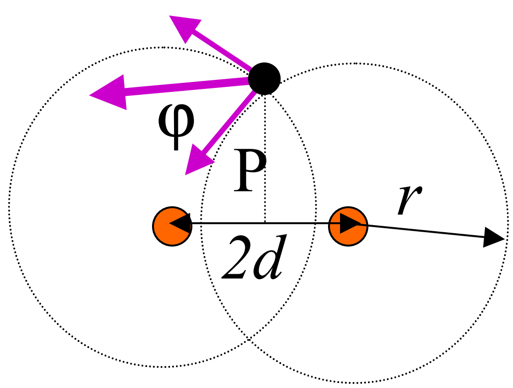
Announcements

1. Problem sets 13 & 14 due on Monday 2/24/03
2. Second exam – Chap. 29-33 – week of March 3rd.
   May pick up exam from physics office between
   9 AM - 5 PM on Monday 3/3 through Thursday
   3/6. Completed exam (**within guidelines of the honor code**) must be turned into the physics office within **24 hours** of when it was picked up.
   (NAWH will be out of town 3/1-3/8.)
3. Mid-term grades will not include 2nd exam.
4. Today's topics –
   Faraday's law
   Inductance

Corrections from Wednesday's lecture –

Field due to two wires:

Suppose that there are two wires perpendicular to the screen both with currents flowing into the screen. What is the magnitude and direction of the magnetic field at the point P?



$$B = \frac{\mu_0 I}{2\pi r} 2\cos\varphi = \frac{2\mu_0 I}{2\pi\sqrt{d^2+P^2}} \frac{P}{\sqrt{d^2+P^2}}$$

Faraday's law:

Define magnetic flux: $\Phi_B \equiv \int \mathbf{B} \cdot d\mathbf{A}$

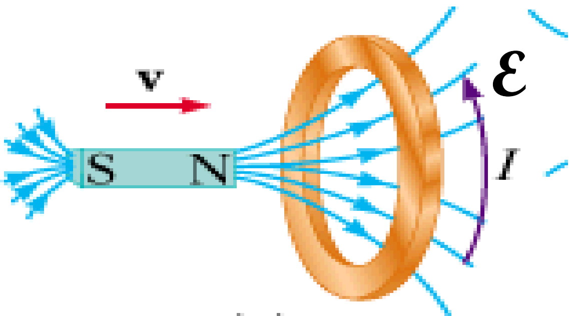
Changing flux induces an emf according to:

$$\boldsymbol{\mathcal{E}} = -\frac{d\Phi_B}{dt}$$

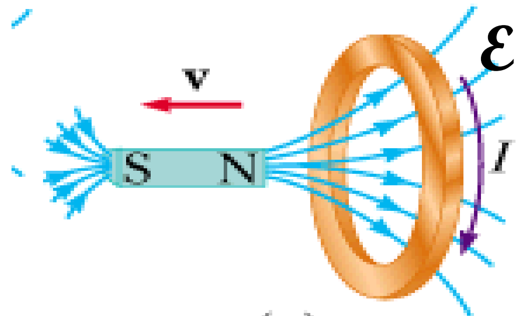
This emf has the same effects on a circuit as an emf from a battery, whoever it can also exist in free space (without the circuit).

Example:

Bar magnetic moving toward (or way) from metal loop inducing emf $\mathcal{E}$ and current I.



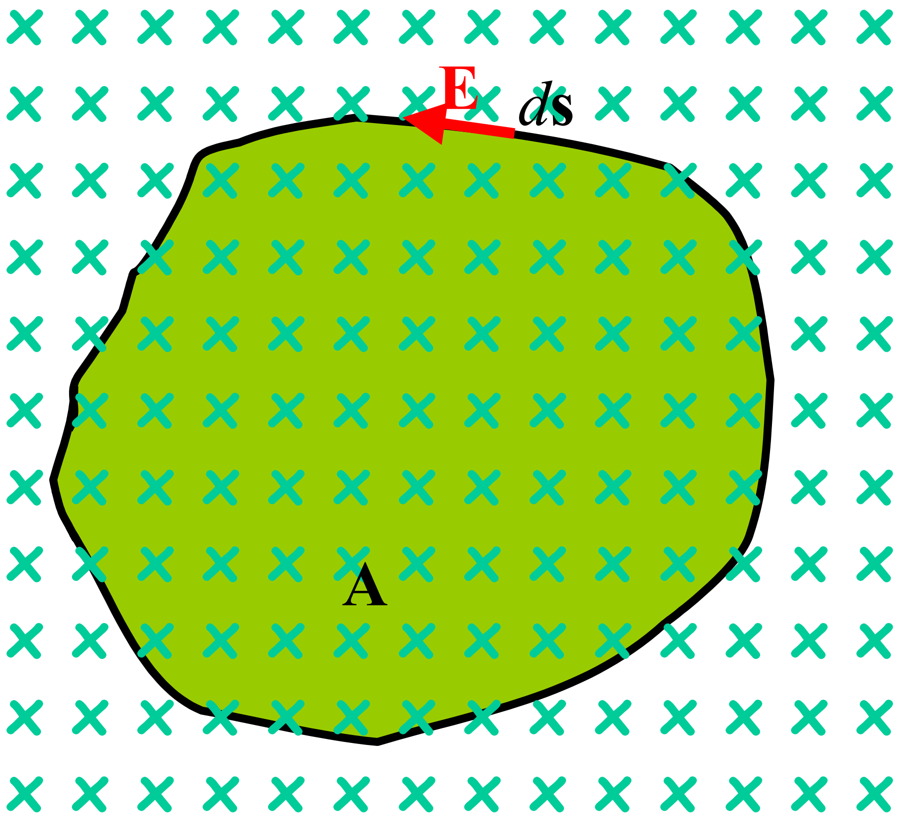
(a)



(c)

Expressing Faraday's law in terms of induced electric field

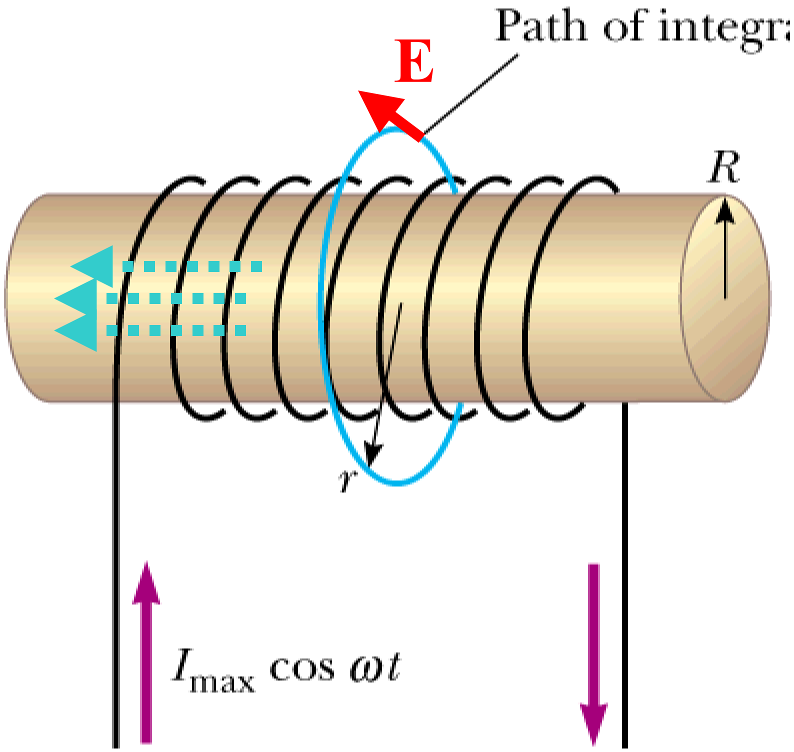
$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}$$



B pointing into screen and changing with time.

Example:

Changing **B** produced by changing I in a solenoid geometry:



$$\oint \mathbf{E}\cdot d\mathbf{s} = E(2\pi r)$$

$$\mathbf{B} = \mu_0 \,\mathrm{n}\, I$$

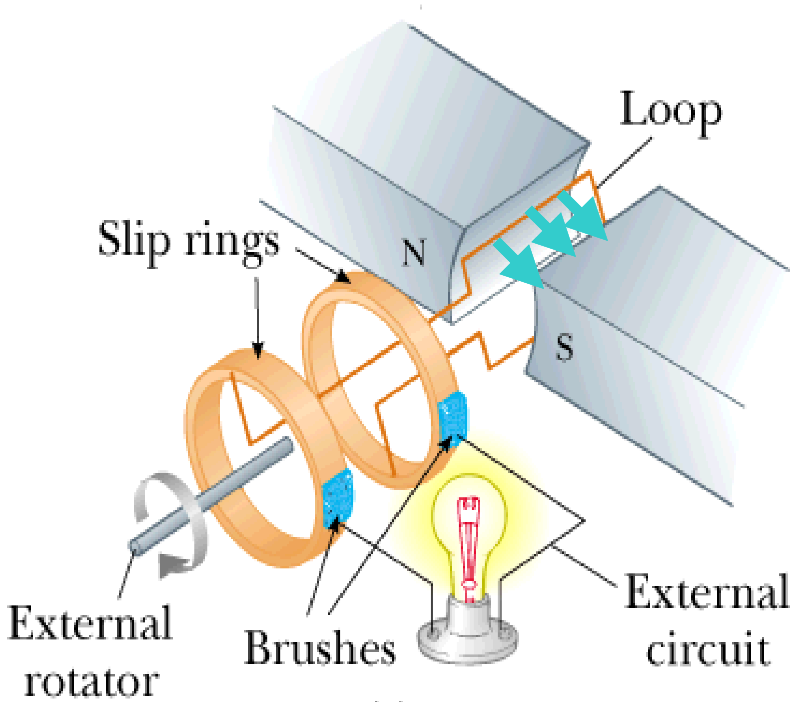
$$\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} = \frac{d}{dt}\left(\mu_0 n I \ \ \pi R^2\right)$$

$$= \mu_0 n \frac{dI}{dt} \ \ \pi R^2$$

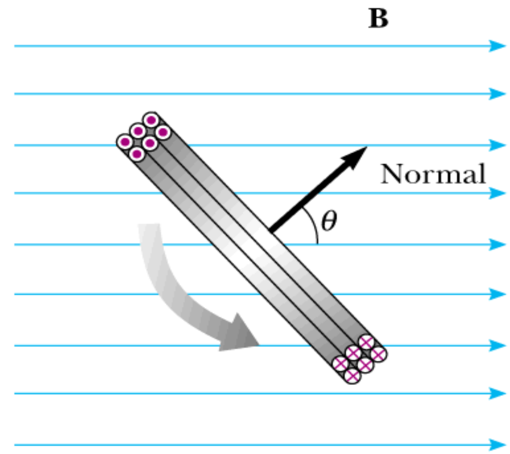
$$E = (\text{constant})\frac{\sin \omega t}{r}$$

$$(\text{constant}) = \frac{1}{2}\mu_0 n I_{\max}\omega R^2$$

Example: AC generator



(a)

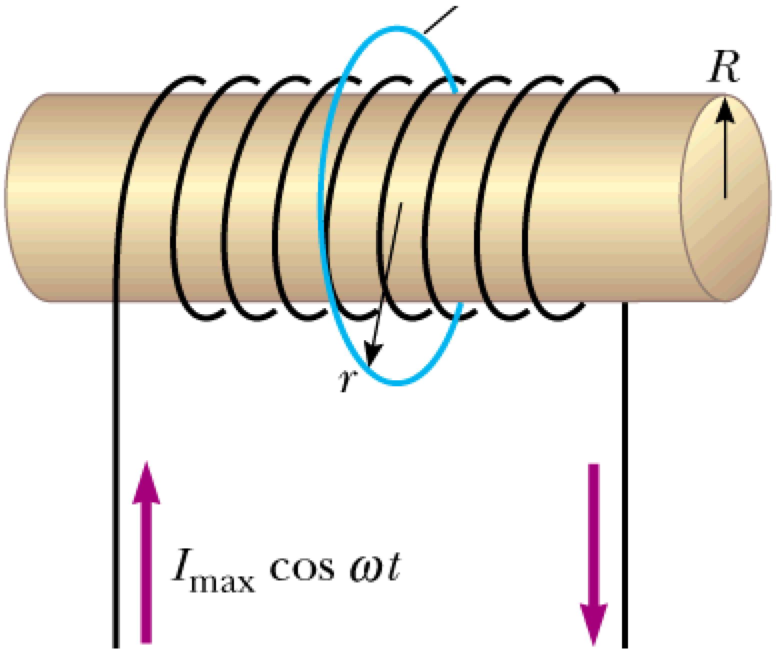


$$\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} = \frac{d}{dt}\left(BA\cos\omega t\right)$$
$$= -\omega BA\sin\omega t$$

$$\Rightarrow \mathcal{E} = \underbrace{\omega BA}_{\mathcal{E}_{\max}}\sin\omega t$$

Inductance as a circuit component:

Recall the solenoid geometry:



when $r = R$, selenoid induces emf in itself ("self inductance").

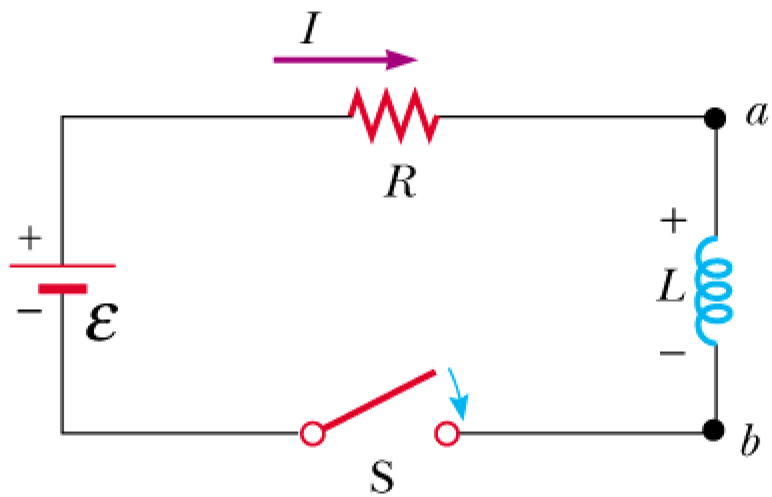
For each coil of the solenoid:

$$\mathcal{E} = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} = -\frac{d}{dt}\left(\mu_0 n I \;\; \pi R^2\right)$$

$$= -\mu_0 n \,\pi R^2 \frac{dI}{dt}$$

$$\mathcal{E}_{\text{total}} = -\underbrace{N\mu_0 n \,\pi R}_{L}{}^2 \frac{dI}{dt}$$

Inductors in a circuit:

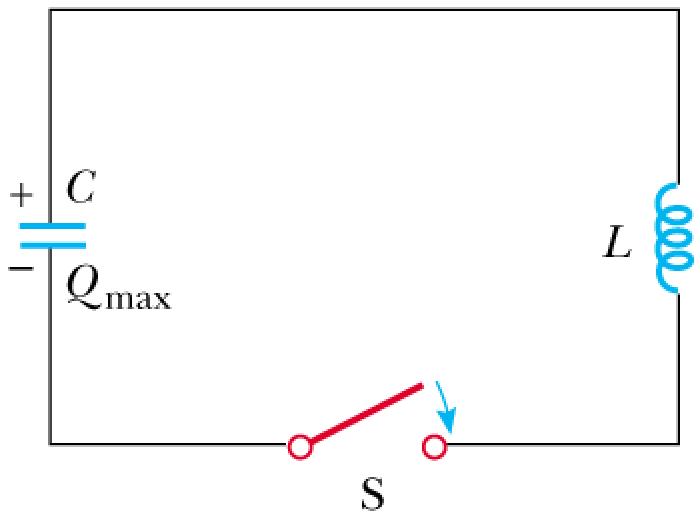


$$\mathcal{E}_{\text{battery}} - IR - L\frac{dI}{dt} = 0$$

solution for $I(t)$ assuming $I(t=0)=0$:

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/(L/R)}\right)$$

LC – circuits:



$$-\frac{q}{C} - L\frac{dI}{dt} = 0$$

$$\text{or:} \quad -\frac{q}{C} - L\frac{d^2q}{dt^2} = 0$$

Peer instruction question

The LC circuit equation can be written in the form:

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q$$

What does this remind of?

(A) A bad dream.

(B) A point mass moving in response to a constant force.

(C) A mass on a spring.

(D) A charged mass moving in a magnetic field.

Solutions to the LC circuit equations

Recall that:

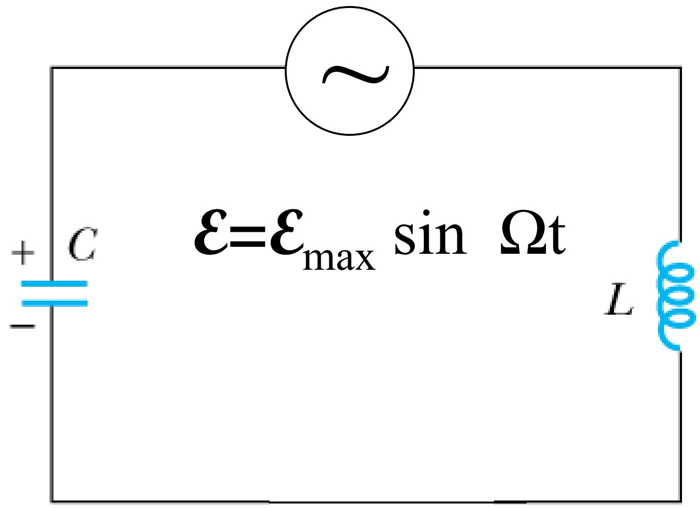
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad \Rightarrow\ x(t) = X_0 \cos(\omega t + \varphi)\ \text{ with }\ \omega \equiv \sqrt{\frac{k}{m}}$$

Therefore:

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q \qquad \Rightarrow\ q(t) = Q_0 \cos(\omega t + \varphi)\ \text{ with }\ \omega \equiv \sqrt{\frac{1}{LC}}$$

$$I(t) = \frac{dq}{dt} = -\omega Q_0 \sin(\omega t + \varphi)$$

Challenge question:



$\Omega=50\pi$ rad/s

L=0.03 H

C=0.0003 F

$\mathcal{E}_{max}$=120V

What is the largest charge on the capacitor and how many times a second does the capacitor have that charge?