Announcements

1. Presentation schedule
   Thursday, 3/20/03 @ 5 PM
   Friday, 3/21/03 @ 3 PM
   Sunday, 3/23/03 @ 1:30 PM

2. Exam 2 revisions – due Monday ≤ 3/24/03

3. Anonymous questionnaires

4. Today's topic – Maxwell's equations and electromagnetic waves – Chap. 34 continued

Maxwell's equations:

Coulomb-Gauss law: $$\oint_{\text{area}} \mathbf{E}\cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$$

Gauss's for magnetic field: $$\oint_{\text{area}} \mathbf{B}\cdot d\mathbf{A} = 0$$

Biot-Savart-Ampere-Maxwell law: $$\oint_{\text{line}} \mathbf{B}\cdot d\mathbf{s} = \mu_0 I + \mu_0\varepsilon_0 \frac{d\left(\int \mathbf{E}\cdot d\mathbf{A}\right)}{dt}$$

Faraday's law: $$\oint_{\text{line}} \mathbf{E}\cdot d\mathbf{s} = -\frac{d\left(\int \mathbf{B}\cdot d\mathbf{A}\right)}{dt}$$

Maxwell's equations far away from sources (*Q=0, I=0*):

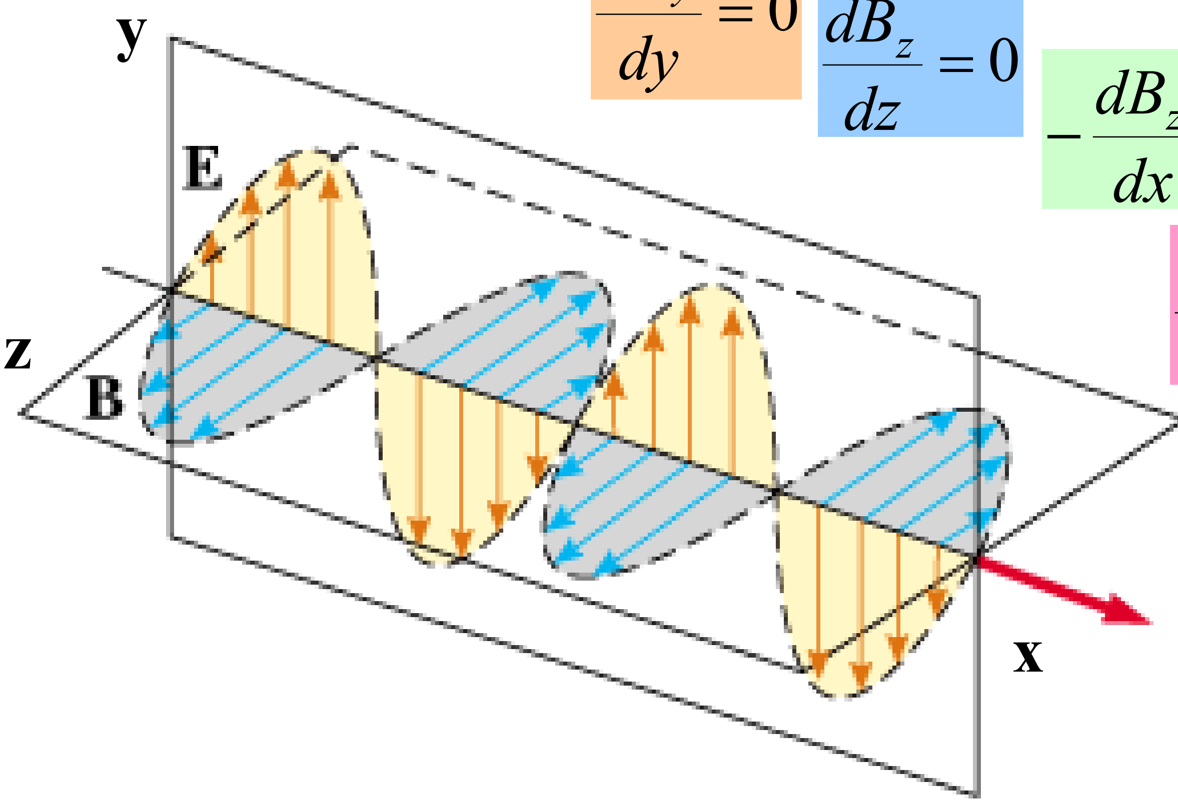
$$\oint_{\text{area}} \mathbf{E}\cdot d\mathbf{A} = 0 \qquad \oint_{\text{area}} \mathbf{B}\cdot d\mathbf{A} = 0 \qquad \oint_{\text{line}} \mathbf{B}\cdot d\mathbf{s} = \mu_0\varepsilon_0 \frac{d\left(\int \mathbf{E}\cdot d\mathbf{A}\right)}{dt} \qquad \oint_{\text{line}} \mathbf{E}\cdot d\mathbf{s} = -\frac{d\left(\int \mathbf{B}\cdot d\mathbf{A}\right)}{dt}$$

Plane-polarized solution:

$$\frac{dE_y}{dy} = 0 \qquad \frac{dB_z}{dz} = 0$$

$$-\frac{dB_z}{dx} = \mu_0\varepsilon_0 \frac{dE_y}{dt}$$

$$\frac{dE_y}{dx} = -\frac{dB_z}{dt}$$



Results:

$$-\frac{dB_z}{dx} = \mu_0 \varepsilon_0 \frac{dE_y}{dt}$$

$$\frac{dE_y}{dx} = -\frac{dB_z}{dt}$$

Wave equations for $E_y$ and $B_z$:

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 E_y}{\partial x^2}$$

$$\frac{\partial^2 B_z}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 B_z}{\partial x^2}$$

$$\Rightarrow v \equiv c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = 2.99792458 \times 10^8 \, m/s$$

Periodic solution:

$$E_y(x,t) = E_{\max} \sin\left( \frac{2\pi}{\lambda}(x - vt) + \varphi \right)$$

$$B_z(x,t) = \frac{E_{\max}}{v} \sin\left( \frac{2\pi}{\lambda}(x - vt) + \varphi \right)$$

Summary of significant properties of electromagnetic waves:

- "Self-sustaining" electric and magnetic fields which can propagate in vacuum at a velocity of $c = 2.99792458 \times 10^8$ m/s (or within matter at a velocity of $v=c/n$) .

- **E** and **B** fields are perpendicular to each other and perpendicular to propagation direction (transverse waves).

- $|\mathbf{E}(x,t)| = \dfrac{|\mathbf{B}(x,t)|}{v}$

- Periodic waves have the form:

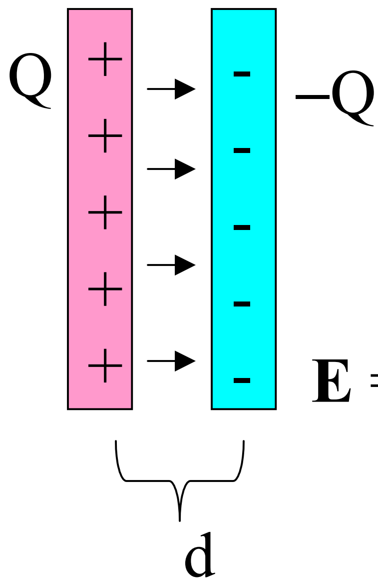
$$E_y(x,t) = E_{\max}\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t + \varphi\right)$$

← polarization direction (**y**)

$$B_z(x,t) = \frac{E_{\max}}{v}\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t + \varphi\right)$$

Energy ~~and forces~~ associated with electromagnetic waves:

Energy associated with electric field




$U = Q^2/(2C) = \frac{1}{2}\,\varepsilon_0\, E^2/(Ad)$

volume

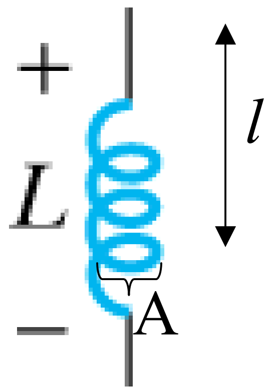
$\mathbf{E} = Q/(A\varepsilon_0)$

Energy density associated with an electric field:

$u_E = U/(\text{volume}) = \frac{1}{2}\,\varepsilon_0\, E^2$

## Energy and forces associated with electromagnetic waves:

### Energy associated with magnetic field




$U = \frac{1}{2}LI^2$

number of turns in coil

$L = \mu_0 N^2 A l$

$I = B/(\mu_0 N/l)$

$\rightarrow U = \frac{1}{2}\,(Al)\, B^2/\,\mu_0$

volume

Energy density associated with magnetic field:

$u_B = U/(\text{volume}) = \frac{1}{2}\, B^2/\mu_0$

Energy and forces associated with electromagnetic waves:

Energy density of electromagnetic wave

$$u \equiv u_E + u_B = \tfrac{1}{2}\varepsilon_0 E^2 + \tfrac{1}{2} B^2/\mu_0$$

$$= \tfrac{1}{2}\left(\varepsilon_0 E_{\max}^2 + B_{\max}^2/\mu_0\right)\sin^2\left(\frac{2\pi x}{\lambda} - 2\pi f t + \varphi\right)$$

Time averaged energy density:

(noting that $B_{\max} = E_{\max}/c = \sqrt{\varepsilon_0\mu_0}E_{\max}$ )

$$\langle u \rangle_{avg} = \tfrac{1}{2}\varepsilon_0 E_{\max}^2 = \tfrac{1}{2}B_{\max}^2/\mu_0$$

units: joules/m$^3$

## Peer instruction question

Suppose an electromagnetic field has an electric field amplitude of $E_{max}$ = 40 N/C, what is the average energy density associated with this radiation (in units of Joules/$m^3$)?

(A) $8x10^{-26}$ (B) $7x10^{-9}$ (C) 800 (D) None of these

Energy and forces associated with electromagnetic waves:

"Flow" of energy -- Poynting vector

$$\mathbf{S} \equiv \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$$

$$\mathbf{S} = \frac{\hat{\mathbf{x}}}{\mu_0} E_{\max} B_{\max} \sin^2\left(\frac{2\pi x}{\lambda} - 2\pi f t + \varphi\right)$$

$$\langle \mathbf{S} \rangle_{avg} = \frac{\hat{\mathbf{x}}}{2\mu_0 c} E_{\max}^2 = \frac{\hat{\mathbf{x}} c}{2\mu_0} B_{\max}^2$$

units: (Joules/s)/m$^2$

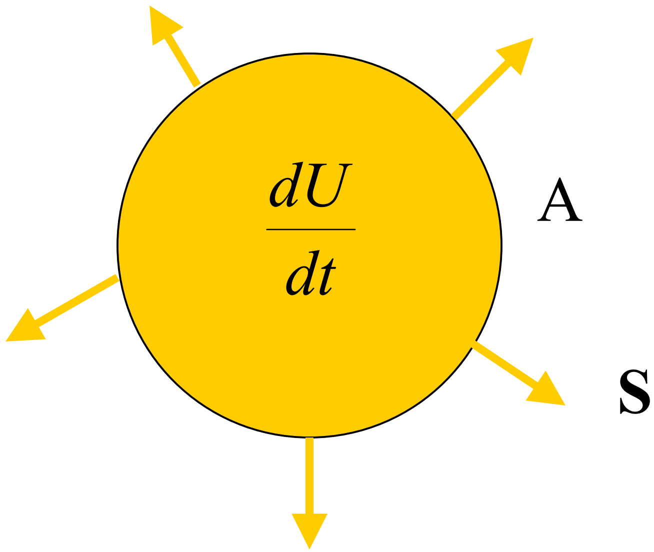
=Watts/m$^2$

Recalling that :

$$\langle u \rangle_{avg} = \tfrac{1}{2}\varepsilon_0 E_{\max}^2 = \tfrac{1}{2} B_{\max}^2 / \mu_0$$

$$\Rightarrow \langle u \rangle_{avg} = \langle |\mathbf{S}| \rangle_{avg} / c$$

Note that : $\dfrac{dU}{dt} = SA$



Momentum carried by an electromagnetic wave

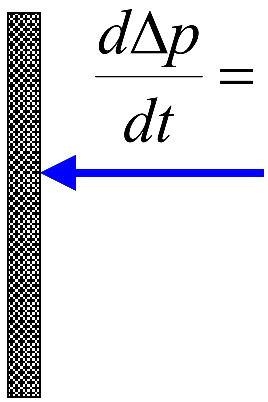
$$\mathbf{p} = \hat{\mathbf{S}} \frac{U}{c}$$

Force associated with electromagnetic wave

$$\frac{d\mathbf{p}}{dt} = \hat{\mathbf{S}} \frac{1}{c} \frac{dU}{dt}$$

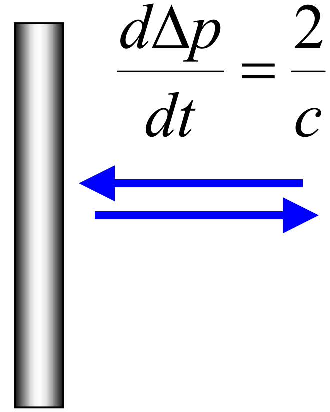
Absorbing:

$$\frac{d\Delta p}{dt} = \frac{1}{c} \frac{dU}{dt}$$



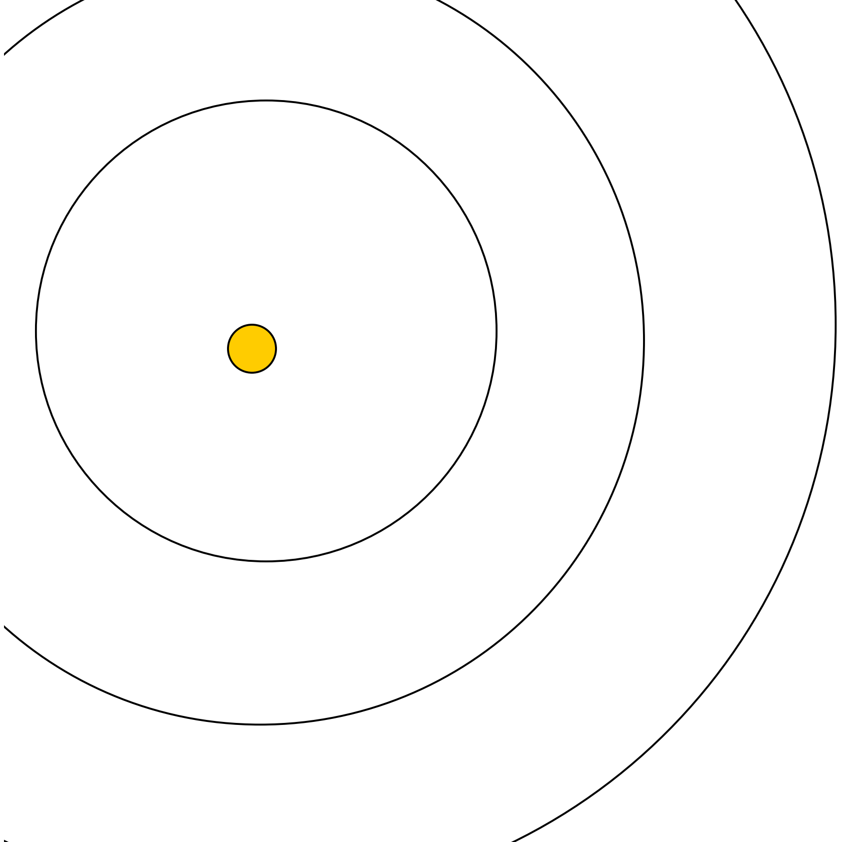
Reflecting::

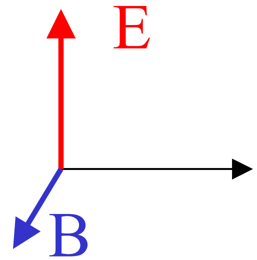
$$\frac{d\Delta p}{dt} = \frac{2}{c} \frac{dU}{dt}$$



Examples of plane-polarized electromagnetic waves:

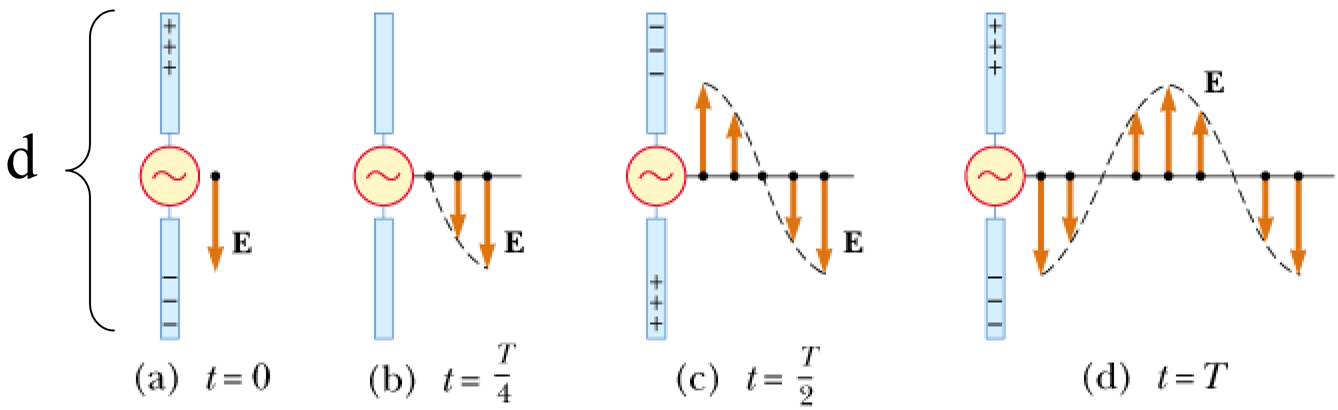
Far from their source, most electromagnetic waves approximate a superposition of plane-polarized electromagnetic waves.






Laser?

Generation of electromagnetic radiation with an antenna:




polarization direction

$f = 1/T$

(large signal when d=nλ/2)