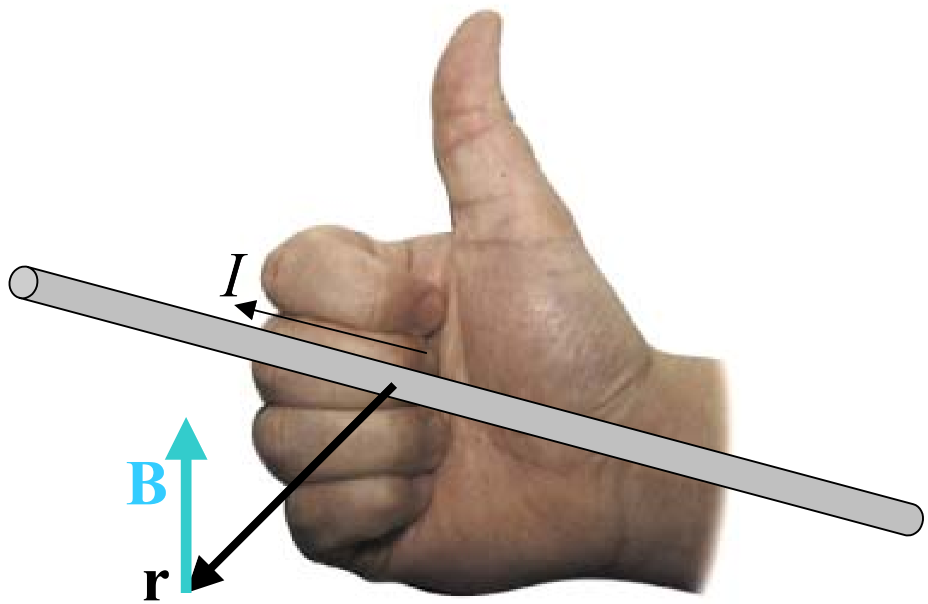


$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$

Field from a single moving charge:

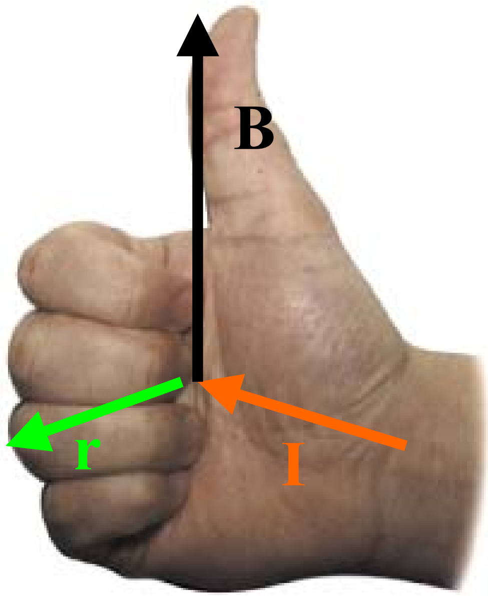
$$\mathbf{B} \approx \frac{\mu_0}{4\pi}\frac{q\mathbf{v}\times\hat{\mathbf{r}}}{r^2}$$

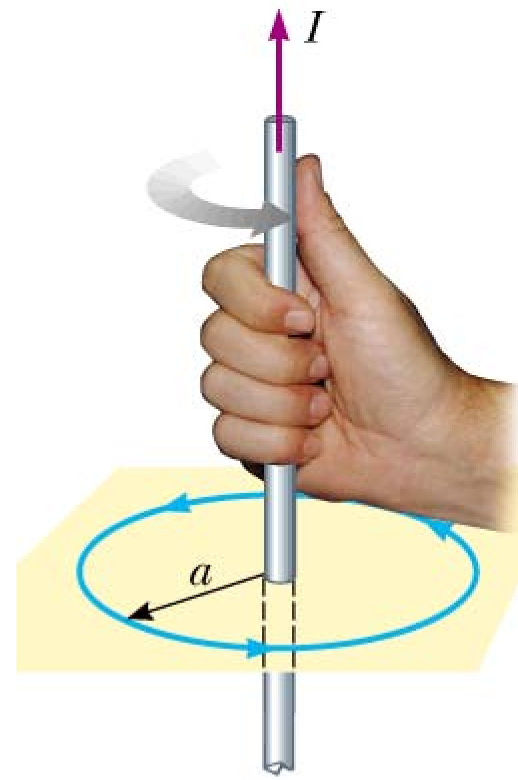
$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Id\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$



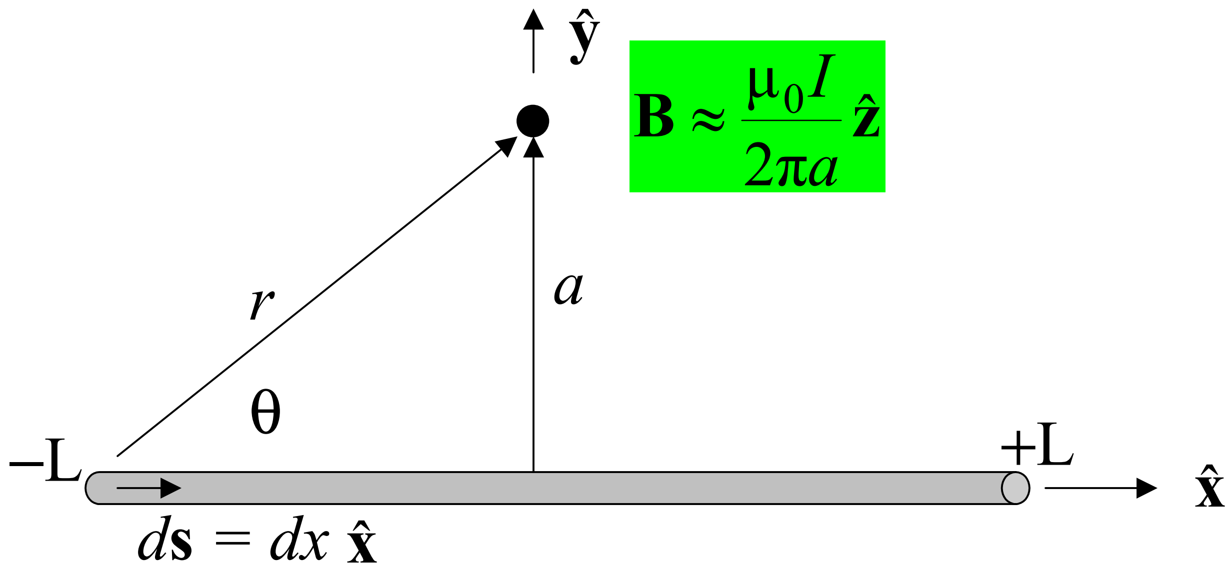
Digression on the right-hand rule:

$\mathbf{B}$	$\rightarrow$ $\mathbf{I}$	$\times$ $\mathbf{r}$
thumb	palm	fingers
palm	fingers	thumb
fingers	thumb	palm





Integrating the Biot-Savart equation

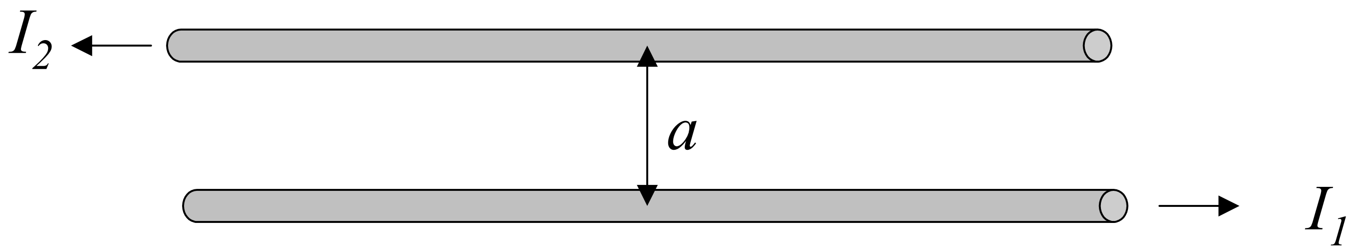


$$\mathbf{B} \approx \frac{\mu_0 I}{2\pi a}\hat{\mathbf{z}}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{I d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} = \frac{\mu_0 I}{4\pi}\hat{\mathbf{z}}\int_{-L}^{+L} dx \frac{1}{x^2 + a^2}\underbrace{\left(\frac{a}{\sqrt{x^2 + a^2}}\right)}_{\sin\theta} = \frac{\mu_0 I}{2\pi a}\hat{\mathbf{z}}\underbrace{\left(\frac{L}{\sqrt{L^2 + a^2}}\right)}_{\approx 1}$$

(when L → ∞)

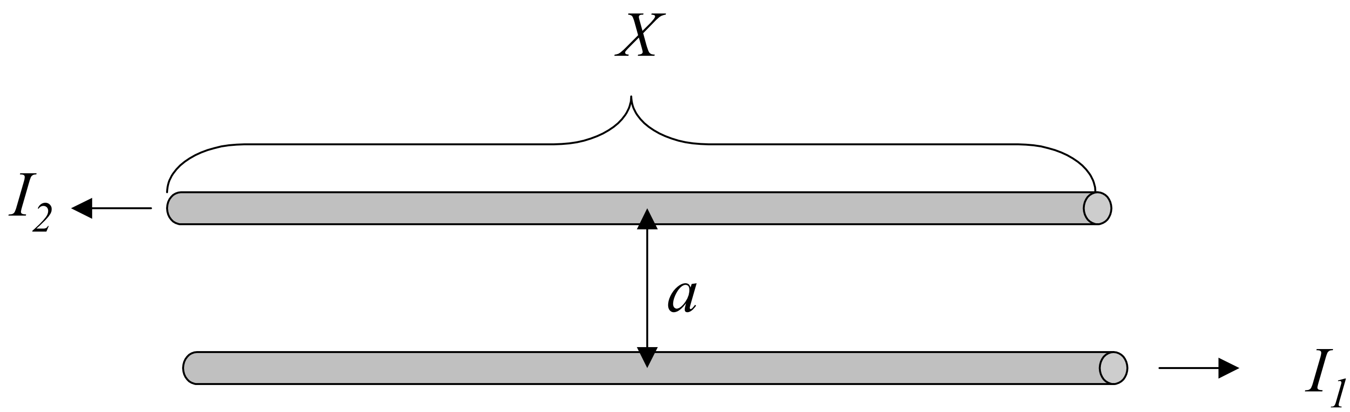
Peer instruction question



Two long wires with currents I_1 and I_2 are separated by a vertical distance a as shown. What is the direction of the magnetic force that I_1 exerts on I_2?

(A) upward (B) downward (C) out of the screen

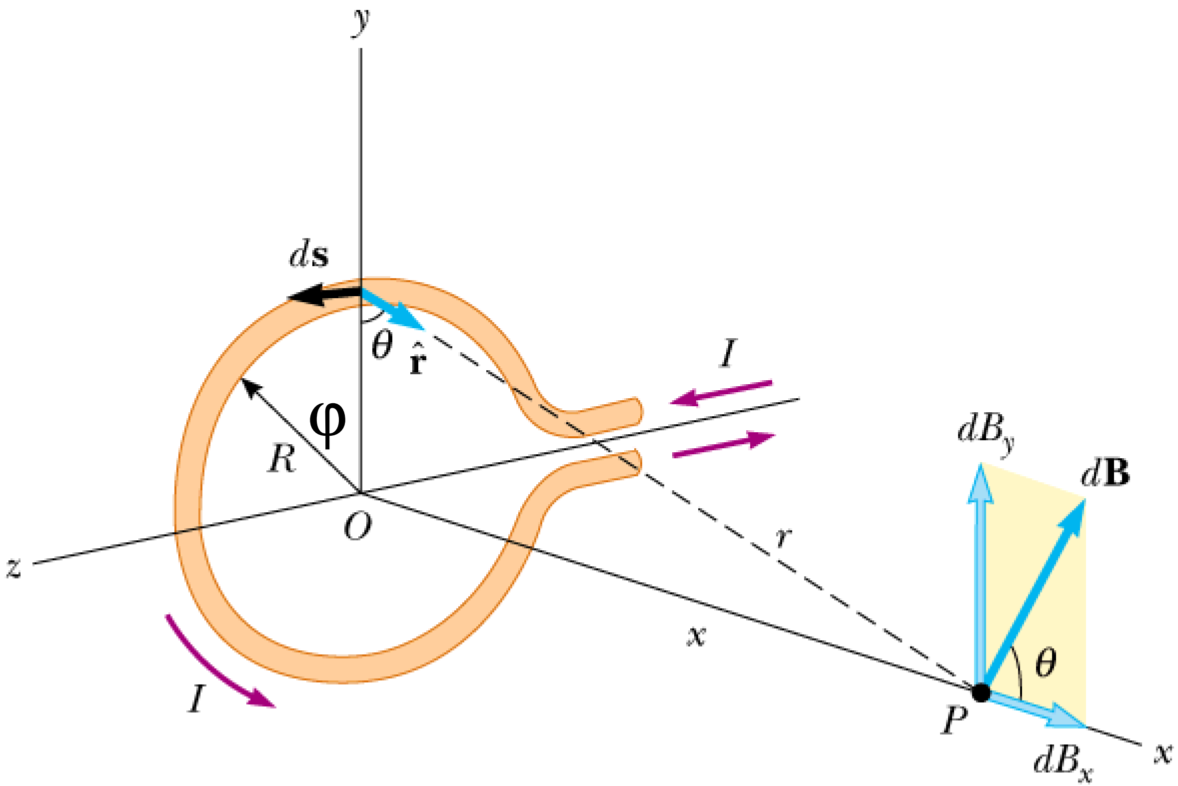
(D) to the left



Magnitude of the magnetic force between two parallel wires:

$$F = \frac{\mu_0 I_1 I_2 X}{2\pi a}$$

Magnetic field from current loop:

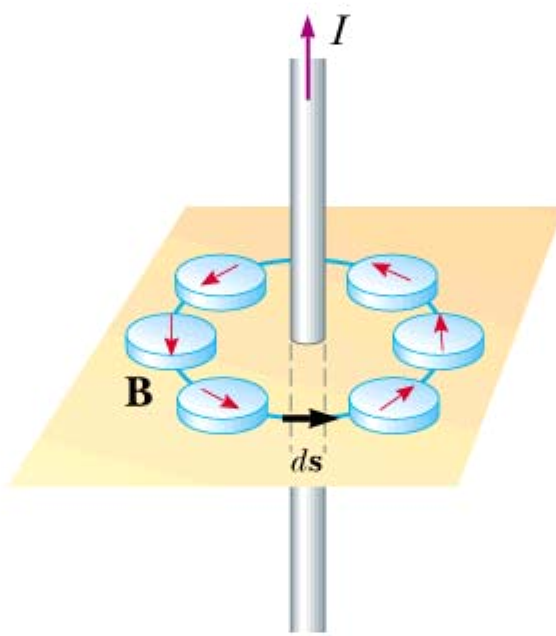


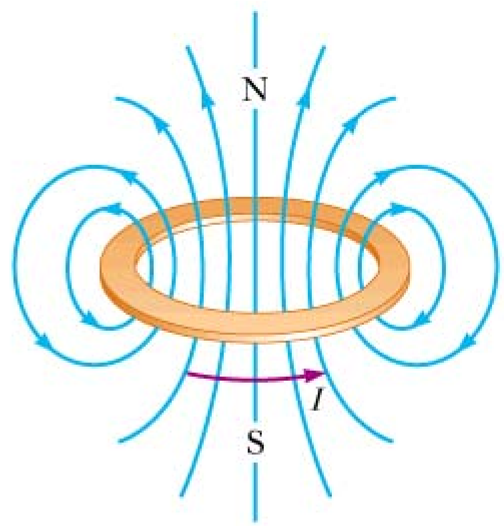
$$\mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{I d\mathbf{s}\times\hat{\mathbf{r}}}{r^2} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} \frac{R d\varphi}{x^2+R^2}\left(\underbrace{\frac{R}{\sqrt{x^2+R^2}}}_{\cos\theta}\right)\hat{\mathbf{x}} = \frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}\,\hat{\mathbf{x}}$$

Visualizing **B** field lines using iron filings

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

filings move until $\tau = 0$ ➔ filings align along **B** field lines



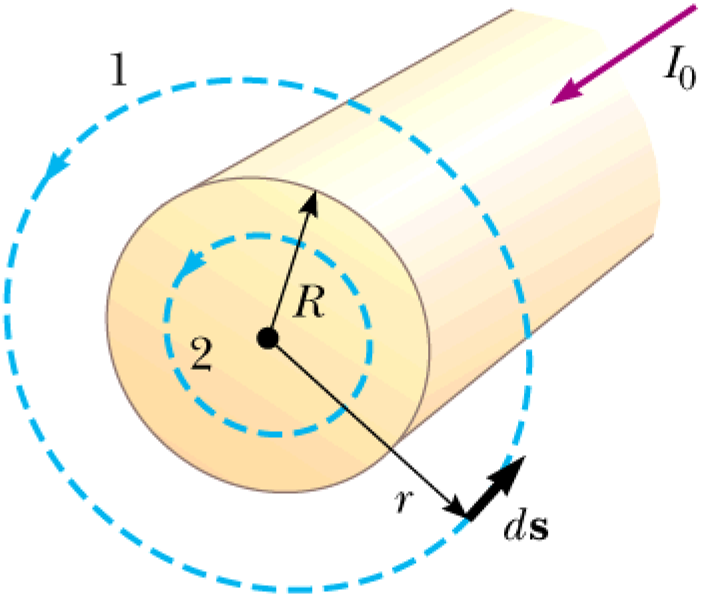


Ampere's law

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{in}$$

1. For $r > R$:

$$B(2\pi r) = \mu_0 I_0 \qquad \Rightarrow \quad B = \frac{\mu_0 I_0}{2\pi r}$$



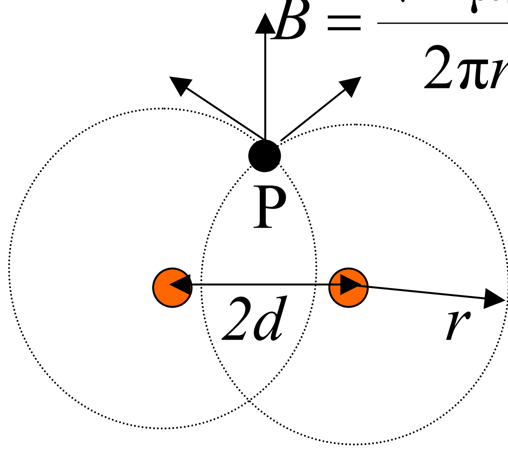
2. For $r < R$:

$$B(2\pi r) = \mu_0 I_0 \frac{r^2}{R^2} \qquad \Rightarrow \quad B = \frac{\mu_0 I_0 r}{2\pi R^2}$$

Field due to two wires:

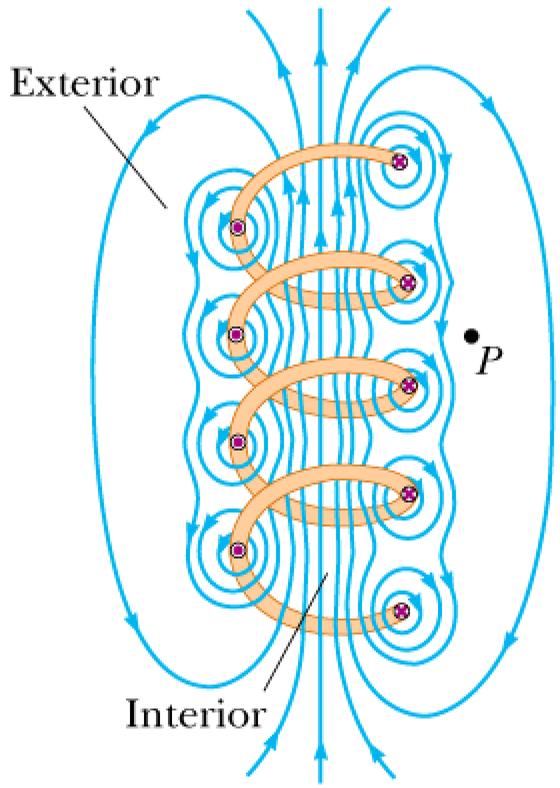
Suppose that there are two wires perpendicular to the screen both with currents flowing into the screen. What is the magnitude and direction of the magnetic field at the point P?

$$B = \frac{\sqrt{2}\,\mu_0 I}{2\pi r} = \frac{\sqrt{2}\,\mu_0 I}{2\pi\sqrt{d^2 + P^2}}$$

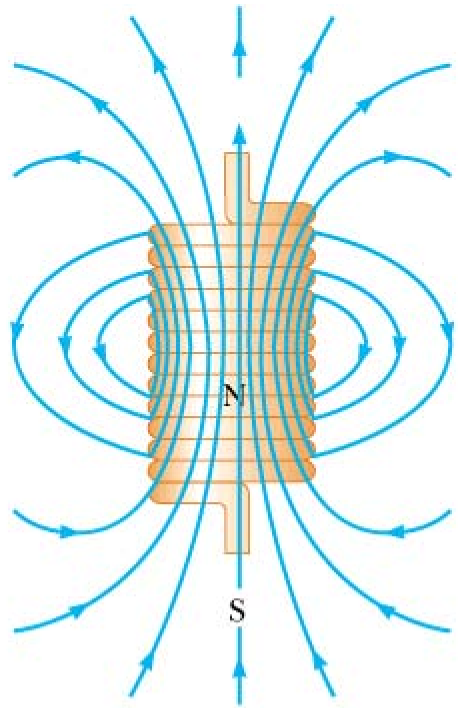


Magnetic field in the solenoid geometry

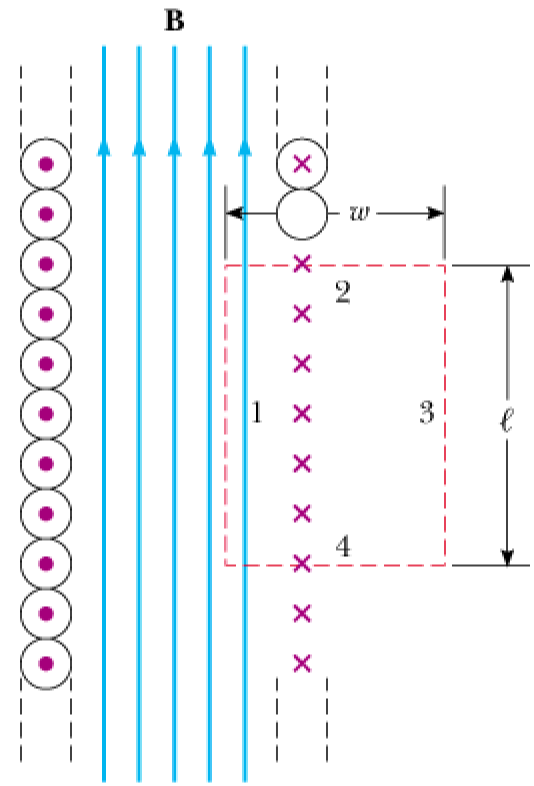
$B_{\text{interior}} = \mu_0 n I$ number of coils/unit length



Helical form

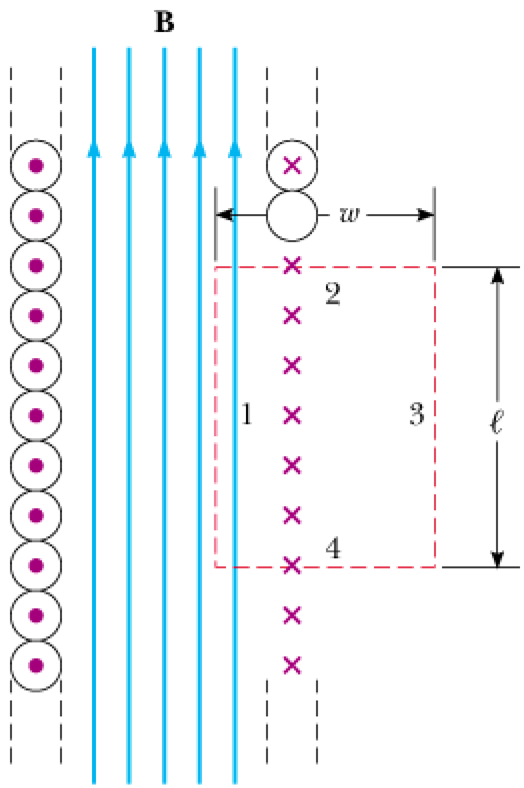


Tight coil form



Ideal form

Details of the solenoid field:



Ampere's law:

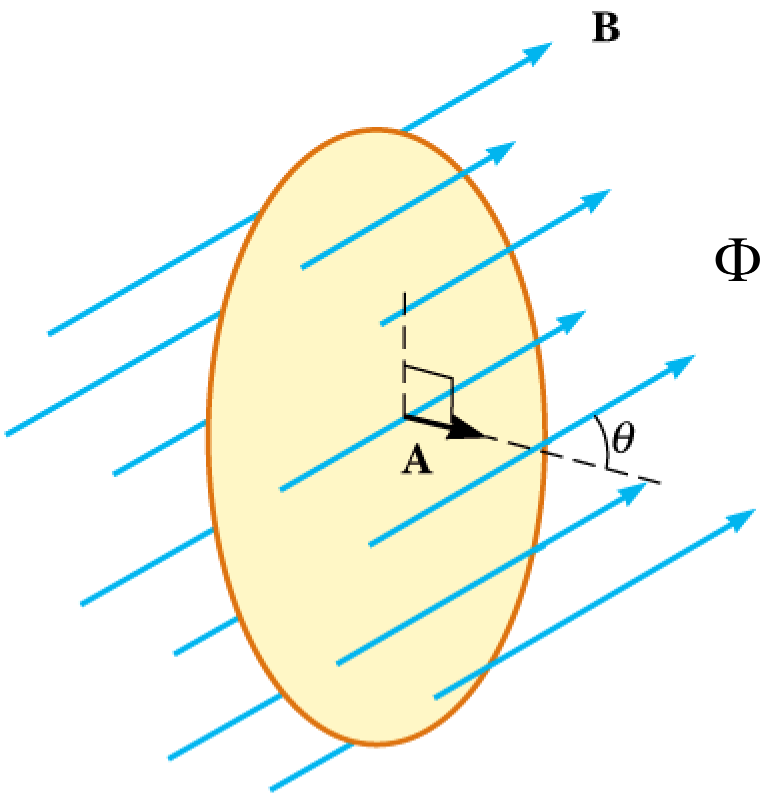
$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in}$$

$$B_{in} l + 0w + 0l = \mu_0 NI$$

$$B_{in} = \frac{\mu_0 NI}{l} = \mu_0 nI$$

$$n \equiv N/l$$

Magnetic flux:



$$\Phi_B \equiv \int \mathbf{B} \cdot d\mathbf{A} = BA\cos\theta$$

Faraday's law:

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

➔A changing magnetic flux produces an emf!

Faraday's law:

$$\frac{d\Phi_B}{dt} = \frac{d\int \mathbf{B}\cdot d\mathbf{A}}{dt} = \frac{d\left(BA\cos\theta\right)}{dt} = -\boldsymbol{\varepsilon}$$

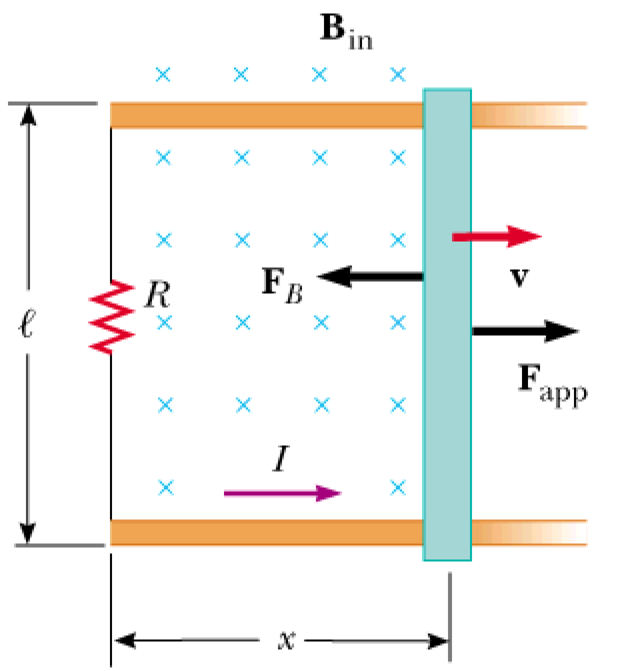
$$\frac{dB}{dt}A\cos\theta + B\frac{dA}{dt}\cos\theta - BA\sin\theta\frac{d\theta}{dt} = -\boldsymbol{\varepsilon}$$

generator

metal detector

inductors

Simple example:



$$\Phi_B = Blx$$

$$\frac{d\Phi_B}{dt} = Blv = -\boldsymbol{\varepsilon}$$

(induced current creates magnetic field opposing B_{in})