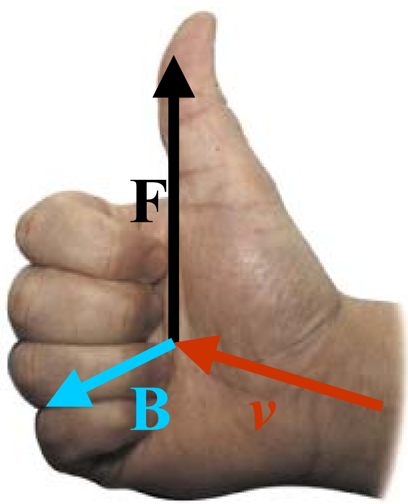
Announcements

1. Next week (Mar. 3-7) –
   No lectures (NAWH will be out of town 3/1-3/8.)
   Second exam – covering Chap. ≤ 33
      May pick up exam from physics office between 9 AM - 5 PM (except during lunch hour) starting Monday 3/3. Completed exam (**within guidelines of the honor code**) must be turned into the physics office within **24 hours** of when it was picked up. **Last deadline for turning in exam is 5 PM Friday, 3/7.** You may use your text and notes (including your HW solutions) but no other resources for working the exam.
2. HW set 16 – deadline extended to Mar. 3 due to power outages.
3. Extra problem solving session ?? Today at 3 PM????

Magnetic forces -- moving charges and currents

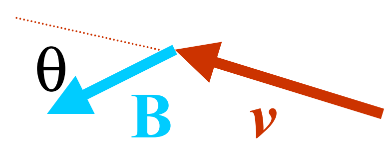
Right hand rule:



$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

$$|\mathbf{F}| = |q||\mathbf{v}||\mathbf{B}|\sin\theta$$

In plane containing $\boldsymbol{v}$ and $\mathbf{B}$:



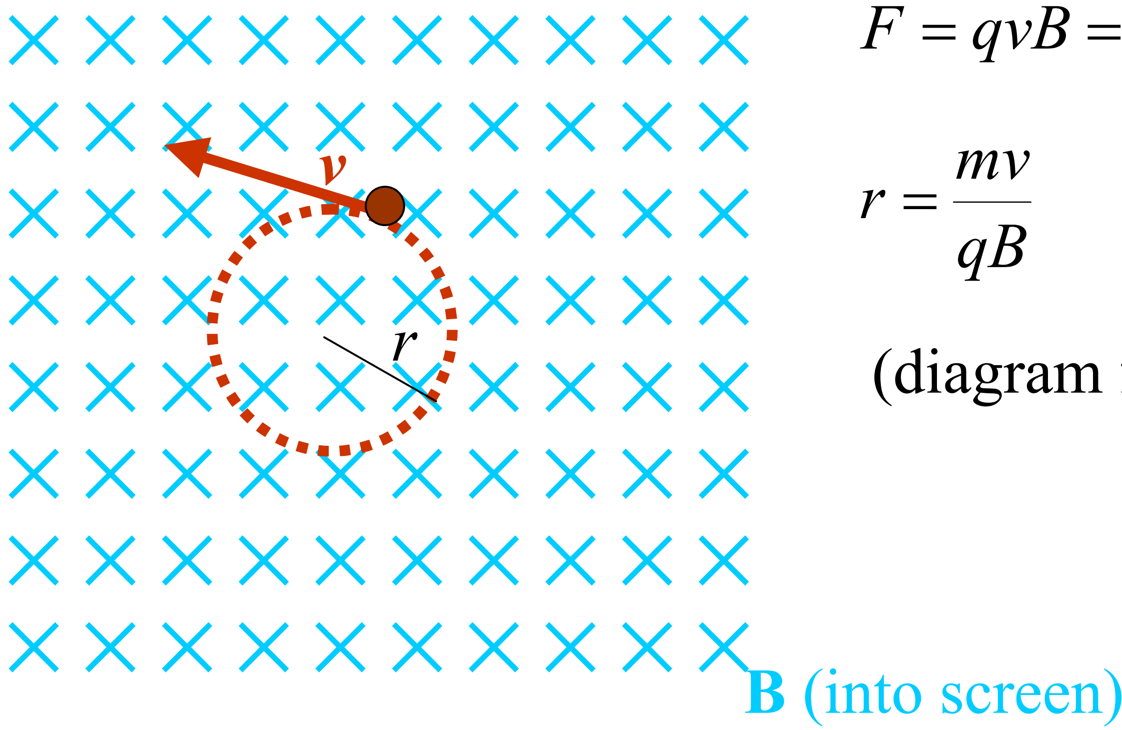
For moving charge:

$$\mathrm{F} = q\,\boldsymbol{v} \times \mathbf{B}$$

For current in wire of length L:

$$\mathrm{F} = L\,\boldsymbol{I} \times \mathbf{B}$$

For point particle of charge q and mass m moving such that there is no component of its velocity vector $\boldsymbol{v}$ of the particle in the direction of the magnetic field, the particle moves in a circular orbit:

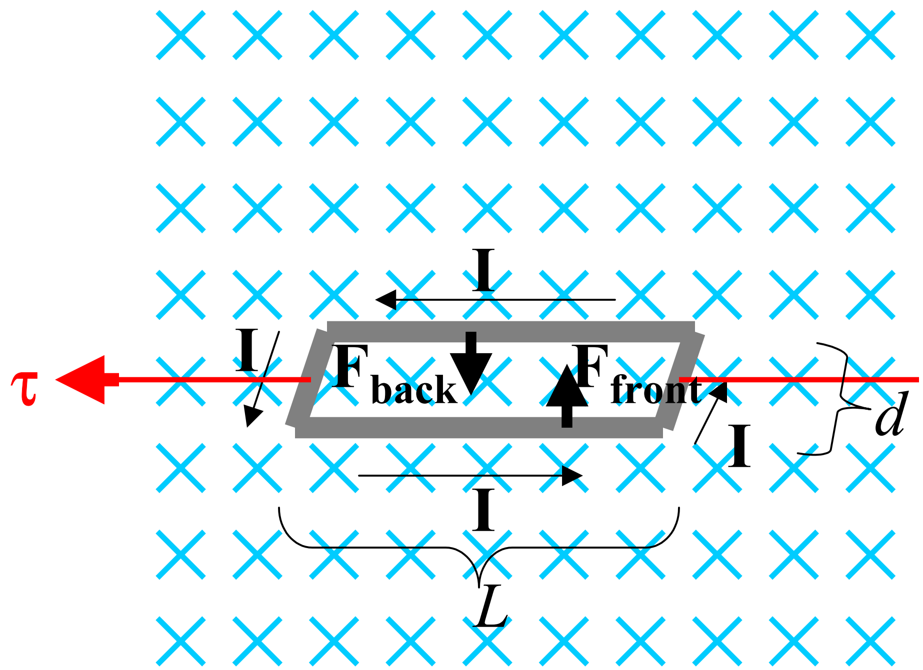


$$F = qvB = ma_r = m\frac{v^2}{r}$$

$$r = \frac{mv}{qB}$$

(diagram for positive charge)

Net forces on a current loop:



B (into screen)

number of loops in coil

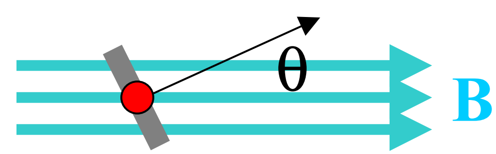
$$\mathbf{F}_{\mathbf{front}} = NL\ \mathbf{IB}\ \textbf{(up)}$$

$$\mathbf{F}_{\mathbf{back}} = NL\ \mathbf{IB}\ \textbf{(down)}$$

Torque on loop:

$$\tau = \underbrace{dNL}_{\mu = ANI}\ IB \sin\theta$$

$$\tau = \mu \times \mathbf{B}$$



New "laws" regarding magnetic fields –

Biot-Savart law: $$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

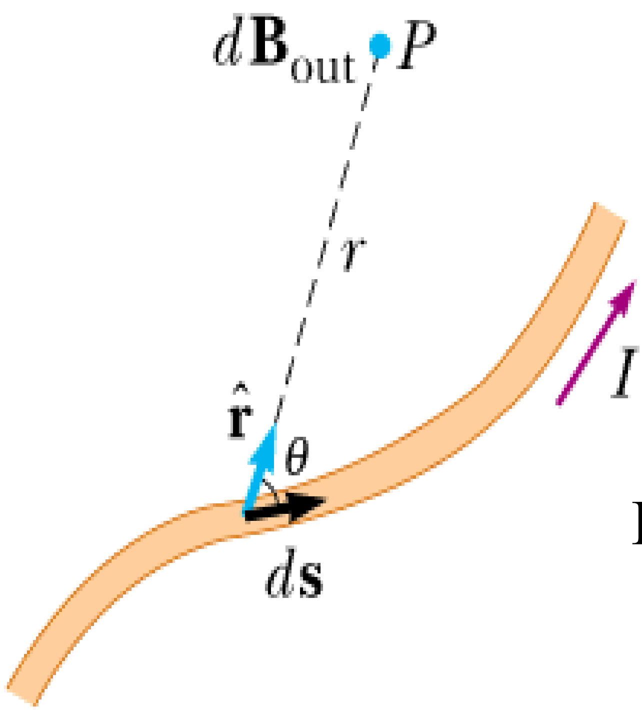
Ampere's law: $$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in}$$

(Ampere's law is equivalent to Biot-Savart law in the same way that Gauss's law is equivalent to Coulomb's law in electrostatics)

Faraday's law: $$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{A}$$

Sources of magnetic field – currents

Biot-Savart law



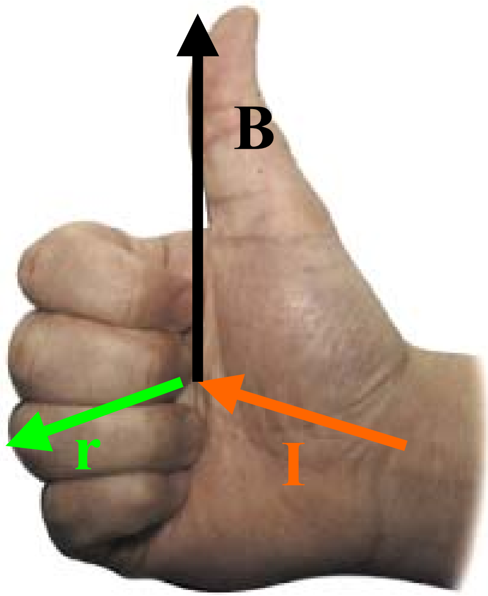
$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

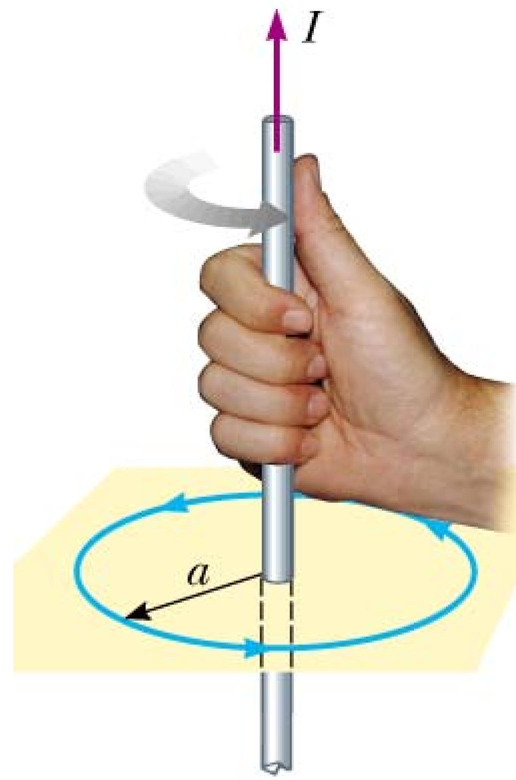
Field from a single moving charge:

$$\mathbf{B} \approx \frac{\mu_0}{4\pi}\frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2}$$

Digression on the right-hand rule:

$\mathbf{I}$ $\times$	$\mathbf{r}$ $\rightarrow$	$\mathbf{B}$
palm	fingers	thumb
fingers	thumb	palm
thumb	palm	fingers



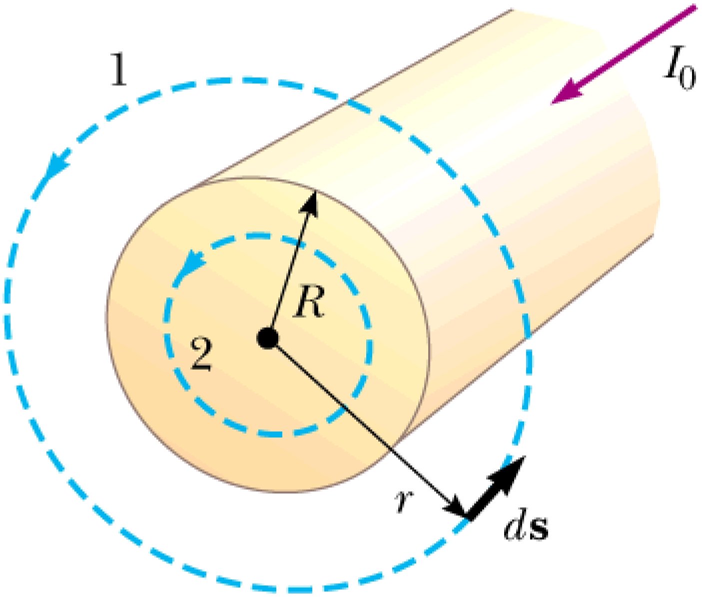


Ampere's law

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in}$$

1. For $r > R$:

$$B(2\pi r) = \mu_0 I_0 \qquad \Rightarrow \quad B = \frac{\mu_0 I_0}{2\pi r}$$



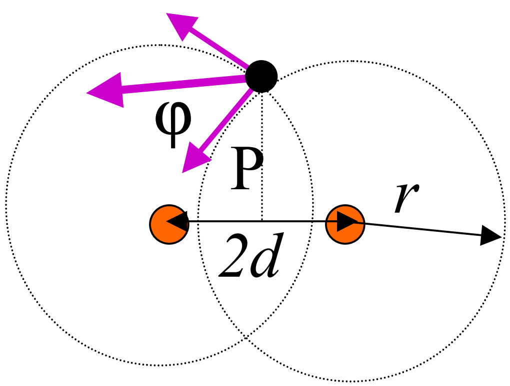
2. For $r < R$:

$$B(2\pi r) = \mu_0 I_0 \frac{r^2}{R^2} \qquad \Rightarrow \quad B = \frac{\mu_0 I_0 r}{2\pi R^2}$$

Example of Ampere's law for two wires:

Suppose that there are two wires perpendicular to the screen both with currents *I* flowing into the screen. What is the magnitude and direction of the magnetic field at the point P?

$$B = \frac{\mu_0 I}{2\pi r} 2\cos\varphi = \frac{2\,\mu_0 I}{2\pi\sqrt{d^2 + P^2}} \frac{P}{\sqrt{d^2 + P^2}}$$



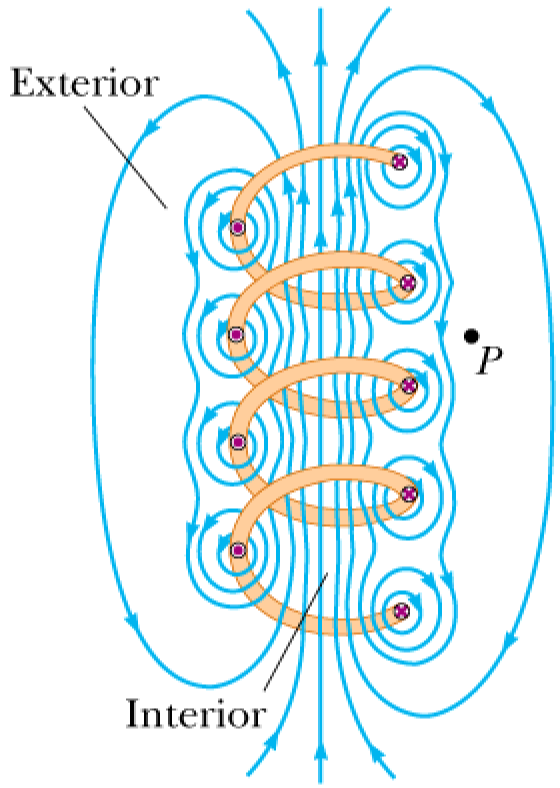
Note: The two wires exert a force on eachother. In this case, the force is attractive and, in terms of the length *l* of the wires, has the magnitude:

$$\frac{F_B}{l} = \frac{\mu_0 I^2}{4\pi d}$$

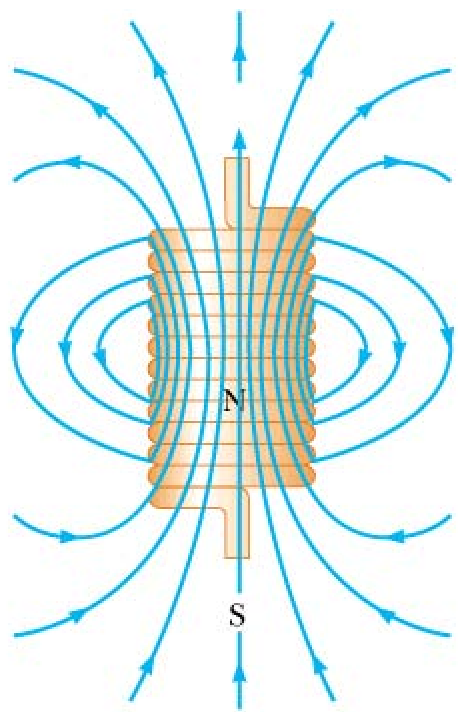
Magnetic field in the solenoid geometry

$$\boxed{B_{interior} = \mu_0\, n\, I}$$

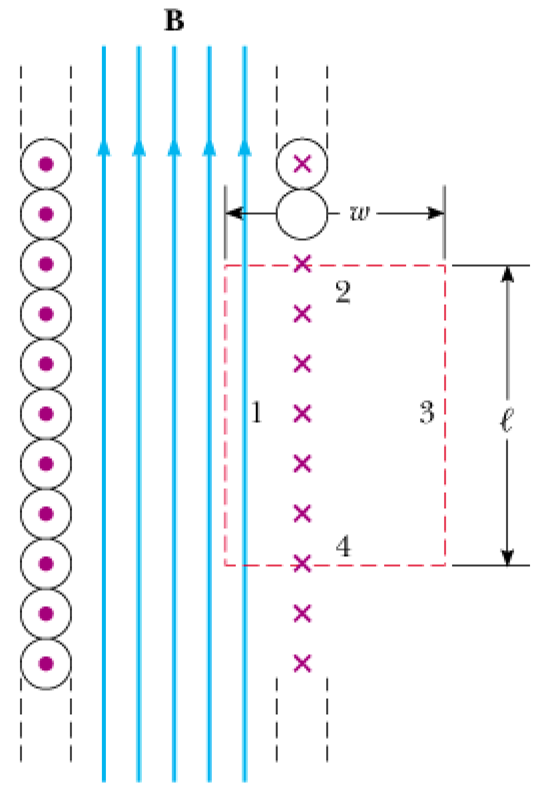
number of coils/unit length



Helical form



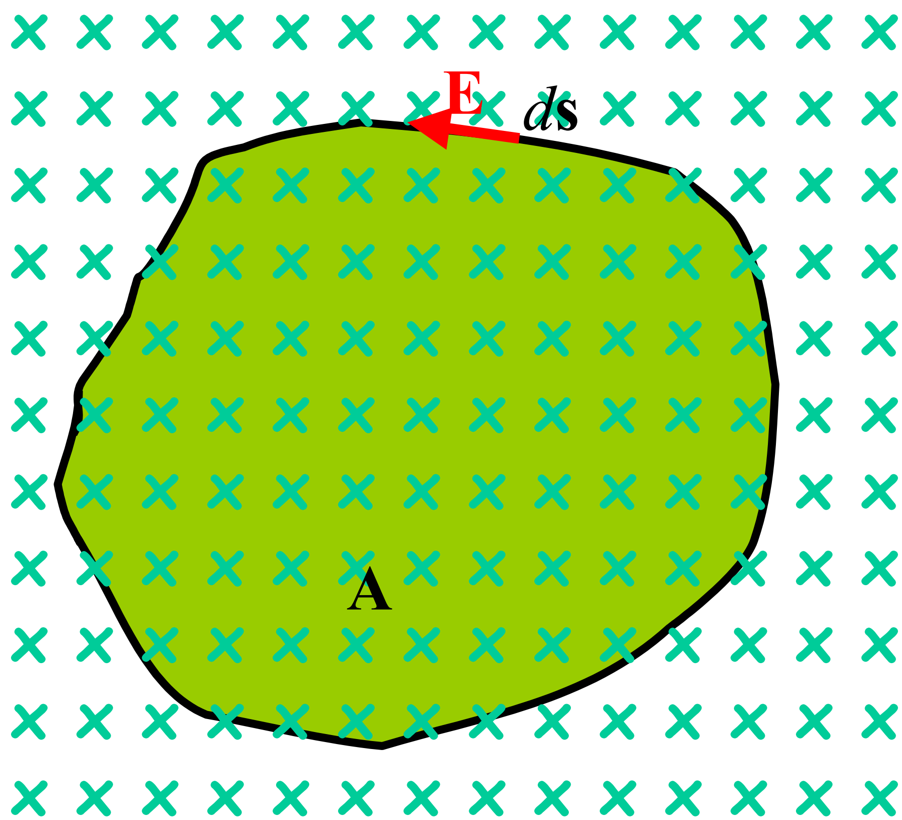
Tight coil form



Ideal form

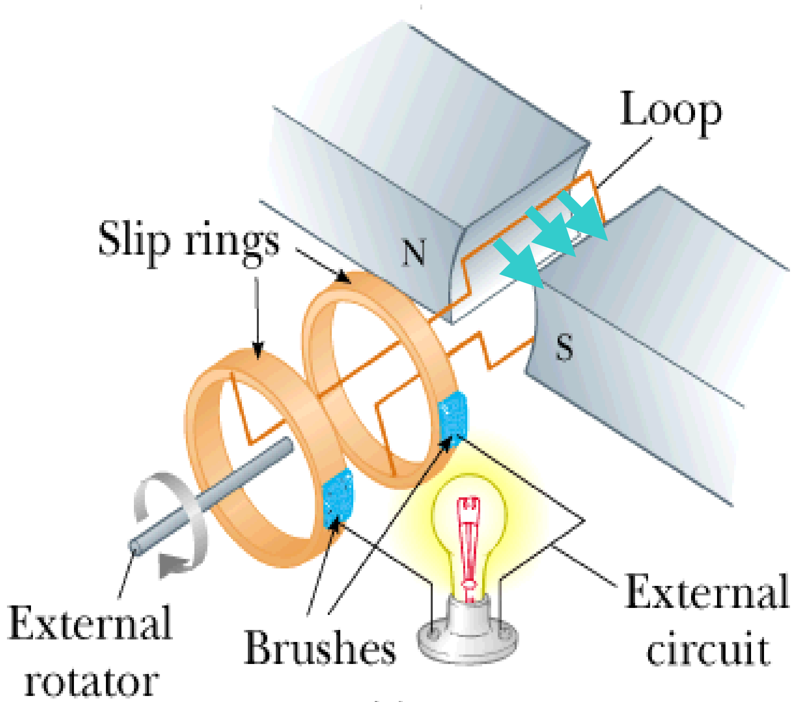
Faraday's law:

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{A}$$

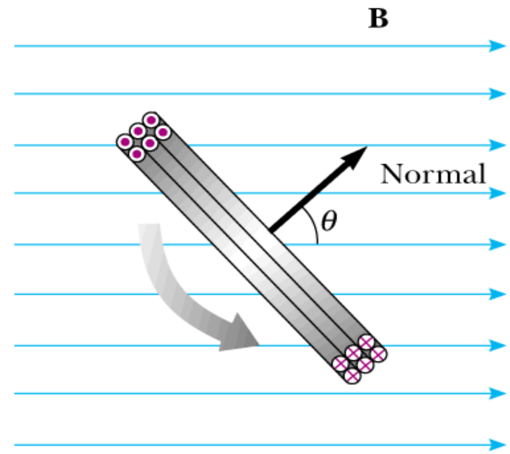


B pointing into screen and changing with time.

Example: AC generator



(a)

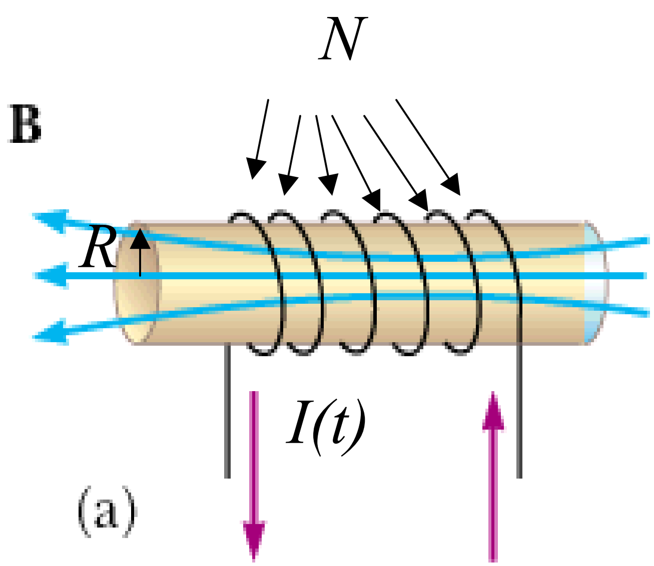


$$\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} = \frac{d}{dt}\left(BA\cos\omega t\right)$$
$$= -\omega BA\sin\omega t$$

$$\Rightarrow \mathcal{E} = \underbrace{\omega BA}_{\mathcal{E}_{\max}}\sin\omega t$$

Inductance as a circuit component:

Recall the solenoid geometry:



Selenoid induces emf in itself ("self inductance").

For each coil of the solenoid:

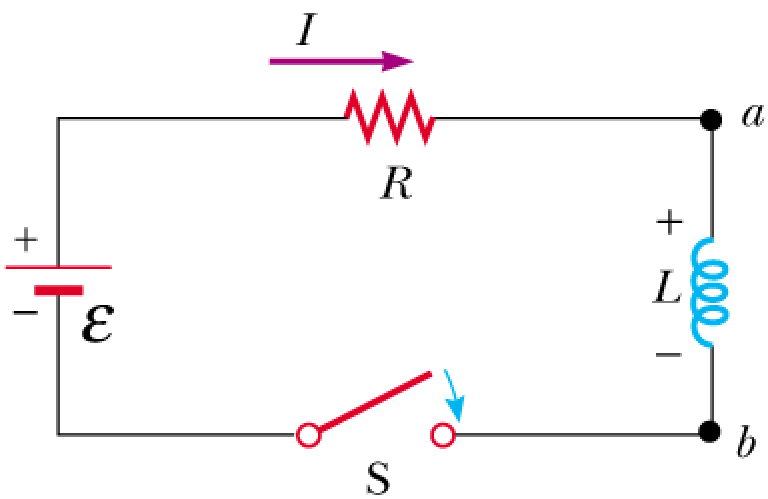
$$\mathcal{E} = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} = -\frac{d}{dt}\left(\mu_0 n I \ \ \pi R^2\right)$$

$$= -\mu_0 n \, \pi R^2 \frac{dI}{dt}$$

$$\mathcal{E}_{\text{total}} = -\underbrace{N\mu_0 n \, \pi R^2}_{L} \frac{dI}{dt}$$

units: 1 Henry = 1 V/(A/s)

Inductors in a circuit:

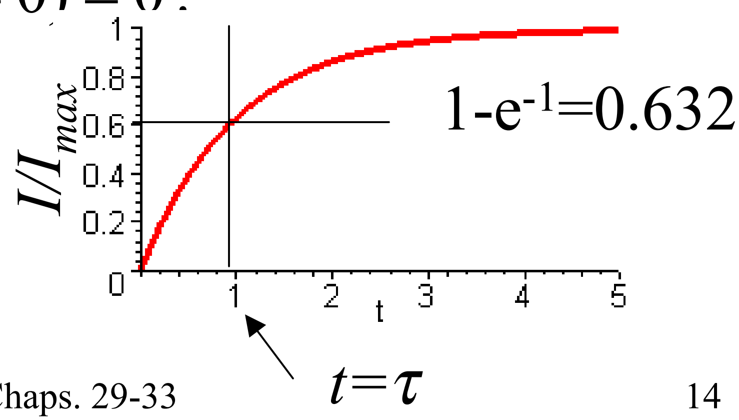


$$\mathcal{E}_{\text{battery}} - IR - L\frac{dI}{dt} = 0$$

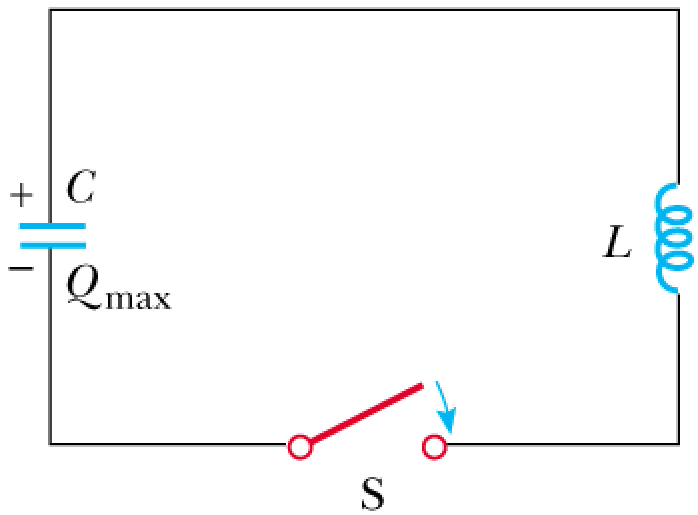
solution for $I(t)$ assuming $I(t=0)=0$:

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/(L/R)}\right)$$

$$\tau = L/R$$



LC – circuits:



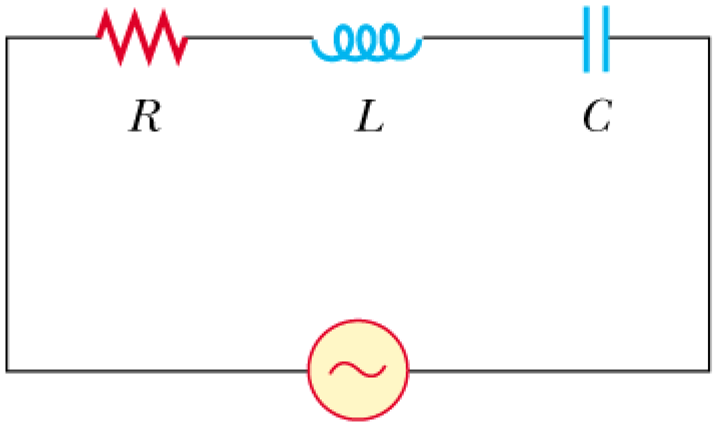
$$-\frac{q}{C}-L\frac{dI}{dt}=0$$

$$\text{or}:\quad -\frac{q}{C}-L\frac{d^2q}{dt^2}=0 \qquad q(t)=Q_0\cos(\omega_0 t+\varphi)\ \text{ with }\ \omega_0\equiv\sqrt{\frac{1}{LC}}$$

$$I(t)=\frac{dq}{dt}=-\omega_0 Q_0\sin(\omega_0 t+\varphi)$$

DC circuits	AC circuits
$\mathcal{E}$ = constant in time	$\mathcal{E} = \mathcal{E}_{max} \sin \omega t$ or $\mathcal{E}_{max} \cos \omega t$ $\mathcal{E}_{rms} = \mathcal{E}_{max}/\sqrt{2}$
$I = \begin{cases} \text{constant in time} \\ \text{constant} + I_0 e^{-t/\tau} \\ \text{damped oscillations} \end{cases}$	I= transients + $I_0 \sin(\omega t - \phi)$
Kirchhoff's rules apply	Kirchhoff's rules apply

Example



Kirchhoff's rule:

$$-RI - L\frac{dI}{dt} - \frac{Q}{C} + \Delta V_{max}\sin\omega t = 0$$

could also be cos ωt

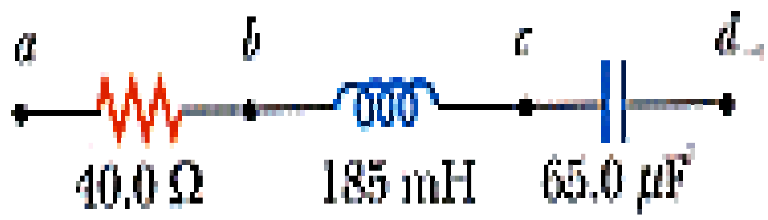
Solution:

$$I(t) = I_{max}\sin(\omega t - \varphi)$$

$$\varphi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \qquad I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Example:

An ac source with Vmax = 170 V and f = 60.0 Hz is connected between points a and d :



What is the maximum voltage between

a and *b*?