Announcements

1. New schedule:
   http://www.wfu.edu/~natalie/s03phy114/homework/
2. Online quiz will resume for Lecture 18
3. Take-home exams will be returned at end of class

   Rework exam for small amount of extra-credit
   (http://www.wfu.edu/~natalie/s03phy114/extrapractice/)
   small corrections – In problem #2 the wires should have been longer in order to use the long wire approximation.

   Presentations? Thursday evening ?
   Friday afternoon ?
   Sunday afternoon ?
4. Today's topic – Chapter 34

   Maxwell's equations and electromagnetic radiation

Maxwell's equations and electromagnetic radiation

- The wave equation – seen last semester for mechanical waves – periodic wave solutions
- Maxwell's equation
  - Coulomb's and Gauss's law
  - Biot-Savart's and Ampere's law
  - Faraday's law
  - Maxwell's contributions
- Properties of electromagnetic waves as a consequence of Maxwell's equations

The mathematical form of the "wave equation":

$$\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$$

velocity of wave

Solution form for a periodic solution:

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right) \equiv A\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t + \varphi\right)$$

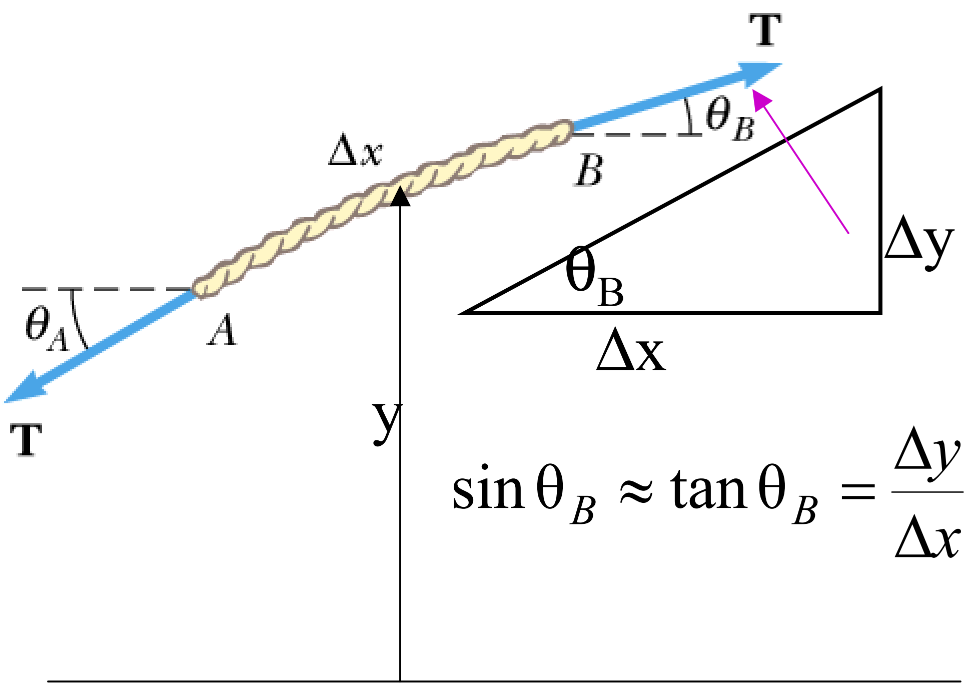
phase

amplitude

$$v = \lambda f$$

Example of mechanical wave motion:

Transverse wave on a string with tension T and mass per unit length μ:



$$\sin\theta_B \approx \tan\theta_B = \left.\frac{\Delta y}{\Delta x}\right|_B$$

$$m\frac{d^2y}{dt^2} = T\sin\theta_B - T\sin\theta_A$$

$$m \approx \mu\Delta x$$

$$\Rightarrow \mu\Delta x\frac{d^2y}{dt^2} \approx T\left(\left.\frac{\Delta y}{\Delta x}\right|_B - \left.\frac{\Delta y}{\Delta x}\right|_A\right)$$

$$\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left(\left.\frac{\Delta y}{\Delta x}\right|_B - \left.\frac{\Delta y}{\Delta x}\right|_A\right) = \frac{\partial^2 y}{\partial x^2}$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2} \qquad \Rightarrow v = \sqrt{\frac{T}{\mu}}}$$

Comparison of mechanical and electromagnetic waves

Mechanical	Electromagnetic
Satisfy wave equation	Satisfy wave equation
v depends upon propagation material.	v depends upon propagation material (or vacuum).
Can be transverse or longitudinal	Can only be transverse
Can only propagate within materials (solids, liquids, gases, strings, etc.)	Can propagate within a vacuum and within some materials.
Doppler effect: $f' = f\frac{1 \pm u_O/v}{1 \mp u_S/v}$	Doppler effect: $f' = f\sqrt{\frac{1+u/v}{1-u/v}}$

Electromagnetic field equations:

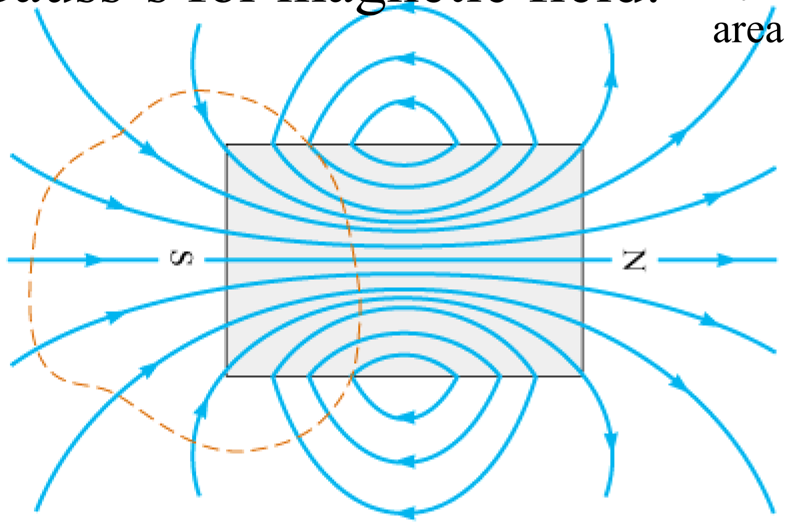
Coulomb-Gauss law: $\oint_{\text{area}} \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$

Biot-Savart-Ampere law: $\oint_{\text{line}} \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$

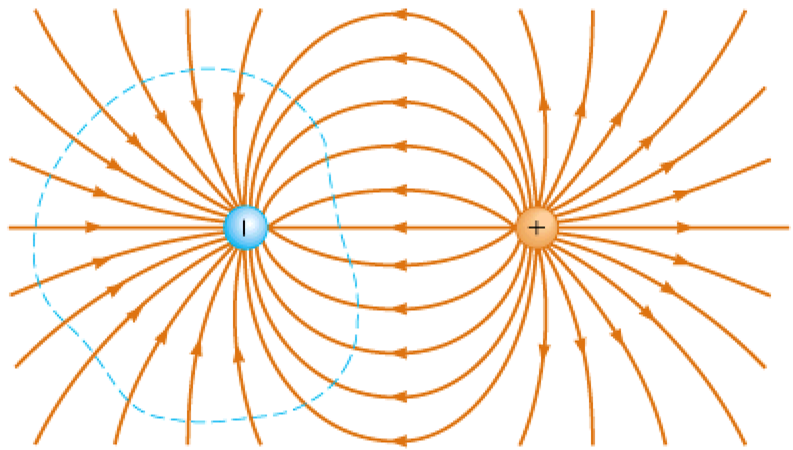
Faraday's law: $\oint_{\text{line}} \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$

Additional law:

Gauss's for magnetic field: $\oint_{\text{area}} \mathbf{B} \cdot d\mathbf{A} = 0$



Recall Gauss's law for electric field: $\oint_{\text{area}} \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$



Electromagnetic field equations:

Coulomb-Gauss law: $\oint_{\text{area}} \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$

Gauss's for magnetic field: $\oint_{\text{area}} \mathbf{B} \cdot d\mathbf{A} = 0$

Biot-Savart-Ampere law: $\oint_{\text{line}} \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$

Faraday's law: $\oint_{\text{line}} \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$

Maxwell's re-analysis of Gauss's and Ampere's laws:

$$\oint_{\text{area}} \mathbf{E}\cdot d\mathbf{A} = \frac{Q}{\varepsilon_0} \qquad\qquad \oint_{\text{line}} \mathbf{B}\cdot d\mathbf{s} = \mu_0 I$$

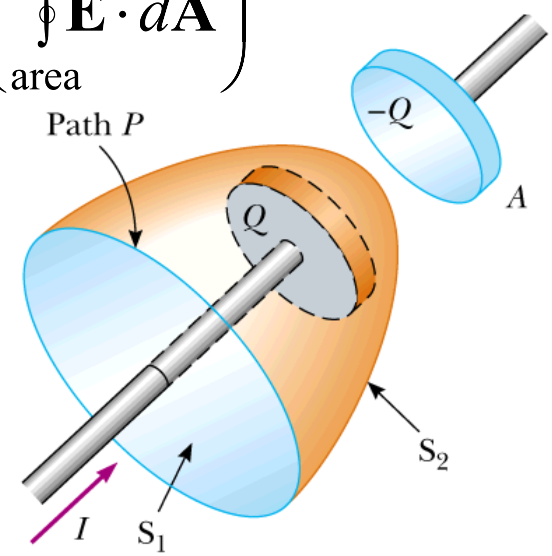
Note that $I_Q = \dfrac{dQ}{dt}$

$$\oint_{\text{area}} \mathbf{E}\cdot d\mathbf{A} = \frac{Q}{\varepsilon_0} \quad \Rightarrow \quad \frac{d}{dt}\left(\oint_{\text{area}} \mathbf{E}\cdot d\mathbf{A}\right) = \frac{1}{\varepsilon_0}\frac{dQ}{dt} = \frac{I_Q}{\varepsilon_0}$$

Maxwell-Ampere law:

$$\oint_{\text{line}} \mathbf{B}\cdot d\mathbf{s} = \mu_0\left(I + I_Q\right) = \mu_0 I + \mu_0\varepsilon_0 \frac{d}{dt}\left(\oint_{\text{area}} \mathbf{E}\cdot d\mathbf{A}\right)$$

$$\equiv \mu_0 I + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$



Maxwell's equations:

Coulomb-Gauss law: $\oint_{\text{area}} \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$

Gauss's for magnetic field: $\oint_{\text{area}} \mathbf{B} \cdot d\mathbf{A} = 0$

Biot-Savart-Ampere-Maxwell law: $\oint_{\text{line}} \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \boxed{\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}$

Faraday's law: $\oint_{\text{line}} \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$

Maxwell's equations in absence of sources (*Q=0, I=0*):

Coulomb-Gauss law: $\oint_{\text{area}} \mathbf{E} \cdot d\mathbf{A} = 0$

Gauss's for magnetic field: $\oint_{\text{area}} \mathbf{B} \cdot d\mathbf{A} = 0$

Biot-Savart-Ampere-Maxwell law: $\oint_{\text{line}} \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

Faraday's law: $\oint_{\text{line}} \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$

Peer instruction question

How can we have electric and/or magnetic fields without sources? Which of the following statements is not true?

(A) Charges and/or currents are necessary to create electric and magnetic fields.

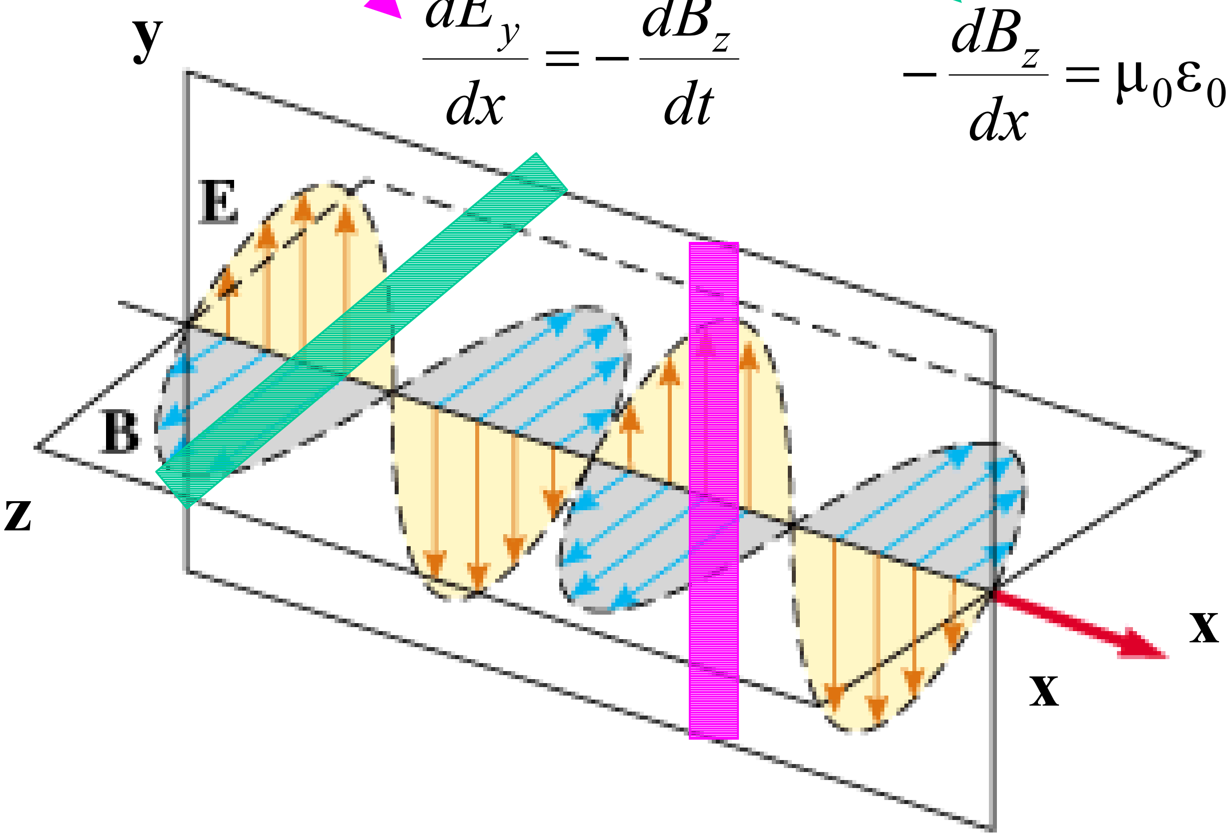
(B) Electric and magnetic fields can exist if there are no sources present.

(C) Statements (A) and (B) are both false.

“Plane wave” Maxwell’s equations:

$$\oint_{\text{line}} \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \qquad \oint_{\text{line}} \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\frac{dE_y}{dx} = -\frac{dB_z}{dt} \qquad -\frac{dB_z}{dx} = \mu_0 \varepsilon_0 \frac{dE_y}{dt}$$



Results:

$$\frac{dE_y}{dx} = -\frac{dB_z}{dt} \qquad -\frac{dB_z}{dx} = \mu_0\varepsilon_0 \frac{dE_y}{dt}$$

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\mu_0\varepsilon_0}\frac{\partial^2 E_y}{\partial x^2} \qquad \frac{\partial^2 B_z}{\partial t^2} = \frac{1}{\mu_0\varepsilon_0}\frac{\partial^2 B_z}{\partial x^2}$$

$$v \equiv c = \sqrt{\frac{1}{\mu_0\varepsilon_0}} = 2.99792458 \times 10^8 \, m/s$$

Periodic solution: $E_y(x,t) = E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$

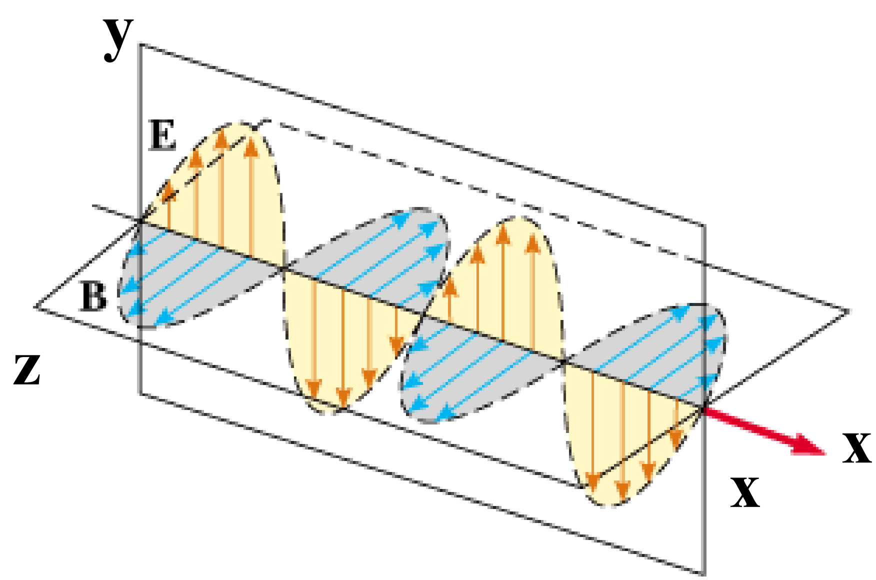
$$B_z(x,t) = \frac{E_{\max}}{v} \sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$$

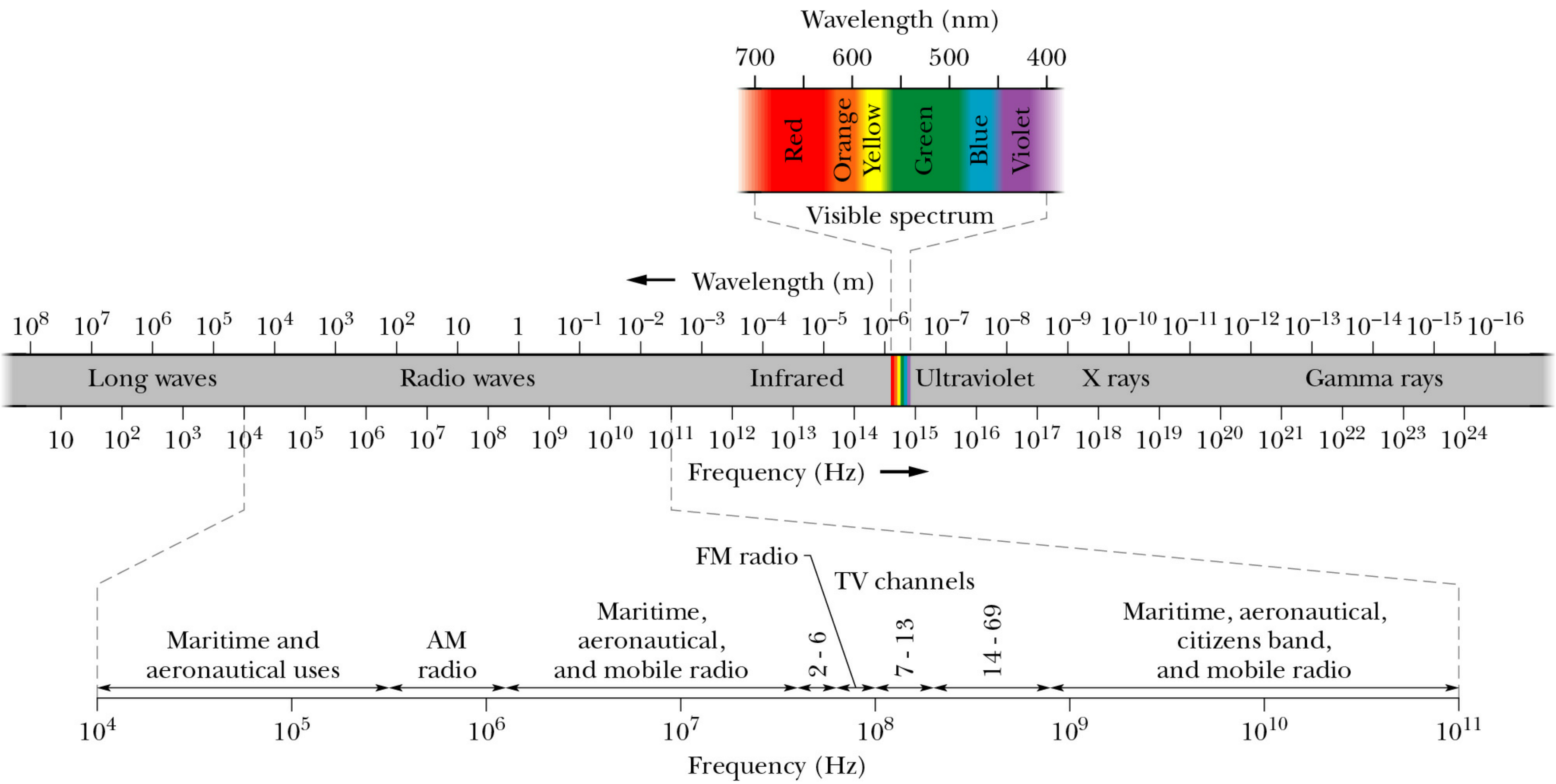
Summary of results for linearly polarized electromagnetic plane waves:

$$E_y(x,t) = E_{\max}\sin\left(\frac{2\pi}{\lambda}(x-ct)\right) = E_{\max}\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right) = E_{\max}\sin(kx-\omega t)$$

$$B_z(x,t) = \frac{E_{\max}}{c}\sin\left(\frac{2\pi}{\lambda}(x-ct)\right) = \frac{E_{\max}}{c}\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right) = \frac{E_{\max}}{c}\sin(kx-\omega t)$$

$$\lambda f = \frac{\omega}{k} = c = \sqrt{\frac{1}{\mu_0\varepsilon_0}} = 2.99792458\times 10^8\, m/s$$





Wavelength (nm)
700
600
500
400
Red
Orange
Yellow
Green
Blue
Violet
Visible spectrum
Wavelength (m)
10^8 10^7 10^6 10^5 10^4 10^3 10^2 10 1 10^−1 10^−2 10^−3 10^−4 10^−5 10^−6 10^−7 10^−8 10^−9 10^−10 10^−11 10^−12 10^−13 10^−14 10^−15 10^−16
Long waves
Radio waves
Infrared
Ultraviolet
X rays
Gamma rays
10 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17 10^18 10^19 10^20 10^21 10^22 10^23 10^24
Frequency (Hz)
FM radio
TV channels
Maritime and aeronautical uses
AM radio
Maritime, aeronautical, and mobile radio
2 - 6
7 - 13
14 - 69
Maritime, aeronautical, citizens band, and mobile radio
10^4 10^5 10^6 10^7 10^8 10^9 10^10 10^11
Frequency (Hz)