Announcements

1. Plan for next week – no class, take-home exam
2. Extra problem session? Thursday 9 PM?
   - Friday 3 PM?
3. Today's topics

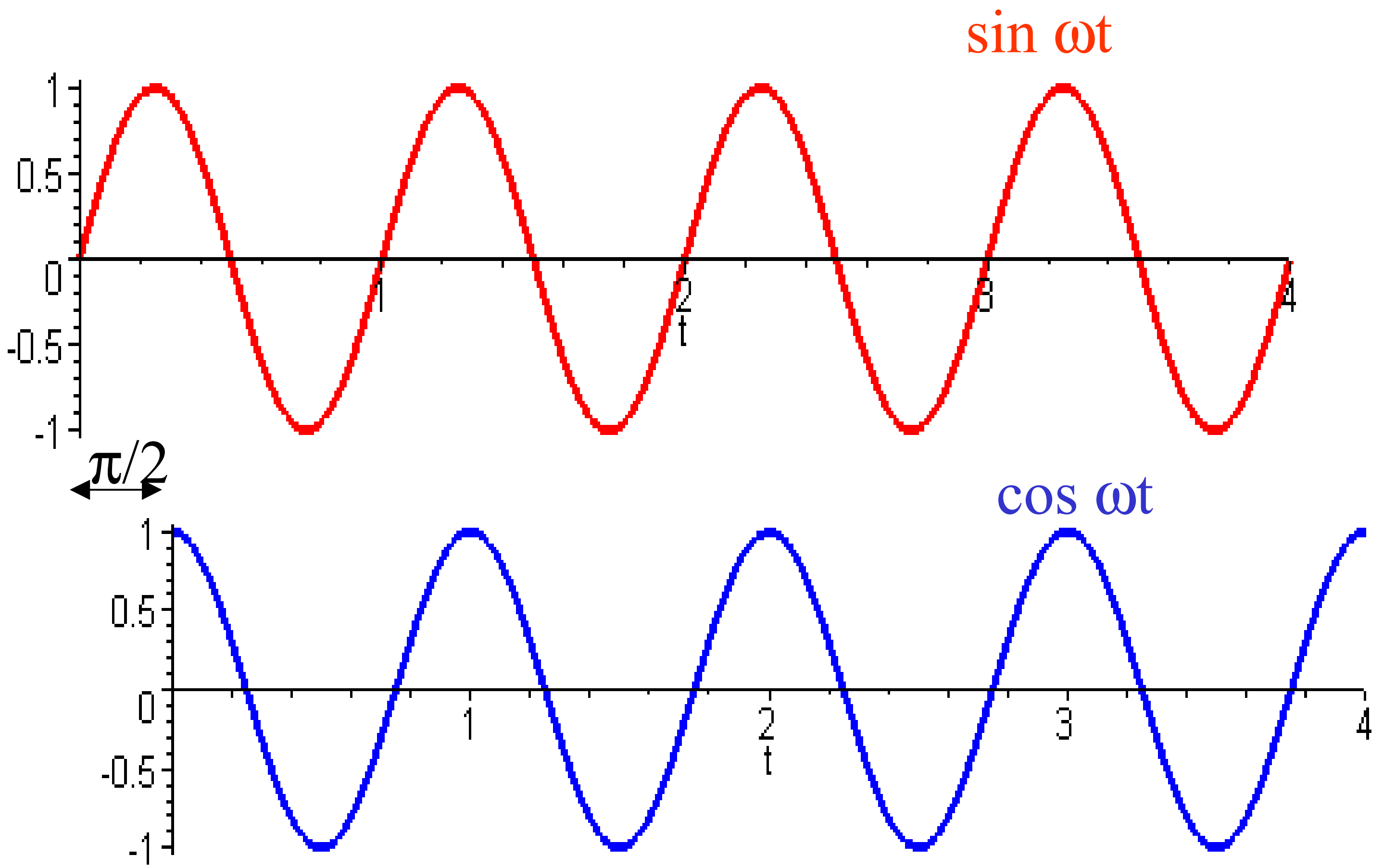
   AC circuits

   Transformers
4. Convocation Thursday – former Senator, Presidential candidate Bill Bradley will speak

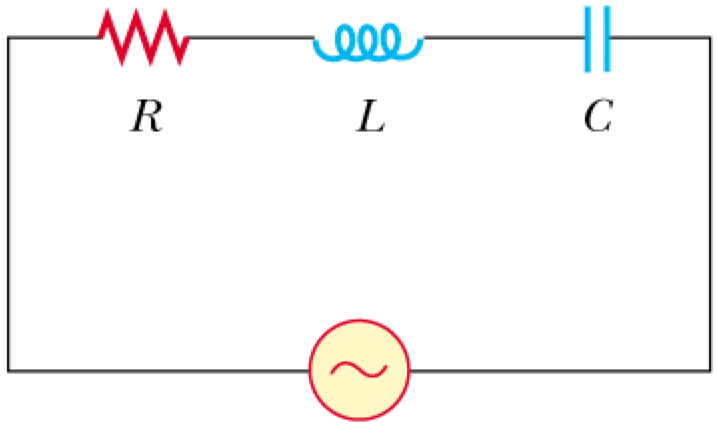
Important facts to remember in analyzing AC circuits

- $\Delta V_{max} = \sqrt{2}\ \ \Delta V_{rms}$
- Steady-state currents and voltages will have sinusoidal time dependence such as sin ωt or cos ωt or a linear combination of the two.
- Inductors and capacitors change the "phase" of the current relative to that of the emf.
- Solution of the circuit equations for *I(t)* → equations must be satisfied for all times *t*.

DC circuits	AC circuits
$\mathcal{E}$ = constant in time	$\mathcal{E} = \mathcal{E}_{max} \sin \omega t$ or $\mathcal{E}_{max} \cos \omega t$
$I = \begin{cases} \text{constant in time} \\ \text{constant} + I_0 e^{-t/\tau} \\ \text{damped oscillations} \end{cases}$	I= transients + $I_0 \sin(\omega t - \phi)$
Kirchhoff's rules apply	Kirchhoff's rules apply


sin ωt
1
0.5
0
-0.5
-1
1
2
t
3
4
π/2
cos ωt
1
0.5
0
-0.5
-1
1
2
t
3
4

Example



could also be cos ωt

Kirchhoff's rule:

$$-RI - L\frac{dI}{dt} - \frac{Q}{C} + \Delta V_{\max} \sin \omega t = 0$$

Differential equation:

$$-RI - L\frac{dI}{dt} - \frac{Q}{C} + \Delta V_{\max}\sin\omega t = 0$$

Solution form: $I(t) = I_{\max}\sin(\omega t - \varphi)$

Strategy: substitute form into differential equation and demand the equality must be true at all times.

$$-RI_{\max}\sin(\omega t - \varphi) - \omega L I_{\max}\cos(\omega t - \varphi) + \frac{1}{\omega C} I_{\max}\cos(\omega t - \varphi)$$
$$+ \Delta V_{\max}\sin\omega t = 0$$

$$-RI_{\max}\sin(\omega t-\varphi)-\omega LI_{\max}\cos(\omega t-\varphi)+\frac{1}{\omega C}I_{\max}\cos(\omega t-\varphi)$$
$$+\Delta V_{\max}\sin\omega t=0$$

Strategy: group terms as factors of sin ωt and cos ωt, noting that:

$$\sin(\omega t-\varphi)=\left(\sin(\omega t)\cos\varphi-\cos(\omega t)\sin\varphi\right)$$
$$\cos(\omega t-\varphi)=\left(\cos(\omega t)\cos\varphi+\sin(\omega t)\sin\varphi\right)$$

$$\left(-RI_{\max}\cos(\varphi)-\omega LI_{\max}\sin(\varphi)+\frac{1}{\omega C}I_{\max}\sin(\varphi)+\Delta V_{\max}\right)\sin\omega t=0$$

$$\left(+RI_{\max}\sin(\varphi)-\omega LI_{\max}\cos(\varphi)+\frac{1}{\omega C}I_{\max}\cos(\varphi)\right)\cos\omega t=0$$

$$\Rightarrow\varphi=\tan^{-1}\left(\frac{\omega L-\frac{1}{\omega C}}{R}\right)\qquad I_{\max}=\frac{\Delta V_{\max}}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

Summary:

$$-RI - L\frac{dI}{dt} - \frac{q}{C} + \Delta V_{\max}\sin\omega t = 0$$

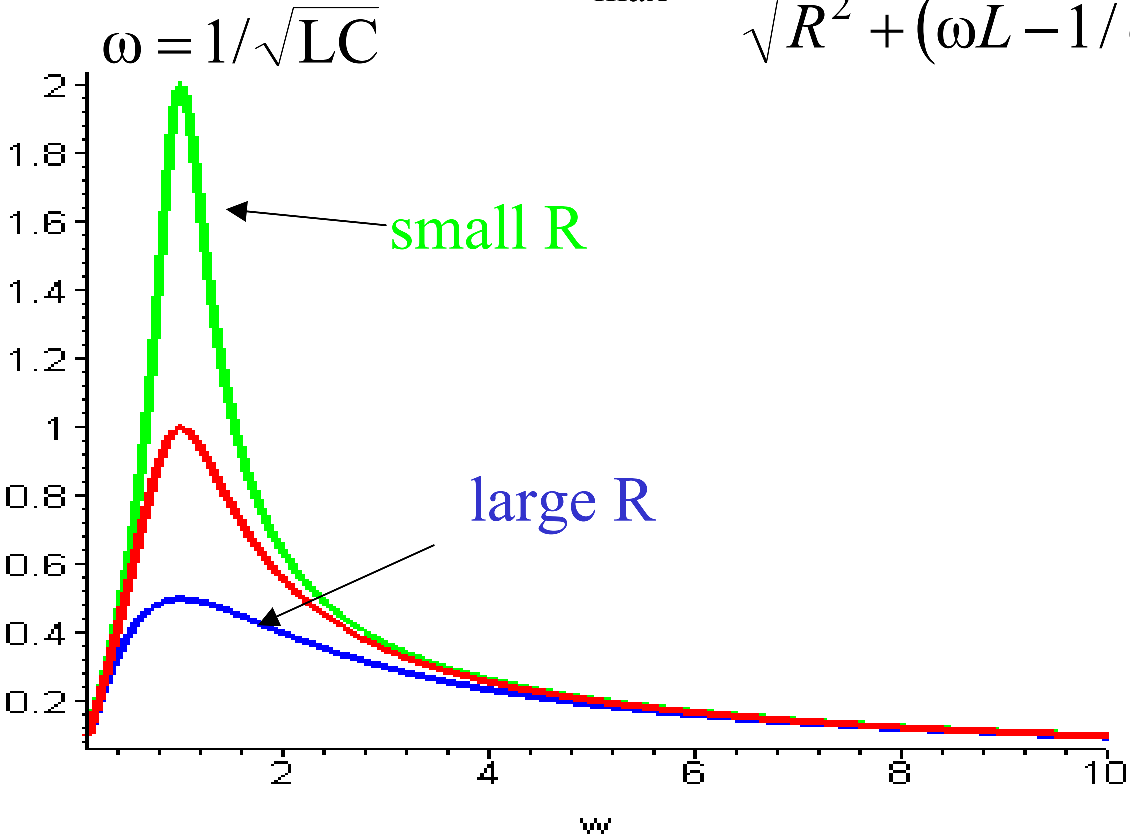
$$I = I_{\max}\sin(\omega t - \varphi)$$

$$I_{\max} = \frac{\Delta V_{\max}}{Z} \qquad Z \equiv \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

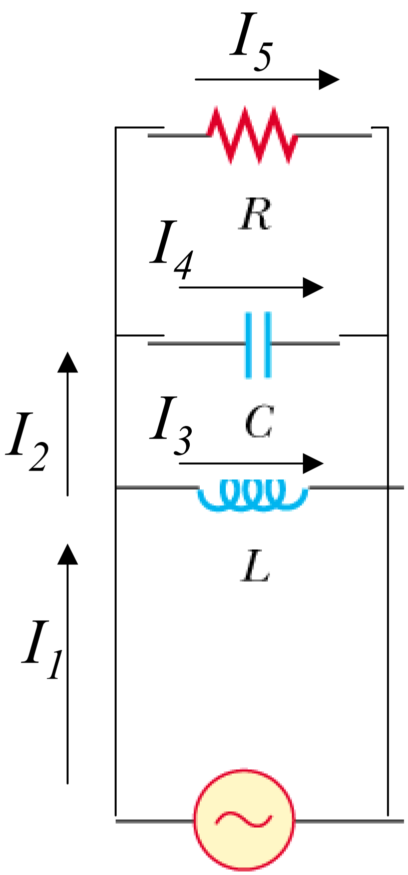
$$\tan\varphi = \frac{\omega L - 1/\omega C}{R}$$

Behavior of I_{max}

$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$



More complicated circuits can be analyzed in a similar way



$$-L\frac{dI_3}{dt}+\Delta V_{\max}\sin\omega t=0$$

$$-L\frac{dI_3}{dt}+\frac{Q_4}{C}=0$$

$$\frac{Q_4}{C}-RI_5=0$$

$$I_1=I_2+I_3$$

$$I_2=I_4+I_5$$

PHY 230 -- Electronics

Power delivered by generator in a circuit:

$\mathcal{P}(t) = I(t)\ \Delta V(t) = I_{max} \sin(\omega t - \phi)\ \Delta V_{max} \sin(\omega t)$

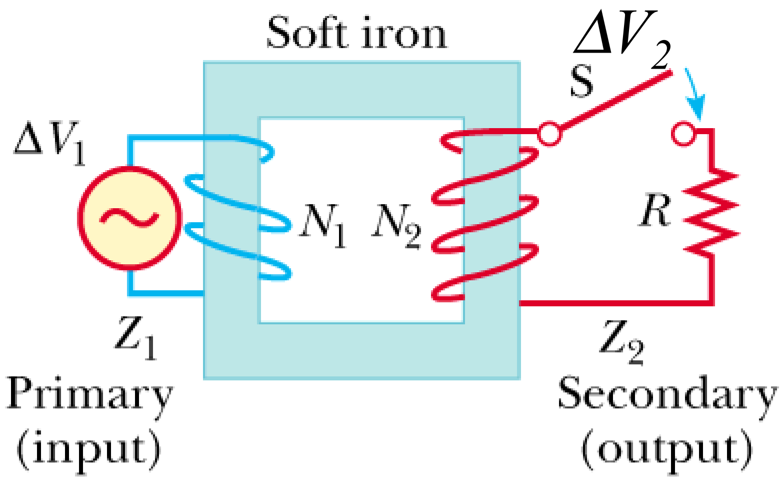
$\langle \mathcal{P} \rangle_{avg} = I_{max}\ \Delta V_{max}\ \frac{1}{2}\ \cos(\phi)$

Note that:

$$\cos\varphi = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

➔ $\langle \mathcal{P} \rangle_{avg} = \frac{1}{2}\ R\ I_{max}{}^2 = R\ I_{rms}{}^2$

Transformer devices



$$\Delta V_1 = -N_1 \frac{d\Phi_B}{dt}$$

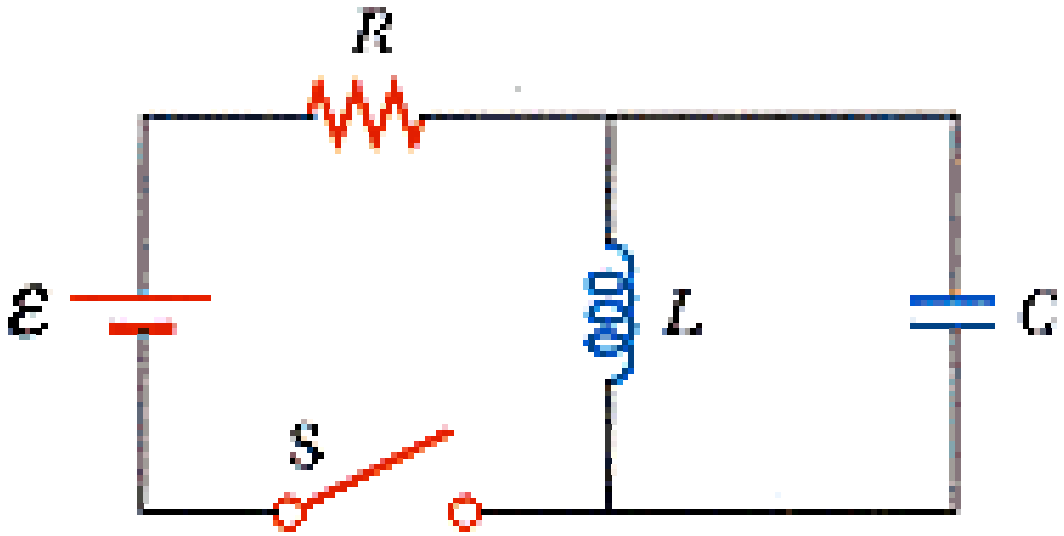
$$\Delta V_2 = -N_2 \frac{d\Phi_B}{dt}$$

$$-\frac{d\Phi_B}{dt} = \frac{\Delta V_1}{N_1} = \frac{\Delta V_2}{N_2}$$

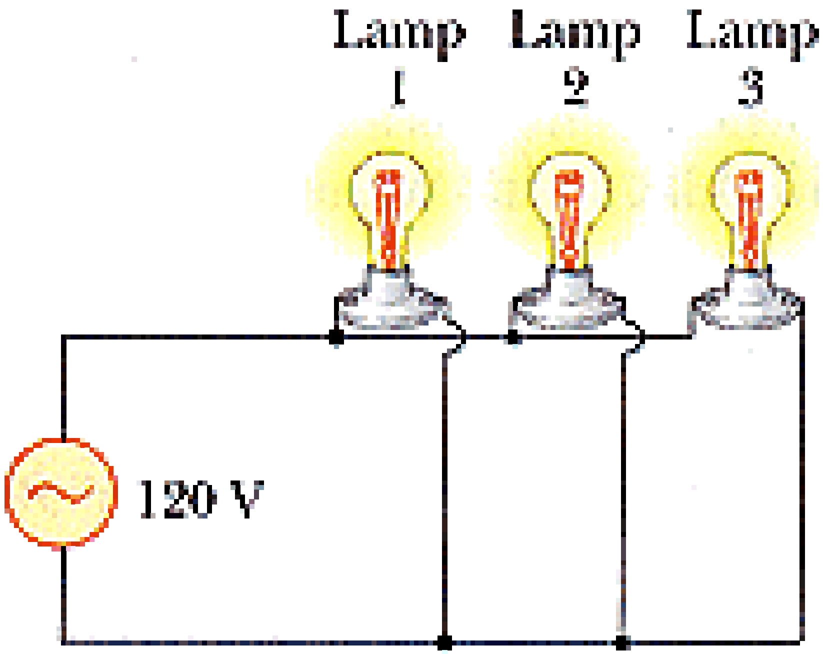
$$\Delta V_2 = \frac{N_2}{N_1}\Delta V_1$$

➔ step up or step down the voltage

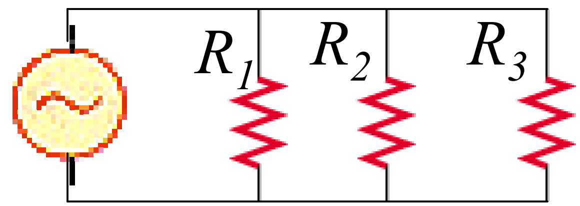
Homework problem -- DC circuits



Homework problem – AC circuits



➔



$$I_{rms} = \frac{\Delta V_{rms}}{Z}$$

$$Z = R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$